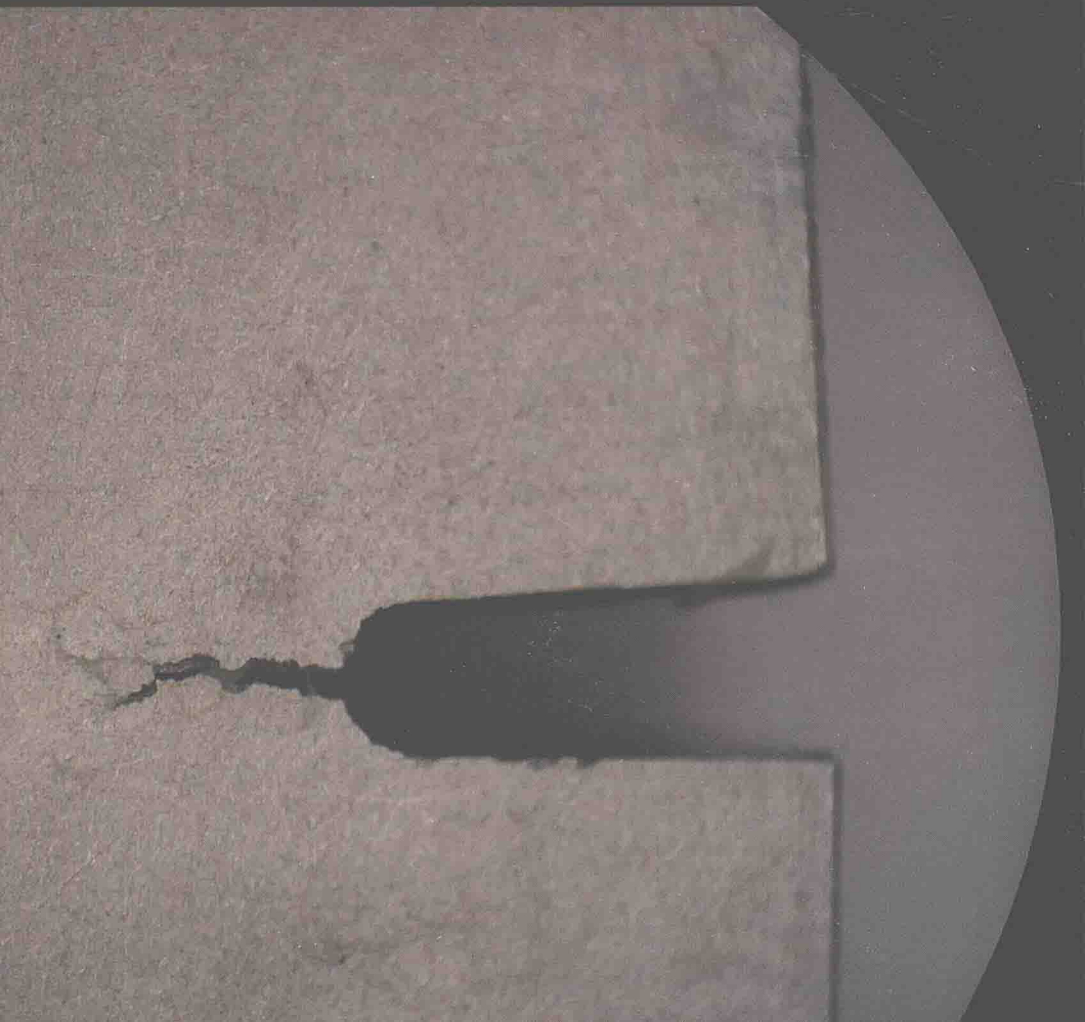
Fracture Mechanics

Arthur Bloomberg



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Edited by **Arthur Bloomberg**

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Preface

In my initial years as a student, I used to run to the library at every possible instance to grab a book and learn something new. Books were my primary source of knowledge and I would not have come such a long way without all that I learnt from them. Thus, when I was approached to edit this book; I became understandably nostalgic. It was an absolute honor to be considered worthy of guiding the current generation as well as those to come. I put all my knowledge and hard work into making this book most beneficial for its readers.

Fracture mechanics is the field of mechanics related to the study of the propagation of cracks in materials. This book provides a collection of researches regarding the functions of fracture mechanics procedures in materials science, medicine and engineering. Major topics discussed in this book include the strength of biological tissues, safety of nuclear reactor components, effects of fatigue in pipelines, environmental effects on fracture etc. The book also discusses mathematical and computational steps underlying fracture mechanics functions, and the advancements in statistical modeling of fatigue. It will be most beneficial to mechanical and civil engineers, and material scientists from industry, research and education.

I wish to thank my publisher for supporting me at every step. I would also like to thank all the authors who have contributed their researches in this book. I hope this book will be a valuable contribution to the progress of the field.

Editor

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Lucas Máximo Alves

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1. Introduction

A wide variety of natural objects can be described mathematically using fractal geometry as, for example, contours of clouds, coastlines , turbulence in fluids, fracture surfaces, or rugged surfaces in contact, rocks, and so on. None of them is a real fractal, fractal characteristics disappear if an object is viewed at a scale sufficiently small. However, for a wide range of scales the natural objects look very much like fractals, in which case they can be considered fractal. There are no true fractals in nature and there are no real straight lines or circles too. Clearly, fractal models are better approximations of real objects that are straight lines or circles. If the classical Euclidean geometry is considered as a first approximation to irregular lines, planes and volumes, apparently flat on natural objects the fractal geometry is a more rigorous level of approximation. Fractal geometry provides a new scientific way of thinking about natural phenomena. According to Mandelbrot [1], a fractal is a set whose fractional dimension (Hausdorff-Besicovitch dimension) is strictly greater than its topological dimension (Euclidean dimension).

In the phenomenon of fracture, by monotonic loading test or impact on a piece of metal, ceramic, or polymer, as the chemical bonds between the atoms of the material are broken, it produces two complementary fracture surfaces. Due to the irregular crystalline arrangement of these materials the fracture surfaces can also be irregular, i.e., rough and difficult geometrical description. The roughness that they have is directly related to the material microstructure that are formed. Thus, the various microstructural features of a material (metal, ceramic, or polymer) which may be, particles, inclusions, precipitates, etc. affect the topography of the fracture surface, since the different types of defects present in a material can act as stress concentrators and influence the formation of fracture surface. These various microstructural defects interact with the crack tip, while it moves within the material, forming a totally irregular relief as chemical bonds are broken, allowing the microstructure to be separated from grains (transgranular and intergranular fracture) and microvoids are joining (coalescence of microvoids, etc..) until the fracture surfaces depart. Moreover, the characteristics of macrostructures such as the size and shape of the sample and notch from which the fracture is initiated, also influence the formation of the fracture surface, due to the type of test and the stress field applied to the specimen.

After the above considerations, one can say with certainty that the information in the fracture process are partly recorded in the "story" that describes the crack, as it walks inside the material [2]. The remainder of this information is lost to the external environment in a form of dissipated energy such as sound, heat, radiation, etc. [30, 31]. The remaining part of the information is undoubtedly related to the relief of the fracture surface that somehow describes the difficulty that the crack found to grow [2]. With this, you can analyze the fracture phenomenon through the relief described by the fracture surface and try to relate it to the magnitudes of fracture mechanics [3 , 4 , 5 , 6 , 7, 8, 9 - 11, 12, 13]. This was the basic idea that brought about the development of the topographic study of the fracture surface called fractography.

In fractography anterior the fractal theory the description of geometric structures found on a fracture surface was limited to regular polyhedra-connected to each other and randomly distributed throughout fracture surface, as a way of describing the topography of the irregular surface. Moreover, the study fractographic hitherto used only techniques and statistical analysis profilometric relief without considering the geometric auto-correlation of surfaces associated with the fractal exponents that characterize the roughness of the fracture surface.

The basic concepts of fractal theory developed by Mandelbrot [1] and other scientists, have been used in the description of irregular structures, such as fracture surfaces and crack [14], in order to relate the geometrical description of these objects with the materials properties [15].

The fractal theory, from the viewpoint of physical, involves the study of irregular structures which have the property of invariance by scale transformation, this property in which the parts of a structure are similar to the whole in successive ranges of view (magnification or reduction) in all directions or at least one direction (self-similarity or self-affinity, respectively) [36]. The nature of these intriguing properties in existing structures, which extend in several scales of magnification is the subject of much research in several phenomena in nature and in materials science [16 , 17 and others]. Thus, the fractal theory has many contexts, both in physics and in mathematics such as chaos theory [18], the study of phase transitions and critical phenomena [19, 20, 21], study of particle agglomeration [22], etc.. The context that is more directly related to Fracture Mechanics, because of the physical nature of the process is with respect to fractal growth [23, 24, 25, 26]. In this subarea are studied the growth mechanisms of structures that arise in cases of instability, and dissipation of energy, such as crack [27, 28] and branching patterns [29]. In this sense, is to be sought to approach the problem of propagation of cracks.

The fractal theory becomes increasingly present in the description of phenomena that have a measurable disorder, called deterministic chaos [18, 27, 28]. The phenomenon of fracture and crack propagation, while being statistically shows that some rules or laws are obeyed, and every day become more clear or obvious, by understanding the properties of fractals [27, 28].

2. Fundamental geometric elements and measure theory on fractal geometry

In this part will be presented the development of basic concepts of fractal geometry, analogous to Euclidean geometry for the basic elements such as points, lines, surfaces and fractals volumes. It will be introduce the measurement fractal theory as a generalization of Euclidean measure geometric theory. It will be also describe what are the main mathematical conditions to obtain a measure with fractal precision.

2.1. Analogy between euclidean and fractal geometry

It is possible to draw a parallel between Euclidean and fractal geometry showing some examples of self-similar fractals projected onto Euclidean dimensions and some self-affine fractals. For, just as in Euclidean geometry, one has the elements of geometric construction, in the fractal geometry. In the fractal geometry one can find similar objects to these Euclidean elements. The different types of fractals that exist are outlined in Figure 1 to Figure 4.

2.1.1. Fractais between $0 \leq D \leq 1$ (similar to point)

An example of a fractal immersed in Euclidean dimension $I = d + 1 = 1$ with projection in $d = 0$, similar to punctiform geometry, can be exemplified by the Figure 1.

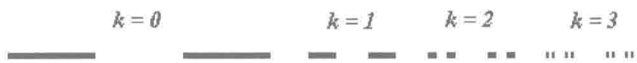

Figure 1. Fractal immersed in the one-dimensional space where $D \cong 0,631$.

This fractal has dimension $D \cong 0,631$. This is a fractal-type "stains on the floor." Other fractal of this type can be observed when a material is sprayed onto a surface. In this case the global dimension of the spots may be of some value between $0 \leq D \leq 1$.

2.1.2. Fractais between $1 \leq D \leq 2$ (similar to straight lines)

For a fractal immersed in a Euclidean dimension $I = d + 1 = 2$, with projectionin $d = 1$, analogous to the linear geometry is a fractal-type peaks and valleys (Figure 2). Cracks may also be described from this figure as shown in Alves [37]. Graphs of noise, are also examples of linear fractal structures whose dimension is between $1 \leq D \leq 2$.

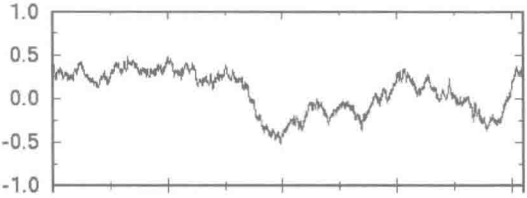

Figure 2. Fractal immersed in dimension $d = 2$. rugged fractal line.

2.1.3. *Fractals between* $2 \leq D \leq 3$ *(similar to surfaces or porous volumes)*

For a fractal immersed in a Euclidean dimension, $I = d + 1 = 3$ with projection in $d = 2$, analogous to a surface geometry is fractal-type "mountains" or "rugged surfaces" (Figure 3). The fracture surfaces can be included in this class of fractals.

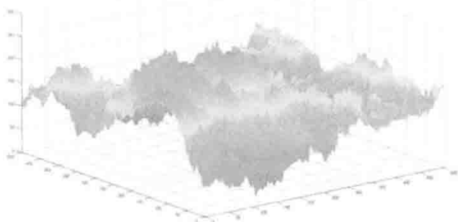

Figure 3. Irregular or rugged surface that has a fractal scaling with dimension D between $2 \leq D \leq 3$.

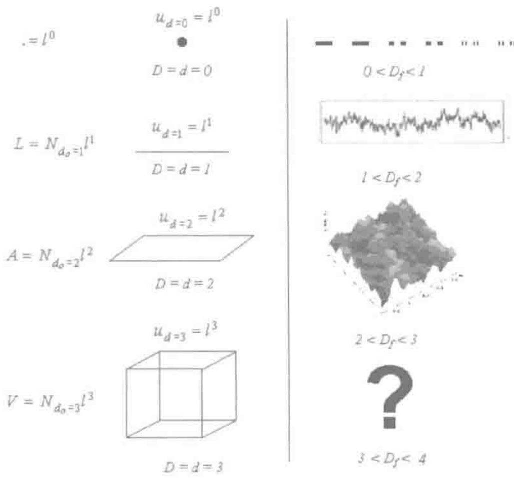

Figure 4. Comparison between Euclidean and fractal geometry. D, d and D_f represents the topological, Euclidean and fractal dimensions, of a point, line segment, flat surface, and a cube, respectively

Making a parallel comparison of different situations that has been previously described, one has (Figure 4)

2.2. Fractal dimension (non-integer)

An object has a fractal dimension, $D,(d \leq D \leq d+1=I)$, where I is the space Euclidean dimension which is immersed, when:

$$F(\varepsilon L_0)=\varepsilon^{-D}F(L_0) \tag{1}$$

where L_0 is the projected length that characterizes an apparent linear extension of the fractal ε, is the scale transformation factor between two apparent linear extension, $F(L_0)$ is a function of measurable physical properties such as length, surface area, roughness, volume, etc., which follow the scaling laws, with homogeneity exponent is not always integers, whose geometry that best describe, is closer to fractal geometry than Euclidean geometry. These functions depend on the dimensionality, I, of the space which the object is immersed. Therefore, for fractals the homogeneity degree n is the fractal dimension D (non-integer) of the object, where ε is an arbitrary scale.

Based on this definition of fractal dimension it can be calculates doing:

$$\varepsilon^{-D}=\frac{F(\varepsilon L_o)}{F(L_o)} \tag{2}$$

taking the logarithm one has

$$D=-\frac{\ln\left[\frac{F(\varepsilon L_o)}{F(L_o)}\right]}{\ln(\varepsilon)} \tag{3}$$

From the geometrical viewpoint, a fractal must be immersed into a integer Euclidean dimension, $I=d+1$. Its non-integer fractal dimension, D, it appears because the fill rule of the figure from the fractal seed which obeys some failure or excess rules, so that the complementary structure of the fractal seed formed by the voids of the figure, is also a fractal.

For a fractal the space fraction filled with points is also invariant by scale transformation, i.e.:

$$P(L_o)=\frac{F(\lambda L_o)}{F(L_o)}=\frac{1}{N(L_o)} \tag{4}$$

Thus,

$$\varepsilon^{D}=P(L_0) \text{ ou } N(L_0)=\varepsilon^{-D} \tag{5}$$

where $P(L_0)$ is a probability measure to find points within fractal object

Therefore, the fractal dimension can be calculated from the fllowing equation:

$$D = -\frac{\ln N(L_0)}{\ln \varepsilon} \tag{6}$$

If it is interesting to scale the holes of a fractal object (the complement of a fractal), it is observed that the fractal dimension of this new additional dimension corresponds to the Euclidean space in which it is immersed less the fractal dimension of the original.

2.3. A generalized monofractal geometric measure

Now will be described how to process a general geometric measure whose dimension is any. Similarly to the case of Euclidean measure the measurement process is generalized, using the concept of Hausdorff-Besicovitch dimension as follows.

Suppose a geometric object is recovered by α -dimensional, geometric units, u_D, with extension, δ_k and $\delta_k \leq \delta$, where δ is the maximum α -dimensional unit size and α is a positive real number. Defining the quantity:

$$M_D(\alpha, \delta, \{\delta_k\}) = \sum_k \delta_k{}^\alpha \tag{7}$$

Choosing from all the sets $\{\delta_k\}$, that reduces this summation, such that:

$$M_D(\alpha, \delta) = \inf_{\{\delta_k\}} \sum_k \delta_k{}^\alpha \tag{8}$$

The smallest possible value of the summation in (8) is calculated to obtain the adjustment with best precision of the measurement performed. Finally taking the limit of δ tending to zero, $(\delta \to 0)$, one has:

$$M_D(\alpha) = \lim_{\delta \to 0} M_D(\alpha, \delta) \tag{9}$$

The interpretation for the function $M_D(\alpha)$ is analogous to the function for a Euclidean measure of an object, i.e. it corresponds to the geometric extension (length, area, volume, etc.) of the set measured by units with dimension, α. The cases where the dimension is integer are same to the usual definition, and are easier to visualize. For example, the calculation of $M_D(\alpha)$ for a surface of finite dimension, $D = 2$, there are the cases:

- For $\alpha = 1 < D = 2$ measuring the "length" of a plan with small line segments, one gets $M_D = \infty$, because the plan has a infinity "length", or there is a infinity number of line segment inside the plane.
- For, $\alpha = 2 = D = 2$ measuring the surface area of small square, one gets $M_D = A_{d=2} = A_0$. Which is the only value of α where M_D is not zero nor infinity (see Figure 5.)
- For $\alpha = 3 > D = 2$ measuring the "volume" of the plan with small cubes, one gets $M_D = 0$, because the "volume" of the plan is zero, or there is not any volume inside the plan.

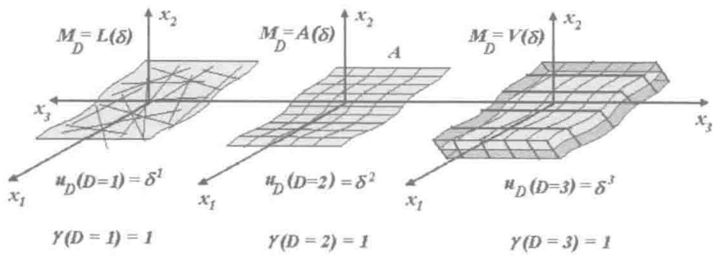

Figure 5. Measuring, $M_D(\delta)$ of an area A with a dimension, $D=2$ made with different measure units u_D for $D=1,2,3$.

Therefore, the function, M_D possess the following form

$$M_D(\alpha)=\begin{cases}0 \; para \; \alpha > D \\ M \; para \; \alpha = D \\ \infty \; para \; \alpha < D\end{cases}. \tag{10}$$

That is, the function M_D only possess a different value of 0 and ∞ at a critical point $\alpha = D$ defining a generalized measure

2.4. Invariance condition of a monofractal geometric measure

Therefore, for a generalized measurement there is a generalized dimension which the measurement unit converge to the determined value, M, of the measurement series, according to the extension of the measuring unit tends to zero, as shown in equations equações (9) and (10), namely:

$$M_D\left(\alpha,\delta,\{\delta_k\}\right)=\sum_k \delta_k^{\ \alpha}=M_{Do}\left(\delta\right)\varepsilon^{\alpha-D} \tag{11}$$

where $M_{D0}(\delta)$ is the Euclidean projected extension of the fractal object measured on α - dimensional space

Again the value of a fractal measure can be obtain as the result of a series.

One may label each of the stages of construction of the function $M_D(\delta)$ as follows:

i. the first is the *measure itself*. Because it is actually the step that evaluates the extension of the set, summing the geometrical size of the recover units. Thus, the extension of the set is being overestimated, because it is always less or equal than tthe size of its coverage.
ii. The next step is the *optimization* to select the arrangement of units which provide the smallest value measured previously, i.e. the value which best approximates the real extension of the assembly.
iii. The last step is the *limit*. Repeat the previous steps with smaller and smaller units to take into account all the details, however small, the structure of the set.

As the value of the generalized dimension is defined as a critical function, $M_{\alpha=D}(\delta)$ it can be concluded, wrongly, that the optimization step is not very important, because the fact of not having all its length measured accurately should not affect the value of critical point. The optimization step, this definition, serves to make the convergence to go faster in following step, that the mathematical point of view is a very desirable property when it comes to numerical calculation algorithms.

2.5. The monofractal measure and the Hausdorff-Besicovitch dimension

In this part we will define the dimension-Hausdorf Besicovicth and a fractal object itself. The basic properties of objects with "anomalous" dimensions (different from Euclidean) were observed and investigated at the beginning of this century, mainly by Hausdorff and Besicovitch [32,34]. The importance of fractals to physics and many other fields of knowledge has been pointed out by Mandelbrot [1]. He demonstrated the richness of fractal geometry, and also important results presented in his books on the subject [1, 35, 36].

The geometric sequence, S is given by:

$$S=\sum_{k} S_k \quad onde\ k=0,1,2,.... \tag{12}$$

represented in Euclidean space, is a fractal when the measure of its geometric extension, given by the series, $M_{\alpha}(\delta_k)$ satisfies the following Hausdorf-Besicovitch condition:

$$M_d(\delta_k)=\sum_{k}\gamma(d)\delta_k^{\alpha}=N_d(\delta_k)\gamma(d)\delta_k^{\alpha}\underset{\delta_k\to 0}{}\begin{cases}0; & \alpha>D\\ M_D; & \alpha=D\\ \infty; & \alpha<D\end{cases}, \tag{13}$$

where:

$\gamma(d)$ is the geometric factor of the unitary elements (or seed) of the sequence represented geometrically.

δ : is the size of unit elements (or seed), used as a measure standard unit of the extent of the spatial representation of the geometric sequence.

$N(\delta)$: is the number of elementary units (or seeds) that form the spatial representation of the sequence at a certain scale

α : the generalized dimension of unitary elements

D : is the Hausdorff-Besicovitch dimension.

2.6. Fractal mathematical definition and associated dimensions

Therefore, fractal is any object that has a non-integer dimension that exceeds the topological dimension ($D<I$, where I is the dimension of Euclidean space which is immersed) with some invariance by scale transformation (self-similarity or self-affinity), where for any continuous contour that is taken as close as possible to the object, the number of points N_D, forming the fractal not fills completely the space delimited by the contour, i.e., there is

always empty, or excess regions, and also there is always a figure with integer dimension, I , at which the fractal can be inscribed and that not exactly superimposed on fractal even in the limit of scale infinitesimal. Therefore, the fraction of points that fills the fractal regarding its Euclidean coverage is different of a integer. As seen in previous sections - 2.2 - 2.5 in algebraic language, a fractal is a invariant sequence by scale transformation that has a Hausdorff-Besicovitch dimension.

According to the previous section, it is said that an object is fractal, when the respective magnitudes characterizing features as perimeter, area or volume, are homogeneous functions with non-integer. In this case, the invariance property by scaling transformation (self-similar or self-affinity) is due to a scale transformation of at least one of these functions.

The fractal concept is closely associated to the concept of Hausdorff-Besicovitch dimension, so that one of the first definitions of fractal created by Mandelbrot [36] was:

"Fractal by definition is a set to which the Haussdorf-Besicovitch dimension exceeds strictly the topological dimension".

One can therefore say that fractals are geometrical objects that have structures in all scales of magnification, commonly with some similarity between them. They are objects whose usual definition of Euclidean dimension is incomplete, requiring a more suitable to their context as they have just seen. This is exactly the Hausdorff-Besicovitch dimension.

A dimension object, D, is always immersed in a space of minimal dimension $I = d+1$, which may present an excessive extension on the dimension d, or a lack of extension or failures in one dimension $d+1$. For example, for a crack which the fractal dimension is the dimension in the range of $1 \le D \le 2$ the immersion dimension is the dimension $I = 2$ in the case of a fracture surface of which the fractal dimension is in the range $2 \le D \le 3$ the immersion dimension is the $I = 3$. When an object has a geometric extension such as completely fill a Euclidean dimension regular, d, and still have an excess that partially fills a superior dimension $I = d+1$, in addition to the inferior dimension, one says that the object has a dimension in excess, d_e given by $d_e = D-d$ where D is the dimension of the object. For example, for a crack which the fractal dimension is in the range $1 \le D \le 2$ the excess dimension is $d_e = D-1$, in the case of a fracture surface of which the fractal dimension is in the range of $2 \le D \le 3$ the excess dimension is $d_e = D-2$. If on the other hand an object partially fills a Euclidean regular dimension, $I = d+1$ certainly this object fills fully a Euclidean regular dimension, d, so that it is said that this object has a lack dimension $d_{fl} = I - D = d+1-D$, where $d_e = 1 - d_{fl}$. For example, for a crack which the fractal dimension is the range of $1 \le D \le 2$ the lack dimension is $d_{fl} = 2 - D$. In the case of a fracture surface of which the fractal dimension is the range of $2 \le D \le 3$ the lack dimension is $d_{fl} = 3 - D$.

2.7. Classes and types of fractals

One of the most fascinating aspects of the fractals is the extremely rich variety of possible realizations of such geometric objects. This fact gives rise to the question of classification,