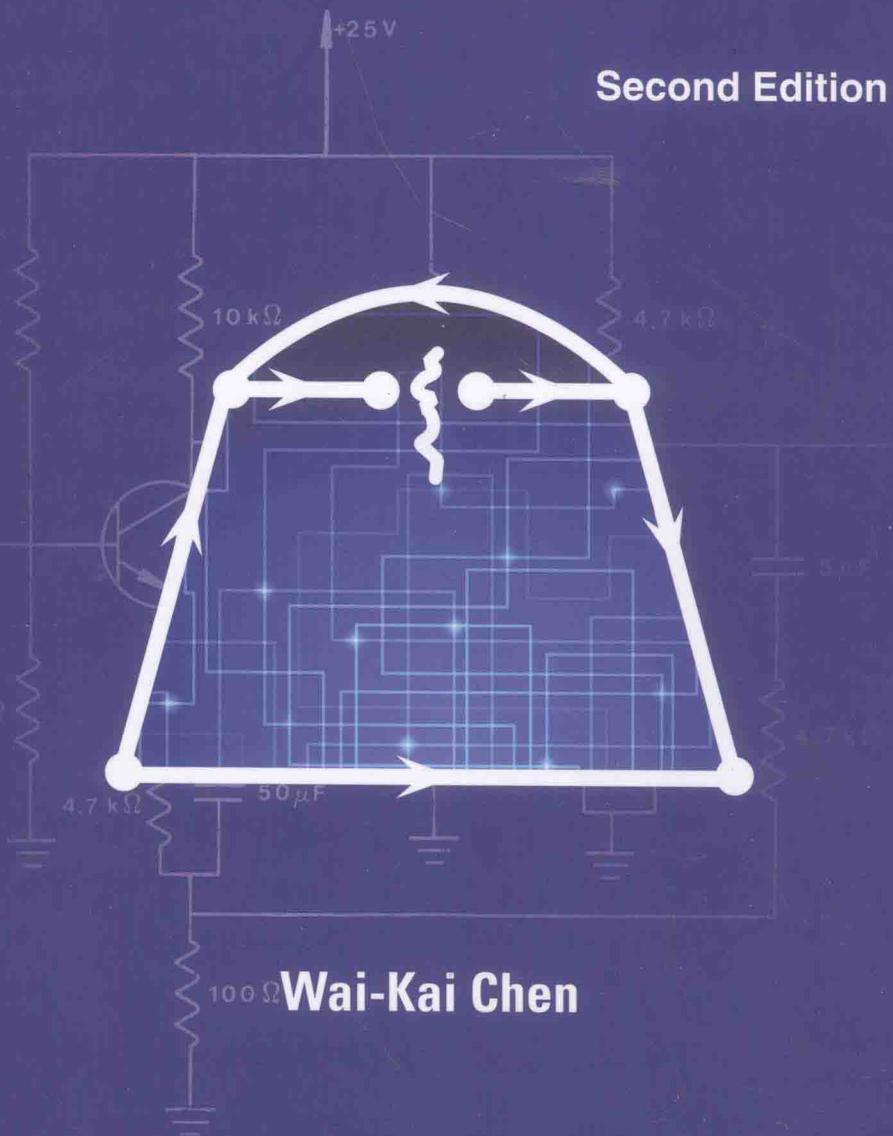


Advanced Series in Electrical and Computer Engineering – Vol. 15

ACTIVE NETWORK ANALYSIS

Feedback Amplifier Theory

Second Edition



Wai-Kai Chen

World Scientific

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University of Illinois, Chicago, USA

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PREFACE TO FIRST EDITION

Since Bode published his classical text “Network Analysis and Feedback Amplifier Design” in 1945, very few books have been written that treat the subject in any reasonable depth. The purpose of this book is to bridge this gap by providing an in-depth, up-to-date, unified, and comprehensive treatment of the fundamentals of the theory of active networks and its applications to feedback amplifier design. The guiding light throughout has been to extract the essence of the theory and to discuss the topics that are of fundamental importance and that will transcend the advent of new devices and design tools. Intended primarily as a text in network theory in electrical engineering for first-year graduate students, the book is also suitable as a reference for researchers and practicing engineers in industry. In selecting the level of presentation, considerable attention has been given to the fact that many readers may be encountering some of these topics for the first time. Thus, basic introductory material has been included. The background required is the usual undergraduate basic courses in circuits and electronics as well as the ability to handle matrices.

The book can be conveniently divided into three parts. The first part, comprising the first three chapters, deals with general network analysis. The second part, composed of the next four chapters, is concerned with feedback amplifier theory. The third part, consisting of the last two chapters, discusses the state-space and topological analyses of active networks and their relations to feedback theory.

Chapter 1 introduces many fundamental concepts used in the study of linear active networks. We start by dealing with general n -port networks and define passivity in terms of the universally encountered physical quantities *time* and *energy*. We then translate the time-domain passivity criteria into the equivalent frequency-domain passivity conditions. Chapter 2 presents a useful description of the external behavior of a multiterminal network in terms of the indefinite-admittance matrix and demonstrates how it can be employed effectively for the computation of network functions. The significance of this approach is that the

indefinite-admittance matrix can usually be written down directly from the network by inspection and that the transfer functions can be expressed compactly as the ratios of the first-and/or second-order cofactors of the elements of the indefinite-admittance matrix. In Chapter 3 we consider the specialization of the general passivity condition for n -port networks in terms of the more immediately useful two-port parameters. We introduce various types of power gains, sensitivity, and the notion absolute stability as opposed to potential instability.

Chapters 4 and 5 are devoted to a study of single-loop feedback amplifiers. We begin the discussion by considering the conventional treatment of feedback amplifiers based on the ideal feedback model and analyzing several simple feedback networks. We then present in detail Bode's feedback theory, which is based on the concepts of return difference and null return difference. Bode's theory is formulated elegantly and compactly in terms of the first- and second-order cofactors of the elements of the indefinite-admittance matrix, and it is applicable to both simple and complicated networks, where the analysis by conventional method for the latter breaks down. We show that feedback may be employed to make the gain of an amplifier less sensitive to variations in the parameters of the active components, to control its transmission and driving-point properties, to reduce the effects of noise and nonlinear distortion, and to affect the stability or instability of the network. The fact that return difference can be measured experimentally for many practical amplifiers indicates that we can include all the parasitic effects in the stability study and that stability problems can be reduced to Nyquist plots.

The application of negative feedback in an amplifier improves its overall performance. However, we are faced with the stability problem in that, for sufficient amount of feedback, at some frequency the amplifier tends to oscillate and becomes unstable. Chapter 6 discusses various stability criteria and investigates several approaches to the stabilization of feedback amplifiers. The Nyquist stability criteria, the Bode plot, the root-locus technique, and root sensitivity are presented. The relationship between gain and phase shift and Bode's design theory is elaborated. Chapter 7 studies the multiple-loop feedback amplifiers that contain a multiplicity of inputs, outputs, and feedback loops. The concepts of return difference and null return difference for a single controlled source are now generalized to the notions of return difference matrix and null return difference matrix for a multiplicity of controlled sources. Likewise, the scalar sensitivity function is generalized to the sensitivity matrix, and formulas for computing multiparameter sensitivity functions are derived.

In Chapter 8, we formulate the network equations in the time domain as a system of first-order differential equations that govern the dynamic behavior of a

network. The advantages of representing the network equations in this form are numerous. First of all, such a system has been widely studied in mathematics and its solution, both analytical and numerical, is known and readily available. Secondly, the representation can easily and naturally be extended to time-varying and nonlinear networks. In fact, nearly all time-varying and nonlinear networks are characterized by this approach. Finally, the first-order differential equations are easily programmed for a digital computer or simulated on an analog computer. We then formulate the general feedback theory in terms of the coefficient matrices of the state equations of a multiple-input, multiple-output and multiple-loop feedback amplifier, and derive expressions relating the zeros and poles of the determinants of the return difference matrix and the null return difference matrix to the eigenvalues of the coefficient matrices of the state equations under certain conditions. Finally, in Chapter 9 we study topological analysis of active networks and conditions under which there is a unique solution. These conditions are especially useful in computer-aided network analysis when a numerical solution does not converge. They help distinguish those cases where a network does not possess a unique solution from those where the fault lies with the integration technique. Thus, when a numerical solution does not converge, it is important to distinguish network instability, divergence due to improper numerical integration, and divergence due to lack of the existence of a unique solution.

The book is an outgrowth of notes developed over the past twenty-five years while teaching courses on active network theory at the graduate level at Ohio University and University of Illinois at Chicago. There is little difficulty in fitting the book into a one-semester or two-quarter course in active network theory. For example, the first four chapters plus some sections of Chapters 5, 6 and 8 would be ideal for a one-semester course, whereas the entire book can be covered adequately in a two-quarter course.

A special feature of the book is that it bridges the gap between theory and practice, with abundant examples showing how theory solves problems. These examples are actual practical problems, not idealized illustrations of the theory. A rich variety of problems has been presented at the end of each chapter, some of which are routine applications of results derived in the text. Others, however, require considerable extension of the text material. In all there are 286 problems.

Much of the material in the book was developed from my research. It is a pleasure to acknowledge publicly the research support of the National Science Foundation and the University of Illinois at Chicago through the Senior University Scholar Program. I am indebted to many graduate students who have made valuable

contributions to this book. Special thanks are due to my doctoral student Hui Tang, who helped proofread Chapters 8 and 9, and to my secretary, Ms. Barbara Wehner, who assisted me in preparing the index. Finally, I express my appreciation to my wife, Shiao-Ling, for her patience and understanding during the preparation of the book.

Wai-Kai Chen
Naperville, Illinois
January 1, 1991

PREFACE TO SECOND EDITION

We are most gratified to find that the first edition of *Active Network Analysis* was well received and is widely used. Thus, we feel that our original goal of providing an in-depth, unified and comprehensive treatment of the fundamentals of the theory of active networks and its applications to feedback amplifier design was, indeed, worthwhile. Since then many changes have occurred, necessitating not only the updating of some of the material, but more startling, the addition and expansion of many topics.

The purpose of the book is to provide in a single volume a comprehensive reference work covering the broad spectrum of active networks and feedback amplifiers. It is written and developed for the practicing electrical engineers in industry, government, and academia. The goal is to provide the most up-to-date information in the classical fields of circuit theory, circuit components and their models, and feedback networks.

The new edition can again be conveniently divided into three parts. The first part, comprising the first three chapters, deals with fundamentals of general network analysis. The second part, composed of the next four chapters, is concerned with feedback amplifier theory and its design. In this part, we also included compact formulas expressing various feedback quantities of a linear multivariable and multiloop feedback network in terms of the first- and the second-order cofactors of the elements of its indefinite-admittance matrix. They are useful in computing the feedback matrices in that they do not require any matrix inversion in computing some of these quantities. Furthermore, they are suitable for symbolical analysis. The third part, consisting of the last four chapters, discusses the general formulations of multiloop feedback systems. In addition to the two original chapters on state-space and topological analyses of active networks and their relations to feedback theory, we added two new chapters. One is on generalization of topological feedback amplifier theory, in which topological formulas are derived. Extensions of topology and the summations of the products of all transmittances

and their associated transfer immittances are also considered. The other chapter is on the indefinite-impedance matrix formulation of feedback amplifier theory. This dual concept as opposed to the more familiar indefinite-admittance matrix is rarely considered in the literature. Perhaps this is due to the fact that measuring the branch voltage is easier than measuring the branch current. However, advances in integrated op-amp circuits have made it possible to measure the branch current on line without opening any branch.

As before, the book stresses fundamental theory behind professional applications. In order to do so, it is reinforced with frequent examples. The reader is assumed to have a certain degree of sophistication and experience. However, brief reviews of theories, principles and mathematics of some subject areas are given. These reviews have been done concisely with perception. The prerequisite knowledge is a typical undergraduate mathematics background of calculus, complex variables, and simple matrix algebra plus a working knowledge in Laplace transform technique.

I am indebted to many of my students over the years who participated in testing the material of this book, and to my colleagues at the University of Illinois at Chicago for providing a stimulating milieu for discussions. Special thanks are due to my graduate students and visiting scholars Jiajian Lu, Jia-Long Lan, Mao-Da Tong, Hui-Yun Wang, and Yi Sheng Zhu, who made significant contributions to the field. In fact, some of the new materials included in the book are based on our joint research.

*Wai-Kai Chen
Fremont, California
March 7, 2016*

CONTENTS

<i>Preface to First Edition</i>	v
<i>Preface to Second Edition</i>	ix
1. Characterizations of Networks	1
1.1 Linearity and Nonlinearity	2
1.2 Time Invariance and Time Variance	7
1.3 Passivity and Activity	9
1.4 Causality and Noncausality	15
1.5 Matrix Characterizations of n -Port Networks	22
1.6 Equivalent Frequency-Domain Conditions of Passivity	28
1.7 Discrete-Frequency Concepts of Passivity and Activity	39
1.8 Summary	61
Problems	62
Bibliography	70
2. The Indefinite-Admittance Matrix	72
2.1 The Indefinite-Admittance Matrix	72
2.2 Rules for Writing Down the Primitive Indefinite-Admittance Matrix	77
2.3 Terminal Contraction and Suppression	84
2.4 Interrelationships of Transistor Models	91
2.5 The First- and Second-Order Cofactors	102
2.6 Computation of Network Functions	105
2.7 Analysis of Constrained Active Networks	115
2.8 Generalized Norton's Theorem	131
2.9 Summary	136

Problems	137
Bibliography	145
3. Active Two-Port Networks	147
3.1 Two-Port Parameters	147
3.2 Power Gains	151
3.3 Sensitivity	153
3.4 Passivity and Activity	157
3.5 The U -Functions	162
3.6 Potential Instability and Absolute Stability	179
3.7 Optimum Terminations for Absolutely Stable Two-Port Networks	193
3.8 Summary	206
Problems	207
Bibliography	216
4. Theory of Feedback Amplifiers I	219
4.1 Ideal Feedback Model	220
4.2 Feedback Amplifier Configurations	222
4.3 General Feedback Theory	263
4.4 The Network Functions and Feedback	282
4.5 Summary	291
Problems	292
Bibliography	298
5. Theory of Feedback Amplifiers II	300
5.1 Sensitivity Function and Feedback	301
5.2 The Return Difference and Two-Port Functions	311
5.3 Return Difference and Null Return Difference with Respect to Two Elements	319
5.4 Extensions to Feedback Concepts	321
5.5 The Network Functions and General Return Difference and General Null Return Difference	327
5.6 The Relative Sensitivity Function and Feedback	336
5.7 Signal-Flow Graph Formulation of Feedback Amplifier Theory	341
5.8 Measurement of Return Difference	347

5.9	Considerations on the Invariance of Return Difference	358
5.10	Summary	372
	Problems	374
	Bibliography	380
6.	Stability of Feedback Amplifiers	382
6.1	The Single-Loop Feedback Amplifiers	383
6.2	The Routh Criterion, the Hurwitz Criterion, and the Liénard-Chipart Criterion	384
6.3	The Nyquist Criterion	392
6.4	Applications of the Nyquist Criterion to Single-Loop Feedback Amplifiers	397
6.5	The Root-Locus Method	404
6.6	Root Sensitivity	423
6.7	Bode Formulas	429
6.8	Bode's Design Theory	442
6.9	Summary	452
	Problems	454
	Bibliography	460
7.	Multiple-Loop Feedback Amplifiers	462
7.1	Matrix Signal-Flow Graphs	463
7.2	The Multiple-Loop Feedback Amplifier Theory	470
7.3	Extensions to Feedback Matrices	505
7.4	The Hybrid-Matrix Formulation of Multiple-Loop Feedback Theory	519
7.5	The Sensitivity Matrix and Multiparameter Sensitivity	529
7.6	Computation of Feedback Matrices	538
7.7	Summary	547
	Problems	549
	Bibliography	556
8.	State-Space Analysis and Feedback Theory	559
8.1	State Equations in Normal Form	560
8.2	Graph Matrices and Kirchhoff's Equations	565
8.3	Trees and Fundamental Cutsets and Circuits	574
8.4	Systematic Procedure in Writing State Equations	584
8.5	State Equations for Degenerate Networks	601

8.6	State-Space Formulation of Feedback Theory	609
8.7	State-Space Formulation of Multiple-Loop Feedback Networks	632
8.8	Summary	649
	Problems	651
	Bibliography	655
9.	Topological Analysis of Active Networks	657
9.1	Natural Frequencies	658
9.2	Digraph Associated with an Active Network	663
9.3	Order of Complexity	671
9.4	Unique Solvability	681
9.5	Topology and the Summation of Return Differences	697
9.6	Topological Analysis of Active Networks	719
9.7	Summary	732
	Problems	734
	Bibliography	735
10.	Generalized Network Matrices and Their Cofactors	738
10.1	Network Determinants	739
10.2	Generalized Cofactors of the Generalized Network Matrix Determinants	757
10.3	The General Null Return Differences	765
10.4	Relations Between the Loop and Cutset Formulations	771
10.5	The Primary Systems of Equations	774
10.6	Invariance and Incidence Functions	775
10.7	Simple Derivations of Topological Formulas	779
10.8	Topological Evaluation of Feedback Matrices in Multiple-Loop Feedback Amplifiers	781
10.9	Summary	796
	Bibliography	798
11.	The Indefinite-Impedance Matrix Formulation of Feedback Amplifier Theory	800
11.1	The Indefinite-Impedance Matrix	801
11.2	Extension to Nonplanar Networks	816

11.3 Extension of a Dual Theorem on the Summation of Return Differences	820
11.4 Dual Topological Theorems of Linear Active Networks	826
11.5 Loop-Impedance Matrix Formulation	833
11.6 Summary	847
Bibliography	848
Appendices	851
I Hermitian Forms	851
II Conversion Chart for Two-Port Parameters	855
III Outline of a Derivation of Eq. (7.224)	856
Indexes	
Symbol Index	859
Subject Index	862

CHARACTERIZATIONS OF NETWORKS

Over the past two decades, we have witnessed a rapid development of solid-state technology with its apparently unending proliferation of new devices. Presently available solid-state devices such as the transistor, the tunnel diode, the Zener diode, and the varactor diode have already replaced the old vacuum tube in most practical network applications. Moreover, the emerging field of integrated circuit technology threatens to push these relatively recent inventions into obsolescence. In order to understand fully the network properties and limitations of solid-state devices and to be able to cope with the applications of the new devices yet to come, it has become increasingly necessary to emphasize the fundamentals of active network theory that will transcend the advent of new devices and design tools.

The purpose of this chapter is to introduce many fundamental concepts used in the study of linear active networks. We first introduce the concepts of portwise linearity and time invariance. Then we define passivity in terms of the universally encountered physical quantities *time* and *energy*, and show that causality is a consequence of linearity and passivity. This is followed by a brief review of the general characterizations of n -port networks in the frequency-domain. The translation of the time-domain passivity criteria into the equivalent frequency-domain passivity conditions is taken up next. Finally, we introduce the discrete-frequency concepts of passivity. The significance of passivity in the study of active networks is that passivity is the formal negation of activity.

1.1 LINEARITY AND NONLINEARITY

A network is a structure comprised of a finite number of interconnected elements with a set of accessible terminal pairs called *ports* at which voltages and currents may be measured and the transfer of electromagnetic energy into or out of the structure can be made. Fundamental to the concept of a port is the assumption that the instantaneous current entering one terminal of the port is always equal to the instantaneous current leaving the other terminal of the port. A network with n such accessible ports is called an n -port network or simply an n -port, as depicted symbolically in Fig. 1.1. In this section we review briefly the concepts of linearity and nonlinearity and introduce the notion of port wise linearity and nonlinearity.

Refer to the general representation of an n -port network N of Fig. 1.1. The port voltages $v_k(t)$ and currents $i_k(t)$ (where $k = 1, 2, \dots, n$) can be conveniently represented by the *port-voltage* and *port-current* vectors as

$$\mathbf{v}(t) = [v_1(t), v_2(t), \dots, v_n(t)]' \quad (1.1a)$$

$$\mathbf{i}(t) = [i_1(t), i_2(t), \dots, i_n(t)]' \quad (1.1b)$$

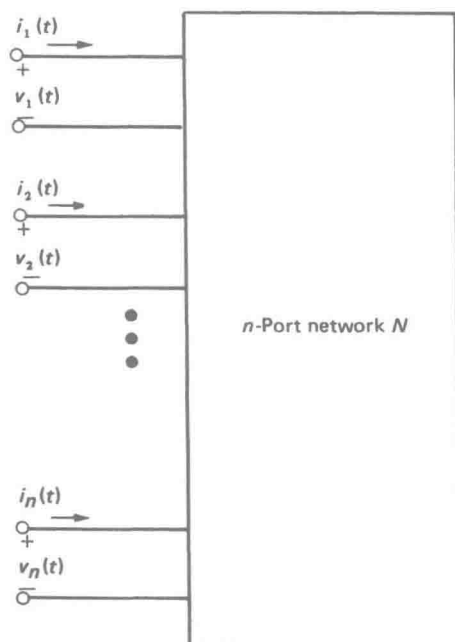


Figure 1.1 The general symbolic representation of an n -port network.

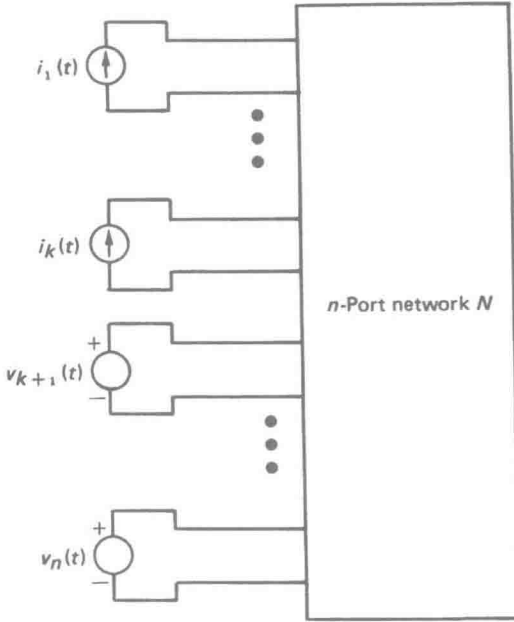


Figure 1.2 A specific input excitation of n -port network.

respectively, where the prime denotes the matrix transpose. There are $2n$ port signals, n port-voltage signals $v_k(t)$, and n port-current signals $i_k(t)$, and each port is associated with two signals $v_k(t)$ and $i_k(t)$. The port vectors $\mathbf{v}(t)$ and $\mathbf{i}(t)$ that can be supported by the n -port network N are said to constitute an *admissible signal pair* for the n -port network. Any n independent functions of these $2n$ port signals, taking one from each of the n ports, may be regarded as the *input* or *excitation* and the remaining n signals as the *output* or *response* of the n -port network. In Fig. 1.1 we may take, for example, $i_1(t), i_2(t), \dots, i_k(t), v_{k+1}(t), \dots, v_n(t)$ to be the input or excitation signals. Then $v_1(t), v_2(t), \dots, v_k(t), i_{k+1}(t), \dots, i_n(t)$ are the output or response signals. This input-output or excitation-response situation is shown in Fig. 1.2. To facilitate our discussion, let $\mathbf{u}(t)$ be the *excitation vector* associated with the excitation signals, and $\mathbf{y}(t)$ the *response vector* associated with the response signals. For the excitation-response situation of Fig. 1.2, the excitation and response vectors are given by

$$\mathbf{u}(t) = [i_1(t), i_2(t), \dots, i_k(t), v_{k+1}(t), \dots, v_n(t)]' \quad (1.2a)$$

$$\mathbf{y}(t) = [v_1(t), v_2(t), \dots, v_k(t), i_{k+1}(t), \dots, i_n(t)]' \quad (1.2b)$$