

# UNIVERSALITY IN CHAOS

Predrag Cvitanović

SECOND EDITION



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# UNIVERSALITY IN CHAOS

Nature provides many examples of physical systems which are described by deterministic equations of motion, but which nevertheless exhibit non-predictable behaviour. The detailed description of turbulent motions remains perhaps the outstanding unsolved problem of classical physics. In recent years, however, a new theory has been formulated which succeeds in making quantitative predictions describing certain transitions to turbulence. Its significance lies in its possible application to large classes (often very dissimilar) of non-linear systems.

The first edition of *Universality in Chaos* has become a standard reference work. Since the book's publication in 1984, progress has continued to be made in our understanding of non-linear dynamical systems and chaos. This second edition extends the collection of articles to cover recent developments in the field, including the use of statistical mechanics techniques in the study of strange sets arising in dynamics. The book concentrates on the universal aspects of chaotic motions: those qualitative and quantitative predictions which apply to large classes of physical systems.

The second edition of *Universality in Chaos* will be an indispensable reference for researchers and graduate students interested in chaotic dynamics in the physical, biological and mathematical sciences, and engineering.



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Suite 300, Boca Raton, FL 33487  
711 Third Avenue  
New York, NY 10017  
2 Park Square, Milton Park  
Abingdon, Oxon OX14 4RN, UK

ISBN 978-1-138-42973-4 90000



9 781138 429734

[www.crcpress.com](http://www.crcpress.com)



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SECOND EDITION

## CVITANOVIC



# Universality in Chaos

Second Edition

*a reprint selection  
compiled and introduced by*

Predrag Cvitanović

Niels Bohr Institute  
Copenhagen

*The classification of the  
constituents of a chaos, nothing  
less is here essayed.*

Herman Melville  
*Moby Dick* chapter 32



**CRC Press**

Taylor & Francis Group

Boca Raton London New York

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CRC Press  
Taylor & Francis Group  
6000 Broken Sound Parkway NW, Suite 300  
Boca Raton, FL 33487-2742

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First issued in hardback 2017

No claim to original U.S. Government works

ISBN: 978-0-85274-260-0 (pbk)  
ISBN: 978-1-138-42973-4 (hbk)

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#### Library of Congress Cataloging-in-Publication Data

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Catalog record is available from the Library of Congress

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# Universality in Chaos

Second Edition



# CLASSICS *Illustrated*

FEATURING IDENTITIES BY THE  
WORLD'S FASTEST THEORISTS

# GRAF VON CHAOS

APPROVED  
BY THE  
CHAOTIC  
CODE  
(R)  
AUTHORITY

HOW STRANGELY  
TURBULENT

WOMEN: WEIBLICHE  
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ZIEHT UNS HINAN



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ASTOUNDING-  
HOLLYWOOD SPECTACULAR  
!!!

## Preface to the Second Edition

The first edition of this reprint selection has origin in those dim times in the past when there was no chaos in physics, or at least not in the minds of most physicists. The people who did this kind of thing were meteorologists, geometers, astronomers, biologists, plasma physicists, field theorists, ..., and there was a clear need for a reprint selection, as the seminal references were scattered far and wide. In those days it was still possible to get a fair overview of the infant field by reading leisurely a few fundamental articles. Already then I found it necessary to narrow the scope to the exciting new development of that time, the universal aspects of chaotic motions. Since then the chaos literature has turned into a veritable torrent, and no fair overview of the field can be attempted within the confines of a single volume. The articles added to the second edition (Part 8 in this selection) reflect to some extent the status of the universality theory today. A particularly fruitful recent development has been the reinjection of techniques of statistical mechanics into the description of dynamically generated strange sets (here represented by the article by Halsey *et al*). Though the methods are old and the original mathematics literature deeper than the recent physics reworking, what once seemed arcane mathematics has turned out to be a very useful tool for the description of observed strange sets, and winged expressions like ' $f$  of  $\alpha$ ' have by now become a part of our conceptual vocabulary.

P. Cvitanović  
Niels Bohr Institute  
Copenhagen  
May 1989





## Preface to the First Edition

This reprint selection presents some of the recent developments in the study of the chaotic behavior of deterministic systems. The problem, posed in its most general form, is old and appears under many guises: Why are clouds the way they are? Is the solar system stable? What determines the structure of turbulence in liquids, the noise in electronic circuits, the stability of plasma in a tokamak? The subject, defined so broadly, could not possibly be covered in a single reprint selection. This selection concentrates on the universal aspects of chaotic motions: those qualitative and quantitative predictions which apply to large classes of (often very different) physical systems. The selection can be divided into roughly four parts. The first part offers a general introduction to deterministic chaos and universality. The second part presents some of the experimental evidence for universality in transitions to turbulence. The third part concentrates on the theoretical investigations of the universality ideas, and the last part gives a glimpse of the further developments stimulated by the success of the one-dimensional universality theory.

This selection originates from a NORDITA reprint selection prepared together with Mogens Høgh Jensen in the fall of 1981. I am grateful to Mogens and to the NORDITA staff, in particular Nils Robert Nilsson, for their help with this project. I thank Harry L Swinney, J Doyne Farmer, David Ruelle, Albert Libchaber, Yves Pomeau, Robert H G Helleman, David Rand, Robert MacKay and Stellan Ostlund for their suggestions and criticisms. And last, but not least, I thank Mitchell J Feigenbaum for teaching me almost all that I know about universality in chaos, and all that I know about Schubert.

P. Cvitanović  
NORDITA  
August 1983

Note: The reference list for articles referred to in the introduction and the comments to reprint selections is placed at the end of this volume.





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