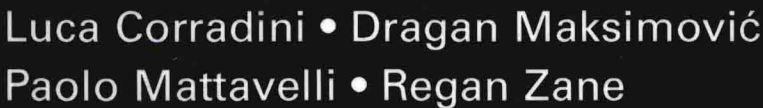


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DIGITAL CONTROL OF HIGH-FREQUENCY SWITCHED-MODE POWER CONVERTERS

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PREFACE

Power electronics fundamentals have been established within the framework of continuous-time analysis, averaged modeling of switched-mode power converters, and analog control theory [1–5]. Ever more often, control and management functions around power converters are implemented digitally, expanding the field of fundamentals to discrete-time modeling and digital control concepts specific to power electronics. Standard textbooks and courses dedicated to digital control of dynamic systems in general provide the necessary background but seldom, if ever, address the specifics necessary to fully understand and successfully practice the design of digitally controlled power converters. We attempt, in this book, to fill the gap by treating the fundamental aspects of digital control of high-frequency switched-mode power converters in a systematic and rigorous manner. Our objectives are to put the reader in the position to understand, analyze, model, design, and implement digital feedback loops around power converters, from system-level transfer function formulations to coding practical designs in one of the mainstream hardware description languages (HDLs) such as VHDL or Verilog.

The book is intended for graduate students of electrical engineering pursuing a curriculum in power electronics and as a reference for engineers and researchers who seek to expand on the expertise in design-oriented knowledge of digital control of power electronics. It is assumed that the reader is well acquainted with foundations of the power electronics discipline, along with associated continuous-time modeling and control techniques [1–6]. Familiarity with sampled-data and discrete-time system analysis topics is helpful but not absolutely essential. Key concepts are developed from the beginning, including a brief review of the necessary discrete-time system fundamentals in Appendix A. For a more comprehensive background, the reader is referred to one of the standard textbooks, such as [7, 8].

The book is composed of eight chapters, structured as follows. The introductory chapter provides an overview of digital control of high-frequency switched-mode power converters, the motivation behind the surge of interest in the area, a summary of analysis, modeling, control, and implementation issues, as well as a summary of recent advances demonstrating potential advantages of digital controllers, including system power management interfaces, programmability of control functions, dynamic response and efficiency improvements, and practical autotuning techniques.

Chapter 1 provides a review of the continuous-time averaged modeling approach for switched-mode power converters. Averaged small-signal modeling is extensively covered by a number of authoritative textbooks [1–3], and the intention is not to replicate this subject in its entirety. Rather, the purpose of Chapter 1 is to focus the reader's attention on the methodology and *assumptions* behind the averaging approach. Understanding of the philosophy and limitations of the

averaging technique is essential to appreciate the need for a different approach when it comes to digitally controlled converters.

Chapter 2 introduces the main elements of a digitally controlled converter, with the purpose of providing the reader with a quick overview of the main differences between analog and digital control without immediately entering into detailed modeling issues. This chapter ends with a discussion about the use of continuous-time averaged modeling for designing digital loops, an approach often employed in practice but which can only account for sampling effects and digital control delays in an approximate manner.

The discussion motivates the formulation of a *discrete-time* modeling approach, developed in Chapter 3, which correctly treats the digitally controlled converter as a sampled-data system and formulates its small-signal dynamics in the z -domain rather than in the Laplace domain. In addition to providing the theoretical framework of discrete-time modeling, a number of modeling examples are discussed in Chapter 3. Furthermore, it is shown that a direct link can be established between continuous-time modeling and discrete-time modeling for the converters that are *topologically invariant*, such as the Buck converter. In such cases, a simple and straightforward discretization rule can be formulated, which translates the converter averaged small-signal model into the exact discrete-time model.

Chapter 4 is devoted to direct digital compensator design, based on the discrete-time models developed in Chapter 3. Among many techniques discussed in the literature, the emphasis is given here to the so-called bilinear transform method. The main advantage of the approach is that the entire design procedure is formulated in an equivalent continuous-time domain, in which both the digitally controlled converter and the compensator under design assume the form of continuous-time systems. As a result, the direct digital design can take advantage of the familiar analog control design techniques with the design specifications formulated in the frequency domain. Standard digital proportional-integral (PI) and proportional-integral-derivative (PID) compensator designs are addressed in a number of examples, including voltage-mode, current-mode, and multiloop control of dc–dc converters and power factor correction (PFC) rectifiers.

Amplitude quantization effects introduced by analog-to-digital (A/D) converters and digital pulse width modulators (DPWMs) are discussed in Chapter 5. This chapter first clarifies how limit cycle oscillations can arise in a digitally controlled dc–dc converter in relation to the existence of a dc operating point for the closed-loop system. Secondly, basic design guidelines—referred to as *no-limit-cycling conditions*—are presented, which aim at preventing such generally undesired phenomena to occur. This chapter ends with a brief overview of DPWM and A/D architectures and associated implementation trade-offs.

The issue of compensator implementation is covered in Chapter 6. Scaling and quantization of compensator coefficients are treated first, with the goal of quantifying the quantization-induced errors on the loop gain magnitude and phase at the desired crossover frequency. Secondly, this chapter addresses implementation of the control law in a fixed-point arithmetic environment, providing a methodology for word length determination of the various signals inside the control structure. Given the focus of the book on high-frequency switched-mode power converter applications, the emphasis

is on *hardwired* implementations of the control law, together with VHDL and Verilog coding examples. Nevertheless, the principles that apply to software-based, microprogrammed realizations are highlighted as well.

Autotuning is an advanced application of digital control, which brings up intriguing potentials and additional challenges. Because of the importance of this emerging topic, Chapter 7 is devoted to an overview of digital autotuning techniques for high-frequency switched-mode power converters. After a brief discussion about digital autotuning basics, two autotuning techniques are presented in more detail: an injection-based approach and a relay-based approach.

An objective in writing the book has been to emphasize the distinction between the fundamental, theoretical aspects of digital control design on one side and the application of these techniques on the other, demystifying the perception about discrete-time models or digital control as being exceedingly complex and difficult to employ in practice. In line with such philosophy, Matlab® script examples are systematically developed alongside the theory. A few Matlab® commands allow, in most situations, to straightforwardly carry out system-level compensator designs and rapidly proceed to HDL coding and implementation steps. Furthermore, throughout the book, a number of design examples are fully worked out and verified by simulations in the Matlab® environment.

CONTENTS

<i>PREFACE</i>	ix
<i>INTRODUCTION</i>	1
CHAPTER 1 <i>CONTINUOUS-TIME AVERAGED MODELING OF DC–DC CONVERTERS</i>	13
1.1	Pulse Width Modulated Converters 14
1.2	Converters in Steady State 16
1.2.1	Boost Converter Example 17
1.2.2	Estimation of the Switching Ripple 19
1.2.3	Voltage Conversion Ratios of Basic Converters 20
1.3	Converter Dynamics and Control 21
1.3.1	Converter Averaging and Linearization 22
1.3.2	Modeling of the Pulse Width Modulator 24
1.3.3	The System Loop Gain 25
1.3.4	Averaged Small-Signal Models of Basic Converters 26
1.4	State-Space Averaging 28
1.4.1	Converter Steady-State Operating Point 28
1.4.2	Averaged Small-Signal State-Space Model 29
1.4.3	Boost Converter Example 30
1.5	Design Examples 32
1.5.1	Voltage-Mode Control of a Synchronous Buck Converter 32
1.5.2	Average Current-Mode Control of a Boost Converter 42
1.6	Duty Ratio $d[k]$ Versus $d(t)$ 48
1.7	Summary of Key Points 50
CHAPTER 2 <i>THE DIGITAL CONTROL LOOP</i>	51
2.1	Case Study: Digital Voltage-Mode Control 52
2.2	A/D Conversion 53
2.2.1	Sampling Rate 53
2.2.2	Amplitude Quantization 56
2.3	The Digital Compensator 58
2.4	Digital Pulse Width Modulation 63
2.5	Loop Delays 65
2.5.1	Control Delays 65
2.5.2	Modulation Delay 66
2.5.3	Total Loop Delay 70
2.6	Use of Averaged Models in Digital Control Design 71

2.6.1	Limitations of Averaged Modeling	71
2.6.2	Averaged Modeling of a Digitally Controlled Converter	74
2.7	Summary of Key Points	78

CHAPTER 3 *DISCRETE-TIME MODELING***79**

3.1	Discrete-Time Small-Signal Modeling	80
3.1.1	A Preliminary Example: A Switched Inductor	82
3.1.2	The General Case	85
3.1.3	Discrete-Time Models for Basic Types of PWM Modulation	87
3.2	Discrete-Time Modeling Examples	88
3.2.1	Synchronous Buck Converter	90
3.2.2	Boost Converter	97
3.3	Discrete-Time Modeling of Time-Invariant Topologies	102
3.3.1	Equivalence to Discrete-Time Modeling	106
3.3.2	Relationship with the Modified Z -Transform	108
3.3.3	Calculation of $T_u(z)$	108
3.3.4	Buck Converter Example Revisited	112
3.4	Matlab® Discrete-Time Modeling of Basic Converters	112
3.5	Summary of Key Points	117

CHAPTER 4 *DIGITAL CONTROL***119**

4.1	System-Level Compensator Design	119
4.1.1	Direct-Digital Design Using the Bilinear Transform Method	120
4.1.2	Digital PID Compensators in the z - and the p -Domains	123
4.2	Design Examples	126
4.2.1	Digital Voltage-Mode Control of a Synchronous Buck Converter	126
4.2.2	Digital Current-Mode Control of a Boost Converter	134
4.2.3	Multiloop Control of a Synchronous Buck Converter	136
4.2.4	Boost Power Factor Corrector	141
4.3	Other Converter Transfer Functions	154
4.4	Actuator Saturation and Integral Anti-Windup Provisions	160
4.5	Summary of Key Points	165

CHAPTER 5 *AMPLITUDE QUANTIZATION***167**

5.1	System Quantizations	167
5.1.1	A/D Converter	167
5.1.2	DPWM Quantization	169
5.2	Steady-State Solution	172
5.3	No-Limit-Cycling Conditions	175
5.3.1	DPWM versus A/D Resolution	175
5.3.2	Integral Gain	178
5.3.3	Dynamic Quantization Effects	181
5.4	DPWM and A/D Implementation Techniques	182
5.4.1	DPWM Hardware Implementation Techniques	182
5.4.2	Effective DPWM Resolution Improvements via $\Sigma\Delta$ Modulation	186

5.4.3	A/D Converters	187
5.5	Summary of Key Points	190

CHAPTER 6 *COMPENSATOR IMPLEMENTATION*
191

6.1	PID Compensator Realizations	194
6.2	Coefficient Scaling and Quantization	197
6.2.1	Coefficients Scaling	198
6.2.2	Coefficients Quantization	200
6.3	Voltage-Mode Control Example: Coefficients Quantization	203
6.3.1	Parallel Structure	204
6.3.2	Direct Structure	206
6.3.3	Cascade Structure	208
6.4	Fixed-Point Controller Implementation	213
6.4.1	Effective Dynamic Range and Hardware Dynamic Range	214
6.4.2	Upper Bound of a Signal and the L^1 -Norm	216
6.5	Voltage-Mode Converter Example: Fixed-Point Implementation	218
6.5.1	Parallel Realization	220
6.5.2	Direct Realization	225
6.5.3	Cascade Realization	229
6.5.4	Linear versus Quantized System Response	233
6.6	HDL Implementation of the Controller	234
6.6.1	VHDL Example	235
6.6.2	Verilog Example	237
6.7	Summary of Key Points	239

CHAPTER 7 *DIGITAL AUTOTUNING*
241

7.1	Introduction to Digital Autotuning	242
7.2	Programmable PID Structures	243
7.3	Autotuning VIA Injection of a Digital Perturbation	247
7.3.1	Theory of Operation	249
7.3.2	Implementation of a PD Autotuner	253
7.3.3	Simulation Example	255
7.3.4	Small-Signal Analysis of the PD Autotuning Loop	261
7.4	Digital Autotuning Based on Relay Feedback	265
7.4.1	Theory of Operation	266
7.4.2	Implementation of a Digital Relay Feedback Autotuner	267
7.4.3	Simulation Example	271
7.5	Implementation Issues	272
7.6	Summary of Key Points	275

APPENDIX A *DISCRETE-TIME LINEAR SYSTEMS AND THE Z-TRANSFORM*
277

A.1	Difference Equations	277
A.1.1	Forced Response	278
A.1.2	Free Response	279
A.1.3	Impulse Response and System Modes	281
A.1.4	Asymptotic Behavior of the Modes	282
A.1.5	Further Examples	283

A.2	Z-Transform	284
A.2.1	Definition	284
A.2.2	Properties	285
A.3	The Transfer Function	287
A.3.1	Stability	287
A.3.2	Frequency Response	288
A.4	State-Space Representation	288

APPENDIX B *FIXED-POINT ARITHMETIC AND HDL CODING*

291

B.1	Rounding Operation and Round-Off Error	291
B.2	Floating-Point versus Fixed-Point Arithmetic Systems	293
B.3	Binary Two's Complement (B2C) Fixed-Point Representation	294
B.4	Signal Notation	296
B.5	Manipulation of B2C Quantities and HDL Examples	297
B.5.1	Sign Extension	298
B.5.2	Alignment	299
B.5.3	Sign Reversal	301
B.5.4	LSB and MSB Truncation	302
B.5.5	Addition and Subtraction	304
B.5.6	Multiplication	305
B.5.7	Overflow Detection and Saturated Arithmetic	307

APPENDIX C *SMALL-SIGNAL PHASE LAG OF UNIFORMLY SAMPLED PULSE WIDTH MODULATORS*

313

C.1	Trailing-Edge Modulators	313
C.2	Leading-Edge Modulators	317
C.3	Symmetrical Modulators	318

REFERENCES 321
INDEX 335

INTRODUCTION

Efficient processing and control of electric power is required in applications ranging from submilliwatt on-chip power management to hundreds of kilowatt and megawatt power levels in motor drives and utility applications. The objectives of high efficiency, as well as static and dynamic control of inputs or outputs under a range of operating conditions, are accomplished using power electronics, that is, switched-mode power converters consisting of passive (capacitive and inductive) components, and power semiconductor devices operated as switches. In high-power applications, control and monitoring tasks are often more complex, while the power semiconductor devices are operated at relatively low switching frequencies, for example, up to tens of kilohertz. The controller cost and power consumption are relatively low compared to the overall system cost and power rating. In these applications, digital control offers clear technical and economic advantages in addressing complex control, management, and monitoring tasks. As a result, for many years now, digital control methods and digital controllers based on general-purpose or dedicated microprocessors, digital signal processor (DSPs), or programmable logic devices have been widely adopted in power electronics applications at relatively high power levels.

In ubiquitous low-to-medium power switched-mode power supply (SMPS) applications, including point-of-load (POL) regulators, nonisolated and isolated dc–dc converters, single-phase power factor correction (PFC) rectifiers, single-phase inverters, and lighting applications, adoption of digital power management and digital control has been slower. In these applications, switching frequencies are often in the range from hundreds of kilohertz to multiple megahertz, and much faster dynamic responses are required. The controller cost and the controller power consumption can easily present significant portions of the system cost and power dissipation. Furthermore, in many applications, control challenges have been successfully met by continuous advances of readily available analog controllers, using well-established analog analysis, modeling, and design techniques [1–5]. Nevertheless, practical digital control of high-frequency switched-mode power converters has moved from proof-of-concept demonstrations to digital pulse width modulation (DPWM) controller chips commercially available from multiple vendors, with growing adoption rates in many applications. Several factors have

contributed to the increasing penetration of the concept of “digital power” in high-frequency power electronics applications:

- Ongoing advances in digital integrated-circuit processes have continued to increase processing capabilities while bringing the cost down.
- The needs for improved system integration and increasingly complex power management and monitoring functions have translated into the needs for digital interfaces and programmability in switched-mode power conversion applications [9–11].
- Practical high-performance digital control techniques have been introduced and demonstrated, together with innovative approaches offering performance gains or entirely new capabilities that would be difficult or impractical to realize using traditional analog techniques [12–14].

The “digital power” concept encompasses several aspects:

1. *Digital power management*, which refers to system-level control and monitoring of power conversion and distribution, usually over a serial communication bus [9–11]. Power management functions include turning on and off or sequencing system power rails, adjusting setpoints for converter control loops, programming control loop parameters, monitoring and reporting of measured status or variables, and so on [15, 16]. These functions are typically performed at timescales that are relatively long compared to a switching period.
2. *Digital control*, which includes time-domain and frequency-domain converter modeling and control techniques, with control actions performed at timescales comparable to a switching period.
3. *Digital implementation techniques*, which can be classified into two main groups:
 - *Software-based controllers*, where control algorithms are designed and implemented in code executed on general-purpose or specialized microcontrollers or DSP chips. An early example of application of microprogrammed digital control to power factor preregulators is presented in [17].
 - *Hardware-based controllers*, based on custom-integrated circuits or programmable logic devices such as field-programmable gate arrays (FPGAs) [18, 19]. Early examples of such hardware-based digital controllers can be found in [20–22].

This book is focused on the fundamental aspects of analysis, modeling, and design of digital control loops around high-frequency switched-mode power converters in a systematic and rigorous manner. The objectives are to enable the reader to understand, analyze, model, design, and implement digital feedback loops around power converters, from system-level transfer function formulations to practical implementation details. The purpose of this chapter is to introduce the topics covered in the book and to motivate the reader to pursue the theoretical and practical concepts covered in the remaining chapters of this book. Furthermore, this introductory chapter points to some of the more advanced digital techniques reported in the literature, including

approaches to dynamic response improvements, system identification, autotuning of digital control loops, and on-line efficiency optimization.

DIGITALLY CONTROLLED SWITCHED-MODE CONVERTERS

A number of DPWM controller architectures and implementation strategies have been investigated and realized in practice. Many standard microcontrollers and DSP chips are now available, featuring multiple high-resolution PWM and analog-to-digital (A/D) channels, which allow software-based implementation of control and management functions. While advances in this area have been rapid, the software-based approaches are still better suited for applications where switching converters operate at relatively low switching frequencies. On the other hand, at switching frequencies in the hundreds of kilohertz to megahertz range, specialized hardware-based control loops are often preferred. This approach is illustrated in the architecture shown in Fig. 1 [12, 13]. The control loop is digital, using specialized, programmable A/D, DPWM, and compensator blocks to achieve high-performance closed-loop dynamic responses, while programmability, power management, and system interface functions are delegated to a microcontroller core. Similar combinations of programmable hardware peripherals specialized for switched-mode power converter applications, with software-based realizations of higher-level management and communication functions are often found in commercially available DPWM controllers.

Controllers of the type shown in Fig. 1 can be developed, realized, and tested using standard digital VLSI design flow starting from logic functions described using

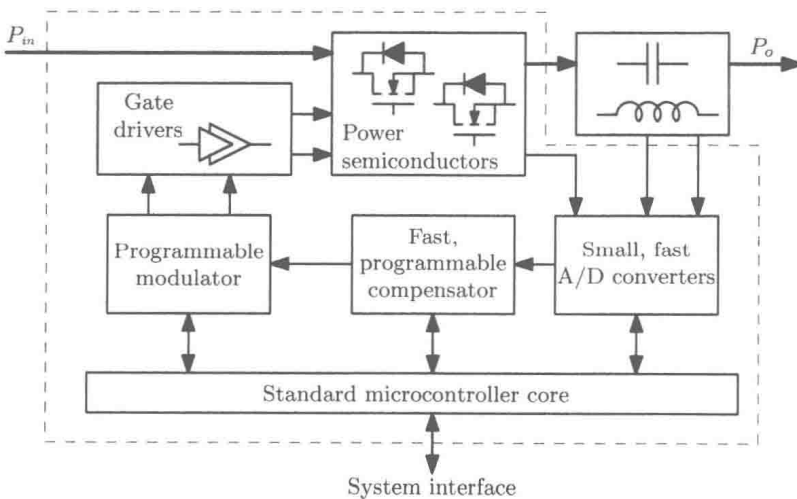


Figure 1 Digital controller architecture for high-frequency switched-mode power converters [13].

hardware description language (VHDL or Verilog), followed by prototyping and experimental verifications using FPGA development platforms, ultimately targeting relatively small, relatively low-gate-count integrated circuits capable of matching or surpassing the state-of-the-art analog solutions in terms of dynamic performance, power consumption, and cost. At the same time, digital PWM controllers offer digital system interface, programmability and flexibility, power management functions, reductions in the number of passive components, reduced sensitivity to process and temperature variations, and potentials for practical realizations of more advanced features.

Figure 2 shows a more detailed block diagram of a hardware-based digital controller around a POL synchronous Buck converter. Output voltage $v_o(t)$ is sampled by an A/D converter and compared to a setpoint reference V_{ref} to produce a digital voltage error signal $e[k]$. The error signal is processed by a discrete-time digital proportional-integral-derivative (PID) compensator to generate a duty cycle command $u_x[k]$. In the basic version of the controller, the compensator gains K_p , K_i , and K_d are found by design to meet control loop specifications, such as the crossover frequency and phase margin, as detailed in Chapters 1–6 of this book. Once a compensator is designed, the gains can be realized using digital multipliers, as shown in Fig. 2. As only a few bits are sufficient to represent the error signal $e[k]$, the entire compensator can also be implemented as a lookup table [21–24]. In a more advanced case, as illustrated by the digital autotuner block in Fig. 2, the compensator gains can

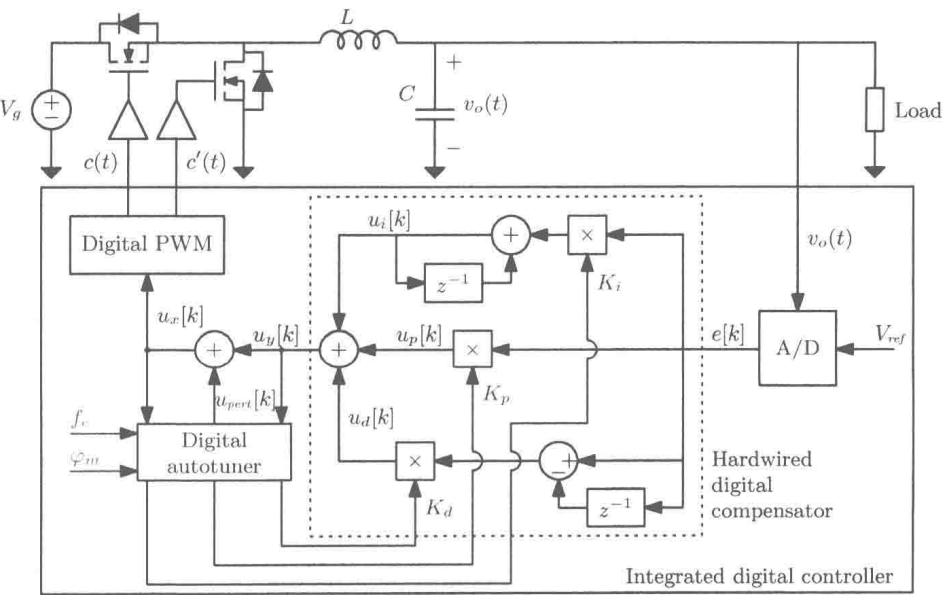


Figure 2 Digitally controlled point-of-load (POL) synchronous Buck dc-dc converter example. Analysis, modeling, design, and implementation of digital control loops are addressed in Chapters 1–6. An additional injection-based autotuning block is shown, which is further discussed in Chapter 7.