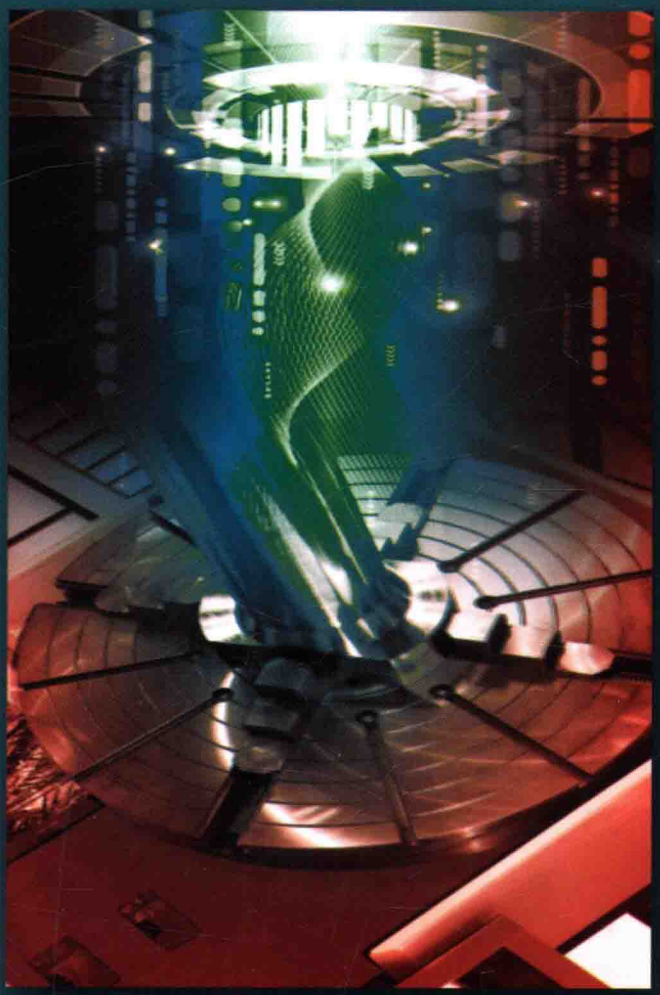
Cartesian Tensors and the Equations of Solid Mechanics

K.B.M. Nambudiripad



Alpha Science

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Cartesian Tensors and the Equations of Solid Mechanics introduces the mechanics of solids at an intermediate level using cartesian tensors. The emphasis is on the physical signicance and understanding behind the tensor formalism. Included in the book are a brief introduction to general tensors, anisotropy, some elementary concepts revisited, and a short early huistory of the mechanics of solids.

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ISBN 978-1-78332-306-7

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Oxford, U.K.

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472 pgs.

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Garsington Road, Oxford OX4 2JZ, U.K.

www.alphasci.com

ISBN 978-1-78332-306-7

Printed from the camera-ready copy provided by the Editor.

Cartesian Tensors and the Equations of Solid Mechanics

Dedicated to the Memory of my Parents and Teachers

Preface

This is a book on cartesian tensors and the equations of solid mechanics as the title indicates. This is meant for those who desire to study something beyond what is traditionally taught in the first (elementary) course on the mechanics of solids. Unfortunately, books on advanced mechanics of solids and the theory of elasticity are generally difficult for engineering students to read. Those on tensor analysis are even more so. With the emergence of more and more engineering colleges, it has become really difficult to find faculty members to teach these crucial courses in the area of mechanics. This difficulty has aggravated in recent years when engineering colleges and universities vie with one another to dilute the mathematical-analytical treatment. More and more non-content courses are introduced which, in my opinion, make engineers professionally disabled. Unless at least a few engineering science courses are taught rigorously, students will never develop the ability to learn whatever they might need in their later professional career. It is the strength of this academic conviction that is the driving force behind writing this book.

This book has the limited objective of being helpful to the young readers by supplying, using cartesian tensors, some material of intermediate difficulty, somewhat half way between the simplified engineers' theory and the more difficult mathematical theory of elasticity. Tensor analysis is notorious; students invariably find it too abstract and difficult. I have tried to lessen their burden by discussing the physical facts in the engineers' style in engineers' language before the results are presented formally in the form of mathematics. I hope that the unfriendly symbols of the tensor formalism will not scare away the young prospective readers.

The book begins with an introduction. After this, the index notation is presented. This is not difficult, although it can be confusing for beginners. A certain amount of practice seems to be necessary before one becomes comfortable with the index notation. The heart and soul of tensors is the coordinate transformation and the corresponding induced transformation. This is discussed quite early in the book. As simple examples are considered, it should not be difficult to understand all this.

At this stage a fairly detailed introduction to the mechanics of solids is given in this book. This material is at an elementary level, but it is given in the spirit of a revisit. The fundamentals do not seem to be emphasised sufficiently strongly in many, perhaps most, engineering schools. In several places the material is still delivered in the outdated semi-empirical style of strength of materials. Now there is a clear need to recast the material in a more scientific setting. The edifice of the mechanics of solids is built on three clearly demarcated pillars: the equations of equilibrium, the strain-displacement relations, and the constitutive equations (generalised Hooke's law).

This introduction is followed by a detailed discussion of the state of stress at a point. This is probably the most important chapter of the book. If only one chapter can be taught, it should surely be this. This material is to be grasped, understood and well

assimilated. It takes time to absorb the ideas; forcing the pace impedes the process of digestion. Once this is accomplished, tensors will not then be conceptually difficult to understand.

Cartesian tensors and tensor operations are next presented before some results and operations are shown using the index notation. Having by now absorbed the physical ideas, readers should have no further difficulty in following the index notation. Other important topics like the differential equations of equilibrium and geometrical visualisation are presented next. It is perhaps true that graphical methods have lost the importance that they once had; nobody uses graphical constructions any more to obtain numerical answers. But this is not to imply that geometrical visualisation is no more important. Visual support is highly beneficial. If, for example, the simple Mohr's circle is understood, most of the qualitative aspects of the states of stress and strain at a point become indelibly clear.

The analysis of deformation and strain is taken up next. Here this author is in a dilemma. On the one hand, continuum mechanics with Almansi and Piola-Kirchhoff tensors, deformation gradient and polar decomposition theorem, etc. is too difficult and abstract. On the other, "change in length divided by the original length" is ridiculously easy and insultingly simple, particularly in a second look at the theory of deformations. Several approaches and difficulties are pointed out, but finally only the simple case is treated in some detail. This is followed by a chapter on the constitutive equations.

All this leads up to a discussion of the field equations of elasticity. It should be understood that this is not a book on the theory of elasticity. Stress analysis problems are not solved (except a few simple illustrative examples).

Given almost at the end of the book is a collection of topics that could not be discussed, but are important. A 44-page introduction to general tensor analysis is attempted here. Also included is orthotropy which has become important in recent years because composites are used more and more. The next and last chapter is devoted to discussing the early history of our subject and a few great scientists.

There are some repetitions here and there. But Sanskrit aestheticians have emphasised that repetition is not only permissible, but is even desirable in teaching and lovemaking!

When we use the index notation, sometimes called the tensor notation, dots and commas can be a distraction, because they can be misunderstood as part of the notation. I have, therefore, taken some liberties with the rules of punctuation of Her Majesty's Queen's English! The absence of commas and full-stops immediately following mathematical equations in index notation should cause no confusion. In most cases, these lapses will hardly be noticed even.

Abstract analytical studies like this are fine as far as they go. However, for the solution of practical problems, these must be supplemented by experimental and numerical methods. Thus, experimental stress analysis and finite element methods are essential sup-

plements to what we shall see in this book. In addition to all such 'bookish' studies, engineers must have some 'field experience'. Without exposure to 'practical' aspects, any study however deep will still be seriously deficient. As these are separate disciplines in their own right, it is not possible even to discuss them briefly in this book.

I am fortunate to have had several outstanding teachers. It is to them that I owe the most. I hope these venerable departed souls, now in their heavenly abode, will approve of what I have ventured to write sometimes with trepidation. Among them it was Professor C.N. Lakshminarayana, a most effective teacher at Indian Institute of Technology Kharagpur, the only IIT at that time, who taught us Advanced Strength of Materials, Theory of Elasticity, and Experimental Stress Analysis. How can I thank him adequately? I thank Dr Gangan Prathap, Dr Somenath Mukherjee and Professor K.N. Ramachandran (my colleague and friend of long standing) for reading some part of the manuscript and giving me helpful suggestions. Professor P.A. Krishnan of NIT Trichy read part of the manuscript and pointed out several mistakes and misprints. I am surprised how he could quickly pick the errors in the subscripts and superscripts in small letters! I express my gratitude to him. I am grateful to the Vidya family. I consider it a privilege to work in Vidya Academy of Science & Technology even at this age, years after my formal retirement from NIT Calicut and later from Amrita. Dr Sudha Balagopalan (Principal & Professor), Professor K.V. Leela, Professor V.N. Krishnachandran, Dr Sooraj K. Prabha, Dr N.K. Sudev, and others (all my colleagues at Vidya); Er P.K. Asokan (Chairman) and Er G. Mohanachandran (Executive Director) at Vidya gave me a lot of support and encouragement. I thank them all most sincerely. I thank Mr Sreerag Srinivasan of Vidya for cheerfully drawing nearly all the figures. And finally I thank M/s Narosa Publishing House Pvt. Ltd, New Delhi and, in particular, Mr N.K. Mehra, Publisher and Managing Director.

It is desirable for young readers to check if their studies are effective, proper and along the royal highway. There is an unmistakable diagnostic tool. Do we enjoy learning? Do we have the thrill, excitement and joy when we learn something new, or when we get a new insight that improves our conceptual understanding? If this book helps the young readers and initiates them into the vast area of engineering mechanics, I shall feel amply rewarded for the labour of writing this book.

As always, critical comments and suggestions are most welcome. They will certainly be acknowledged gratefully.

Calicut, Kerala
Thiruvonam

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