



**Christos Kassapoglou**

# **Modeling the Effect of Damage in Composite Structures**

**Simplified Approaches**

**Aerospace Series**

Editors **Peter Belobaba, Jonathan Cooper  
and Allan Seabridge**

**WILEY**

# **MODELING THE EFFECT OF DAMAGE IN COMPOSITE STRUCTURES**

## **SIMPLIFIED APPROACHES**

**Christos Kassapoglou**

*Delft University of Technology, The Netherlands*

**WILEY**

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# Series Preface

The field of aerospace is multi-disciplinary and wide ranging, covering a large variety of products, disciplines and domains, not merely in engineering but in many related supporting activities. These combine to enable the aerospace industry to produce exciting and technologically advanced vehicles. The wealth of knowledge and experience that has been gained by expert practitioners in the various aerospace fields needs to be passed onto others working in the industry, including those just entering from University.

The *Aerospace Series* aims to be a practical, topical and relevant series of books aimed at people working in the aerospace industry, including engineering professionals and operators, allied professions such commercial and legal executives, and also engineers in academia. The range of topics is intended to be wide ranging, covering design and development, manufacture, operation and support of aircraft, as well as topics such as infrastructure operations and developments in research and technology.

Composite materials are being used increasingly in aerospace structures due to their impressive strength to weight properties, and the ability to manufacture complex integral components. However, in structural design, it is also important to be able to account for the effect of damage on the structure's integrity throughout its lifetime.

This book, *Modelling the Effect of Damage in Composite Structures: Simplified Approaches*, considers the various types of damage that can occur in composite structures and how they can be modelled during preliminary structural design. Analytical models are developed in order to understand and predict the physical phenomena that lead to the onset of damage and its evolution. The techniques provide a set of design tools and rules of thumb for a range of different types of damage in composite structures. The book complements the author's previous book *Design and Analysis of Composite Structures: With Application to Aerospace Structures, Second Edition* which is also in the Wiley Aerospace Series.

Peter Belobaba, Jonathan Cooper and Allan Seabridge

# Preface

One of the main features of a good structural design is the ability to account for damage representative of what may occur during the lifetime of the structure and ensure that performance is not compromised with such damage present. For the case of airframe structures, knowledge of the effects of damage on structural performance is crucial in designing safe but weight-efficient structures.

This book provides a brief discussion of various types of damage and how they can be modelled during preliminary design and analysis of composite structures. It is addressed to graduate-level students and entry-level design and structural engineers. It is the result of a graduate course at Delft University of Technology covering design and analysis of composite structures in the presence of damage. While the emphasis is on aerospace structures, the principles and methods are applicable to all other fields with appropriate adjustment of safety factors and design criteria (e.g. impact damage).

Because the emphasis is on preliminary design and analysis, more accurate, and absolutely necessary during detailed design, computational methods such as finite elements are only briefly touched upon during the discussion. As a result, the accuracy and applicability of some of the approaches presented are not as good as those resulting from detailed finite element analysis. However, the methods are very efficient and provide good starting designs for more detailed subsequent evaluation. They can also be used to compare different designs and thus can be very useful for optimisation.

In a sense, this book is a natural continuation of my previous book on Design and Analysis of Composite Structures. There, the effect of damage was conservatively accounted for by applying appropriate knockdown factors on the allowable strength. Here, an attempt is made to replace these knockdown factors with analytical models that focus on understanding better some of the physical phenomena behind damage creation and evolution, while, at the same time, removing some of the conservatism associated with knockdown factors.

It is also recognised that some of the subjects discussed in this book such as the analysis of structures with impact damage or the fatigue analysis of composite structures are very much the subject of on-going research. As such, the methods presented here should be viewed as good design tools that may be superseded in the future as our understanding of damage creation and evolution in composite structures improves.

Chapter 1 includes a brief overview of types of damage and points out some important characteristics specific to composites manifested by increased notch sensitivity compared to metals. Chapter 2 discusses the effect of holes and provides improved methodology to obtain reliable failure predictions. Chapter 3 discusses through-thickness cracks and gives simple methods of analysis to obtain failure predictions. Delaminations are discussed in Chapter 4 where solutions for different structural details are given. Impact damage, which includes all previous types of damage, matrix cracks, holes (for high impact energies) and delaminations, is addressed in Chapter 5. A brief discussion of fatigue of composite materials with an emphasis on analytical models for predicting cycles to failure is given in Chapter 6. Constant amplitude and spectrum loading are discussed with an emphasis on how damage at different length scales may be accounted for in the analysis. Finally, design guidelines and rules of thumb that can be deduced from all previous chapters are summarised in Chapter 7.

*Christos Kassapoglou*



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# 1

## Damage in Composite Structures: Notch Sensitivity

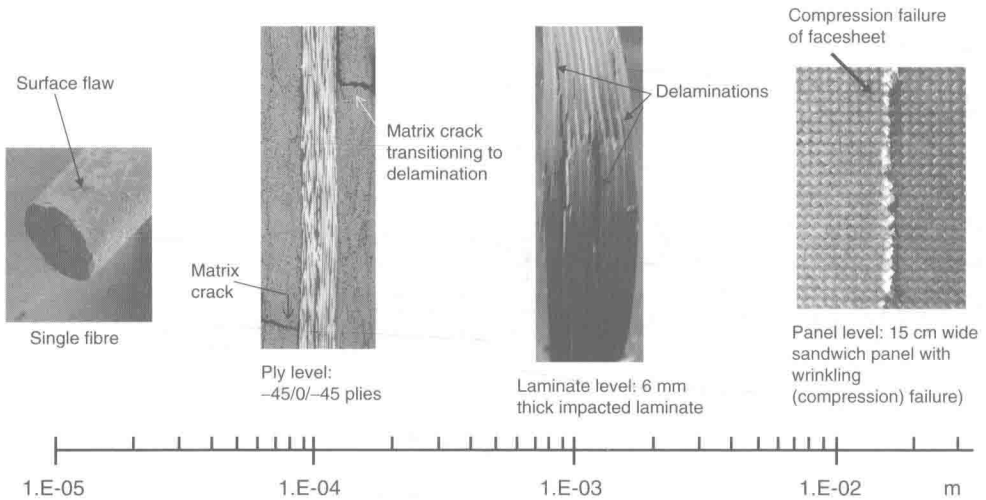
### 1.1 Introduction

Owing to its construction, where two basic constituents, fibres and matrix, are combined, a composite structure shows a wide variety of types of damage. Damage may be specific to one or both of the constituents or involve interaction of the two. Furthermore, depending on the scale over which phenomena are described, damage may have different forms ranging from micro-voids or inconsistencies and cracks of the fibre/matrix interphase to large-scale delaminations, holes and laminate failures.

Here, the emphasis is placed on damage that is no smaller than a few fibre diameters with the understanding that this damage most likely is the result of creation and coalescence of damage at smaller scales, which are beyond the scope of this book. Within this framework, the most common forms of damage are matrix cracks, fibre/matrix interface failures, fibre failures, through-thickness failures (holes and cracks) and inter-ply failures such as delaminations. Of course any combination of these may also occur as in cases of impact damage. Representative forms of damage and their corresponding scales are shown in Figure 1.1.

In advanced composites typical of aerospace structures, the matrix has much lower strength than the fibres. Failure then typically initiates in the matrix and the associated damage is in the form of matrix cracks. These cracks usually appear in plies with fibres not aligned with the directions along which appreciable loads are applied [1]. Matrix cracks may also be present in a composite right after curing due to curing stresses [2] or tooling problems where heat uptake or cool-down during the cure cycle is not uniform [3].

This does not mean that damage may not initiate at a location where a small flaw (resin-rich region, resin-poor region, void and contamination) is present. Ideally, a damage model should start at the lowest possible scale where damage initiated and track the latter as it evolves and grows. As can be seen from Figure 1.1, however, this process may require bridging at least three to four orders of magnitude in the length scale. This means that separate models for the individual constituents are needed at the



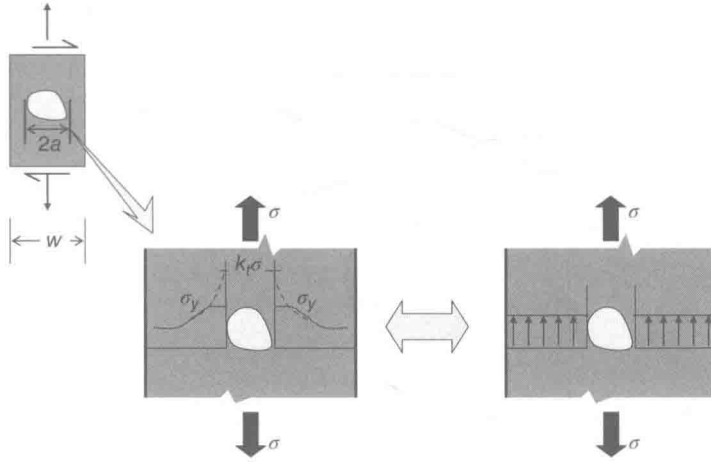
**Figure 1.1** Typical damage at various scales of a composite structure

lower scales at which even the material homogeneity is in doubt. To minimise computational complexity, models that address macroscopic structures start at larger scales, the ply level or, less frequently, at somewhat lower scales and focus on aggregate flaws such as notches.

In general, a notch can be considered any type of local discontinuity such as a crack, hole, and indentation. Here, the definition of a notch is generalised and is not confined to a surface flaw. It can also be a through-the-thickness discontinuity. Notches act as stress risers and, as such, reduce the strength of a structure. The extent of the reduction is a function of the material and its ability to redistribute load around the notch. The possible range of behaviour is bounded by two extremes: (i) notch insensitivity and (ii) complete notch sensitivity.

## 1.2 Notch Insensitivity

This is the limiting behaviour of metals. Consider the notched plate at the top left of Figure 1.2. The shape and type of the notch are not important for the present discussion. Now assume that a purely elastic solution is obtained in the vicinity of the notch for a given far-field loading. Typically, there is a stress concentration factor  $k_t$  and, for an applied far-field stress  $\sigma$ , the stress at the edge of the notch is  $k_t\sigma$ . This is shown in the middle of Figure 1.2. If the material of the plate is metal, then, for sufficiently high values of the far-field stress  $\sigma$ ,  $k_t\sigma$  exceeds the yield stress  $\sigma_y$  of the material. As a first-order approximation, one can truncate the linear stress solution in the region where the local stress exceeds the yield stress (shown by a dashed line in the middle of Figure 1.2) by setting the stress there equal to the yield stress. To maintain force



**Figure 1.2** Stress distribution in the vicinity of a notch-insensitive material

equilibrium, the region where the stress equals  $\sigma_y$  must extend beyond the point of intersection of the horizontal line at  $\sigma_y$  and the linear stress solution such that the areas under the original curve corresponding to the linear solution and the modified 'truncated' curve are equal.

For sufficiently high  $\sigma$  and/or sufficiently low  $\sigma_y$  value, the material on either side of the notch yields and the stress distribution become the one shown on the right of Figure 1.2.

This means that the stress aligned with the load on either side of the notch is constant and there is no stress concentration effect any more. The stress is completely redistributed and only the reduced area due to the presence of the notch plays a role. More specifically, if  $F_{tu}$  is the failure strength of the material (units of stress), the force  $F_{fail}$  at which the plate fails is given by the material strength multiplied by the available cross-sectional area:

$$F_{fail} = F_{tu}(w - 2a)t \quad (1.1)$$

with  $w$  and  $2a$  the plate and notch widths, respectively, and  $t$  the plate thickness.

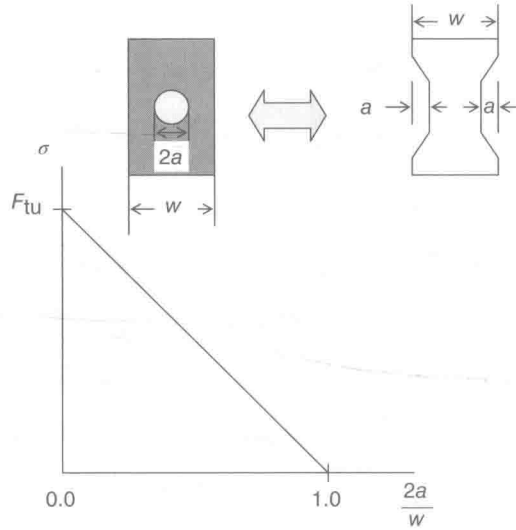
At the far-field, the same force is given by

$$F_{fail} = \sigma wt \quad (1.2)$$

The right-hand sides of Equations 1.1 and 1.2 can be set equal and a solution for the far-field stress that causes failure can be obtained:

$$\sigma = F_{tu} \left( 1 - \frac{2a}{w} \right) \quad (1.3)$$

A plot of the far-field stress as a function of normalised notch size  $2a/w$  is shown in Figure 1.3. The straight line connecting the failure strength  $F_{tu}$  on the y-axis with the



**Figure 1.3** Notch-insensitive behaviour

point  $2a = w$  on the  $x$ -axis gives the upper limit of material behaviour in the presence of a notch.

It should be pointed out that, for this limiting behaviour, the shape of the notch is not important. The specimen with the hole and the dog-bone specimen shown in Figure 1.3 are completely equivalent.

### 1.3 ‘Complete’ Notch Sensitivity

At the other extreme of material behaviour are brittle materials that are notch-sensitive, such as some composites and ceramics. In this case, if there is a stress riser due to the presence of a notch with a stress concentration factor  $k_t$ , failure occurs as soon as the maximum stress in the structure reaches the ultimate strength of the material. For a far-field applied stress  $\sigma$ , this leads to the condition:

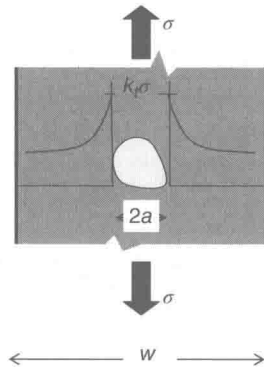
$$k_t \sigma = F_{tu} \quad (1.4)$$

The situation is shown in Figure 1.4. Here, there is no redistribution of stress in the vicinity of the notch. For the case of an infinite plate in Figure 1.4, or a very small notch, the far-field stress to cause failure is given by rearranging Equation 1.4:

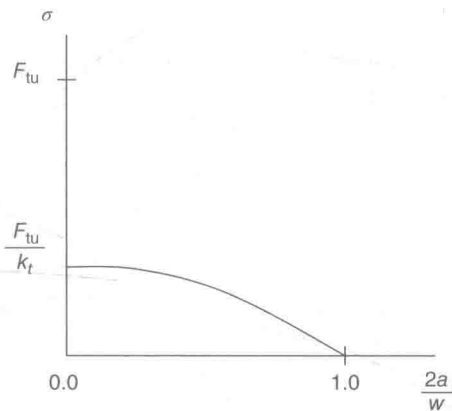
$$\sigma = \frac{F_{tu}}{k_t} \quad (1.5)$$

For finite plates, with larger notches, finite width effects reduce further the strength of the plate. In the limit, as the notch size approaches the width of the plate, the strength goes to zero:

$$\sigma \rightarrow 0 \quad \text{as} \quad 2a \rightarrow w \quad (1.6)$$



**Figure 1.4** Stress distribution in the vicinity of a notch-sensitive material



**Figure 1.5** Notch-sensitive material

Equations 1.5 and 1.6 are combined in Figure 1.5, which shows the notch-sensitive behaviour.

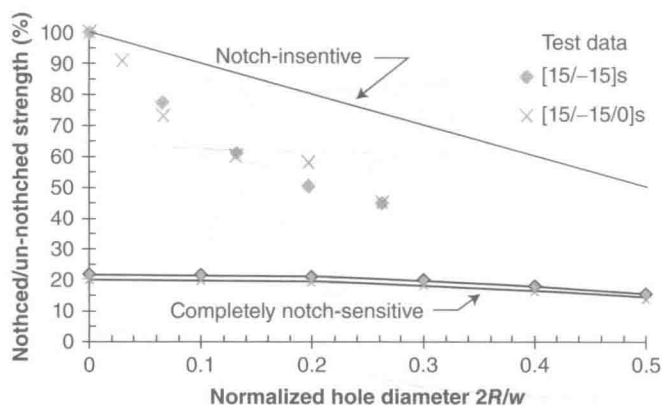
### 1.4 Notch Sensitivity of Composite Materials

The types of behaviour discussed in the previous two sections are the two extremes that bracket all materials. It is interesting to see where typical composite materials lie with respect to these two extremes. Experimental data for various composite laminates with different hole sizes under tension are shown in Figure 1.6. The test data are taken from Ref. [4].

Note that two curves, very close to each other, are shown for the ‘completely notch-sensitive’ behaviour. One corresponds to the [15/–15]s and the other to the [15/–15/0]s laminate.

It is seen from Figure 1.6 that the composite data fall between the two curves. More importantly, even at very small holes, there is a significant drop of the strength towards





**Figure 1.6** Test results for composite laminates with holes under tension

the curve of complete notch sensitivity and, at higher hole diameters ( $2R/w > 0.7$ ), the data tend to follow that curve. However, the fact that the data start at the top curve and drop towards the lower curve suggests that composites have some load redistribution around a notch but the redistribution is limited. A damage zone or process zone is created at the edge of the hole with matrix cracks, broken fibres and delaminations. This process zone limits the stress to a value equal or close to the undamaged failure strength. As the load is increased, the stress inside the process zone stays constant. The size of the process zone increases and the strains in the material next to the hole increase. As the load is increased further, a point is reached where the structure can no longer store energy and fails. In general, therefore, composites are notch sensitive but they do have some limited ability to redistribute load around notches. This will be of some significance in subsequent chapters when stresses in the vicinity of a notch are discussed in more detail.

## Exercises

- 1.1** Discuss how small-scale defects and flaws affect the scatter in the static strength of a composite structure. Then, discuss how the presence of damage of a size sufficient to drive failure may reduce the scatter of the static strength.
- 1.2** Often, but not always, the presence of a notch of sufficient size and severity, not only drives failure but also masks the effects of smaller more benign notches in a way that the scatter of test results is lower with a notch than without it. Table E1.1 gives the un-notched and notched strength values for [45/-45/0]s, [0/45/-45]s and [45/0/-45]s laminates. Lumping all laminates in one data set, determine the B- and A-basis for un-notched specimens and for each hole diameter. Do this as a fraction of the corresponding mean value. Comment on the knockdown due to the hole and how it relates to the scatter of the test data.