



Kim Williams
Michael J. Ostwald
Editors

Architecture and Mathematics from Antiquity to the Future

Volume II:
The 1500s to the Future

 Birkhäuser

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Iidabashi Subway Station, Makoto Sei Watanabe, architect (2000), entrance at street level.
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Preface to Architecture and Mathematics from Antiquity to the Future

In June of 1996, in his keynote address at the conference ‘Nexus’96: Relationships Between Architecture and Mathematics’, the founding international conference of what would become an international community for research in a new interdisciplinary field, eminent engineer Mario Salvadori asked, ‘[c]an there be any relationship between architecture and mathematics?’ Over the next 18 years, the Nexus community came together for a series of bi-yearly conferences in Italy, Portugal, Mexico, Turkey and the USA to examine, debate and celebrate the relationships that exist between architecture and mathematics. The conferences were hosted in locations where important historic connections had been proposed between architecture and mathematics: in Europe these locations include Fucecchio (1996), Mantua (1998), Ferrara (2000), Óbidos (2002), Genoa (2006), Porto (2010) and Milan (2012). Further afield, conferences were held in Mexico City (2004), San Diego (2008) and Ankara (2014). Conference venues were chosen to permit participants to visit local sites of historic importance for architecture and mathematics in post-conference workshops, such as Pompeii and Herculaneum in 1996, the villas of Palladio in 1998 and Teotihuacan in 2004. The speakers at these events include some of the most influential people in architecture, art, mathematics and engineering. Lionel March, Robert Tavenor, Alberto Pérez-Gómez, Marco Frascari, Michele Emmer, Leonard Eaton and Mario Salvadori, amongst many other luminaries, have all presented at the Nexus conferences and taken part in round-table discussions, forums and visits to some of the great architecture of these regions.

The first Nexus conference was actually conceived out of the frustration caused by the difficulty of finding a venue for publishing interdisciplinary research: papers in architecture and mathematics were seen as too mathematical for architectural journals, but not mathematical enough for mathematics journals. At best, such research was viewed as a curiosity, too far from the mainstream to garner much interest. Because there was no single journal that encouraged such research, when authors were fortunate enough to have an article accepted, publications were scattered, and authors seldom knew about the work of others examining similar topics. The Internet was in its infancy at that time, leaving far-flung scholars to

work in isolation. One journal, *The Mathematical Intelligencer*, and its particularly open-minded editor-in-chief, Chandler Davis, had accepted papers by three of the participants at the first conference, Kim Williams, Benno Artmann and Heinz Götze, who subsequently began to correspond. The 23 people who met in 1996 at the first conference knew of each other's work by word of mouth: friends sending their work to friends. But already by the second conference, 2 years later, the growing group felt the need for a publishing venue, and it was decided to found the *Nexus Network Journal*—*Nexus*, from the name chosen for the first conference to represent the idea of interweaving ideas from two disciplines, and *Network*, to describe the group of people whose acquaintances and collaborations were continuing to expand. The first issue of the journal, with Kim Williams as editor-in-chief, was introduced online in 1999, was added to at trimester intervals of the course of that year and was produced in print at its end. The journal continued in that way for its first 2 years, but by volume 3 in 2001, submissions had grown so much that it was published in two issues per year, until with volume 9 in 2010, it grew to three issues per year.

Across 15 volumes, 35 issues and over 500 refereed papers, the international reputation and impact of the journal have grown considerably. Now published jointly in the Birkhäuser programme of Springer-Basel and Kim Williams Books of Torino, Italy, the journal is highly respected and has a growing readership. Beginning with volume 16 in 2014, the *NNJ* will be overseen jointly by the editors of these present two volumes.

Foreseen along with the conferences was the publication of the proceedings. The series entitled 'Nexus: Architecture and Mathematics' comprised seven volumes from the first seven conferences. At the beginning, the conference books were seen as separate from the journal. This changed with the eighth conference, when speakers voiced the desire to see their papers published in the *NNJ*, which was by that time mature and esteemed. Thus, since 2010, papers presented in the Nexus conferences have been published in special issues of the journal and are available online. However, the research presented at the early conferences was only available in a series of limited edition books. With many of these being out of print there has been growing pressure to make the most highly cited works from the early years of the Nexus conferences available. Rather than simply republishing selected works in the order in which they were written, such was the scope of these early Nexus publications that an alternative proposition presented itself.

We, the editors, have assembled almost a hundred papers from the early years of the Nexus conferences, and arranged them both thematically and chronologically to trace key moments in the history and theory of architecture and mathematics, from antiquity to the present day, along with predictions for the future. These chapters describe over 60 major buildings and architectural works, consider more than twenty major theories of geometry and design and cover themes and ideas arising from five continents and spanning over four millennia.

Having said this, the present two-volume work does not pretend to be a comprehensive encyclopaedia of the history and theory of every facet of the relationship between architecture and mathematics. Being works by more than

one hundred authors with backgrounds in not only architecture and mathematics but also engineering, physics, chemistry, philosophy, music and more, there is a rich diversity of approaches to the topic, along with some insightful synergies and informative disagreements. All of the chapters have undergone minor editorial revisions, including, in some cases, updated bibliographies. In a few cases authors have chosen to make more substantial revisions, to bring their chapters up to date, or direct the reader to advances that are currently occurring in their areas. In addition to this, we have provided an overview chapter for each volume (Chap. 1 in vol. I and Chap. 48 in vol. II), to frame the sequence and structure of the whole as well as a chapter entitled 'Mathematics *in, of and for* Architecture: A Framework of Types' (Chap. 3) which seeks to classify, and thereby make more accessible, the myriad connections proposed across this work.

Each of the chapters in the present work have become crucial landmarks in the scholarly landscape of architecture and mathematics. Some represent pioneering research, the first studies of the relationships between architecture and mathematics in a specific period, or in the oeuvre of a given architect. They serve as both points of departure for new voyages of discovery and as destinations for people entering unfamiliar terrain. For the novice researcher these works provide a grounding for their explorations, and for seasoned scholars these chapters offer a critical record of the efforts of fellow travellers. We, the editors, hope that through this two-volume work, these chapters can continue to inspire and guide future generations.

We wish to thank Maria Roberts, Valentina Filemio and Marco Giorgio Bevilacqua for assistance with editing and proofing, and Michael Dawes for support with image preparation and research assistance. We thank all authors for permission to reuse their material, and for their help in updating texts and references. Finally, we thank Anna Mätzener, Editor for Mathematics and History of Science, and Thomas Hempfling, Executive Editor for Mathematics, Birkhäuser, for their support of the Nexus conferences and the *Nexus Network Journal* throughout the years, and especially for their support of this present work.

Torino, Italy
Newcastle, Australia
January 2015

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