ELEMENTS OF ACOUSTICS

SAMUEL TEMKIN

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TO MY SONS

DAVID AND MICHAEL

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PREFACE

This book is an outgrowth of a course in acoustics I have taught for a number of years at Rutgers University. The main reason for adding one more book to an already long list of books on this subject is the lack of modern introductory texts that treat acoustics as a branch of fluid mechanics. In my view, this is the most natural approach, at least for those areas of acoustics dealing with the most common media for sound propagation, namely, air and water. This approach is, of course, not new. It was used by the authors of many of the books now considered classical, including Rayleigh, Lamb, and others. In recent times, however, many of the acoustics texts that have appeared treat the subject as a branch of electrical engineering. There are indeed many instances in which acoustic oscillations are analogous to some phenomena discussed in electrical engineering courses and the analogies are clearly advantageous to those students whose background is in that discipline. For others, the analogies may be a drawback; to them, both the acoustic equations and their electrical analogues are new.

The main subjects discussed in this book are: propagation in uniform fluids at rest; transmission and reflection phenomena; attenuation and dispersion; and emission. These are only some of the main topics in acoustics. To have attempted to cover all of them would have been presumptuous on my part. Nevertheless, there are several topics that, by some, may be considered basic enough to warrant their inclusion in a text of this nature, but that have been omitted. These include aerodynamic sound, diffraction, and propagation in nonuniform media. Some of these are mentioned in the text, but all too briefly in relation to their importance. The reasons are that some of these topics are either outside my areas of competence or are too advanced compared to the general level of the book. In any event, most of them are fully treated in one or more specialized books that have appeared recently, so that their detailed discussion in this book is unnecessary. On the other hand, sound absorption is discussed in more detail than is usual in books on acoustics. To a certain extent, this reflects my personal interest in that subject, but it is also intended to qualify the strongly held notion that dissipation effects in sound waves are unimportant.

The material given here is intended primarily for a beginning graduate course in acoustics, but includes portions suitable for more advanced courses. In writing this book, I have assumed that the student's background includes the usual preparation in undergraduate physics and mathematics, as well as a course in advanced calculus and a course in basic thermodynamics. Prior acquaintance with fluid mechanics is desirable, but not required. The required material on that subject is developed in Chapter 1. Chapter 1 also includes a summary of basic thermodynamics. To make the book self-sufficient, both of these subjects are developed to a greater degree than is needed in an introductory course.

The book contains more material than is possible to cover in one semester. By deleting some of the more advanced material, it can be used in a one-semester course in basic acoustics for students in engineering or in the physical sciences. On the other hand, with some additional material, it may be used in a one-year sequence covering both basics and applications.

Because of the basic nature of the subject of this book, I have attempted to derive each result from basic principles. However, the emphasis throughout is on the physical meaning of the results, and not on the mathematical techniques that were used to derive them. On the other hand, in some of the derivations I have included more detail than customary, since all too often the student's main effort is spent in trying to fill in the mathematical steps missing between main results. Of course, this has some pedagogical value but, more often than not, it merely improves the ability of the student to manipulate equations. In my view, a better way of learning is by doing. To this end, a number of problems have been included in the text.

Each chapter contains a brief list of suggested references. A more complete list is given in the Bibliography at the end of the book. The lists are not exhaustive; their purpose is merely to direct the interested student to other general sources, or to recent articles touching on some of the material discussed in the text.

Although I have included the results of some of my own investigations, the bulk of the material presented may be considered classical. It is therefore difficult to acknowledge the sources of many of the results that are presented. I have, however, profited much from Chapter 8 of *Fluid Mechanics* by L. D. Landau and E. M. Lifshitz and from Chapters 1–3 of *An Introduction to Fluid Dynamics* by G. K. Batchelor. Other books that have influenced this work are *The Theory of Sound* by Lord Rayleigh, *Theoretical Acoustics* by P. Morse and U. Ingard, *Fundamentals of Acoustics* by L. E. Kinsler and A. R. Frey, and *The Foundations of Acoustics* by E. Skudrzyk.

A major portion of this book was written during 1974–1975 while I was on leave at the Technion-Israel Institute of Technology. I wish to thank Rutgers University and The Lady Davis Fellowship Trust for making this leave possible. I also owe much to the faculty of the Department of Mechanical Engineering at the Technion for their kind hospitality.

I would like to express my gratitude to Professor R. A. Dobbins of Brown University, who introduced me to the subject of this book; to my colleagues at Rutgers University for their continued encouragement; to many of my students for their valuable comments and observations; and to Mrs. Rosemarie Boysen, who typed an earlier version of this book. The final manuscript was typed by Mrs. Erma Sutton, to whom I am also indebted for improving the clarity of many passages.

To conclude, I wish to express my gratitude to my wife Judy and to my sons David and Michael, who patiently endured the writing of this book.

S. TEMKIN

CONTENTS

CHAPTER ONE

CHAPTER TWO

CHAPTER FIVE

CHAPTER SIX

APPENDIXES

CHAPTER ONE

BASIC FLUID MECHANICS AND THERMODYNAMICS

Acoustics is the science that studies the emission, transmission, and reception of sound waves. It touches on disciplines as disparate as psychology and meteorology, and includes many subdisciplines such as architectural acoustics, bio-acoustics, environmental acoustics, and musical acoustics.

This book deals only with some of the physical properties of sound waves in fluids. This topic, although limited in scope, covers one of the most important applications of acoustics, namely, the study of sound waves in air, and provides the basis for other branches of acoustics.

Most of the properties of acoustic waves in fluids may be obtained by means of the wave equation, and this can be derived from approximate conservation principles without having to resort to the far more complicated equations that describe general fluid motions. However, in doing so, one is forced to ignore, from the beginning, effects that may sometimes be important, such as dissipation and nonlinear distortion. To be sure, these effects could be included at a later stage when the more common aspects of acoustics have been studied, but this procedure is more useful when one is aware of the degree of approximation used in the initial description of the waves. This awareness can best be achieved by first deriving the general equations of fluid mechanics, an approach that has the additional advantage of introducing the concepts and symbols needed to describe acoustic fields at a relatively slower pace. We will therefore take this more complete approach, and begin by presenting a short derivation of those equations. More detailed derivations can be found in textbooks dealing with that subject, such as those listed at the end of the chapter.

1.1 INDICIAL NOTATION

We will often be interested in describing some physical property at a given point in space. The coordinates of this point will be denoted by the components x_1, x_2, and x_3, with respect to a cartesian system of coordinates, of a position vector $\mathbf{x}$. Thus, if $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ are unit vectors along these coordinate axes, then

$$\mathbf{x}=\mathbf{e}_1x_1+\mathbf{e}_2x_2+\mathbf{e}_3x_3 \tag{1.1.1}$$

The magnitude of the position vector will be denoted by r, where

$$r=|\mathbf{x}|=\sqrt{x_1^2+x_2^2+x_3^2} \tag{1.1.2}$$

The unit vectors $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$ are orthonormal; that is, they are mutually orthogonal and have unit length. They therefore satisfy the following conditions:

$$\mathbf{e}_1\cdot\mathbf{e}_2=\mathbf{e}_1\cdot\mathbf{e}_3=\mathbf{e}_2\cdot\mathbf{e}_3=0$$

$$\mathbf{e}_1\cdot\mathbf{e}_1=\mathbf{e}_2\cdot\mathbf{e}_2=\mathbf{e}_3\cdot\mathbf{e}_3=1$$

These relationships can be written more succinctly by using the so-called indicial notation. In this notation, any component of a vector $\mathbf{A}=(A_1, A_2, A_3)$, for example, may be represented by the symbol A_i, where the index i runs through the values 1 to 3. Therefore, $\mathbf{A}$ may be expressed as

$$\mathbf{A}=\sum_{i=1}^{3}\mathbf{e}_iA_i \tag{1.1.3}$$

This can be simplified even further by adopting the convention that if in an expression where indices are used, an index is repeated twice, a summation over the range of that index is implied. Thus, the scalar product between vectors $\mathbf{A}$ and $\mathbf{B}$,

$$\mathbf{A}\cdot\mathbf{B}=A_1B_1+A_2B_2+A_3B_3 \tag{1.1.4}$$

is simply represented by

$$\mathbf{A}\cdot\mathbf{B}=A_iB_i \tag{1.1.5}$$

Of course, since the result of summing over a repeated index is independent of the symbol used for that index, we could as well have written A_kB_k. Indices that can thus be replaced are called *dummy* indices, and are useful in writing proper indicial expressions.

Using this notation, we can write the six relationships between the unit vectors **e** simply as

$$e_i e_j = \delta_{ij}, \qquad i, j = 1,2,3 \tag{1.1.6}$$

where the quantity δ_{ij}, known as Kronecker's delta, is defined as

$$\delta_{ij} = \begin{cases} 1, & i=j \\ 0, & i \neq j \end{cases} \tag{1.1.7}$$

The quantity δ_{ij} is an example of a type of quantity known as a second-order tensor. To specify such a quantity, one requires $3 \times 3 = 9$ components, which may be arranged in matrix form. Thus, the components of some tensor t_{ij} may be represented by

$$\begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix}$$

The sum of the diagonal elements of this matrix, known as the trace of the matrix, may be obtained by setting $i=j$ in t_{ij}, that is,

$$t_{11} + t_{22} + t_{33} = t_{ii} \tag{1.1.8}$$

The operation of setting one index equal to another in an indicial expression is known as *contraction*.

In some cases, the elements of a matrix that are symmetrically located with respect to the diagonal are equal. In such cases, the matrix (and the tensor it represents) is said to be *symmetric*. Symmetric tensors of second order satisfy the symmetry condition

$$S_{ij} = S_{ji} \tag{1.1.9}$$

On the other hand, a tensor ξ_{ij} for which

$$\xi_{ij} = -\xi_{ji} \tag{1.1.10}$$

is said to be *antisymmetric*.

An arbitrary second-order tensor t_{ij} can be represented in terms of a symmetric part s_{ij} and an antisymmetric part a_{ij} as follows:

$$t_{ij} = s_{ij} + a_{ij} \tag{1.1.11}$$

where

$$s_{ij}=\tfrac{1}{2}(t_{ij}+t_{ji}) \tag{1.1.12}$$

$$a_{ij}=\tfrac{1}{2}(t_{ij}-t_{ji}) \tag{1.1.13}$$

Tensors of higher order may also be needed. For example, a tensor of third order is required to represent the cross product between two vectors. This is the alternating tensor ε_{ijk}, which is equal to zero unless i, j, and k are all different, in which case its value is either $+1$ or -1 depending on whether i, j, and k are in cyclic order. Thus, if $e_{123}=1$, then

$$e_{123}=e_{231}=e_{312}=+1$$

$$e_{213}=e_{132}=e_{321}=-1$$

with the remaining 21 components all being equal to zero. In terms of e_{ijk}, the ith component of the cross product between vectors **A** and **B** can be written as

$$(\mathbf{A}\times\mathbf{B})_i=e_{ijk}A_jB_k \tag{1.1.14}$$

The following relations involving e_{ijk} are often useful:

$$e_{ijk}e_{ilm}=\delta_{jl}\delta_{km}-\delta_{jm}\delta_{kl}$$

$$e_{ijk}e_{ijm}=2\delta_{km} \tag{1.1.15}$$

$$e_{ijk}e_{ijk}=6$$

As an example of the usefulness of these expressions, let us derive the following vector identity:

$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C})=(\mathbf{A}\cdot\mathbf{C})\mathbf{B}-(\mathbf{A}\cdot\mathbf{B})\mathbf{C} \tag{1.1.16}$$

First, denote the left-hand side of this identity by **D**. Then, using the representation given by (1.1.14), the ith component of **D** may be expressed as

$$D_i=e_{ijk}A_j(\mathbf{B}\times\mathbf{C})_k \tag{1.1.17}$$

Again, the kth component of $\mathbf{B}\times\mathbf{C}$ may be written as $e_{kij}B_iC_j$, but if this quantity were to be used in the above equation, it would result in an indicial expression having indices repeated more than twice. Such expressions are not admissible, as they cannot be evaluated properly. To avoid this difficulty, we write, instead, the equivalent expression

$$(\mathbf{B}\times\mathbf{C})_k = e_{klm}B_lC_m \tag{1.1.18}$$

so that

$$D_i = e_{ijk}e_{klm}A_jB_lC_m \tag{1.1.19}$$

Because of its definition, the alternating-symbol tensor is not affected by an even number of permutations of its indices. Therefore, the first of these symbols on the right-hand side of (1.1.19) can be written as e_{kij}, so that using the first of the identities given by (1.1.15) we obtain

$$D_i = (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl})A_jB_lC_m \tag{1.1.20}$$

Remembering the properties of the Kronecker delta, we can write this as

$$D_i = A_jB_iC_j - A_jB_jC_i \tag{1.1.21}$$

This is the indicial equivalent of the identity that was to be derived.

We should notice that each of the terms on the right-hand side of the last expression for D_i above contains a repeated index, and must therefore be summed over it. The other index, i, is not repeated, and is therefore a "free" index. If an indicial expression is properly written, the free indices in each term of the expression must be the same.

Finally, we may also use the indicial notation to simplify expressions involving spatial derivatives of various quantities. Thus, the ith component of the operator ∇ defined by

$$\nabla = \mathbf{e}_1\frac{\partial}{\partial x_1} + \mathbf{e}_2\frac{\partial}{\partial x_2} + \mathbf{e}_3\frac{\partial}{\partial x_3} \tag{1.1.22}$$

is simply denoted by

$$(\nabla)_i = \frac{\partial}{\partial x_i} \tag{1.1.23}$$

Therefore, the ith component of the gradient of a scalar ϕ is

$$(\nabla\phi)_i = \frac{\partial\phi}{\partial x_i} \tag{1.1.24}$$