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To the memory of my beloved mother,
Touraandokht Khoshravaan,
and
my dear brother, Mahdaad

Preface

A helicopter, with its capability of vertical take-off and landing, is a crucial means of aerial transportation. In fire-fighting rescue operations and missions for helping survivors of an earthquake or an avalanche, helicopters have played vital roles. The expansion of the domain of the application of helicopters, however, faces a few serious constraints; among them is the relatively poor ride quality due to severe vibration and noise. Vibration can reduce the fatigue life of structural components and hence increase the operating costs. Furthermore, environmental consequences of noise and vibration have limited the range of application and the velocity of helicopters. That is why reducing noise and vibration is a major goal in the design of helicopters.

Smart materials are good candidates for providing a way to control noise and vibrations in helicopters. Embedded strain sensing and actuation in active structures can be used to reduce blade vibration, minimize blade vortex interaction, decrease noise, and improve stability and response characteristics of the helicopter (Traugott et al. 2005).

Reliable and economically viable design of structures and machine elements is impossible without the use of accurate and efficient methods of structural analysis. Such methods should be capable of analyzing real-world problems that involve different types of materials. While isotropic materials behave identically when loaded in different directions, anisotropic materials are direction-dependent. Fiber-reinforced composites are among the latter type of materials and by proper orientation of fibers with respect to the direction of loading, they can provide higher values of strength-to-weight ratio compared to conventional isotropic materials.

Materials may also be classified as passive or active. The usual characteristic of active materials is that they deform in response to electrical stimuli. The conversion of electrical input to mechanical output corresponds to the actuator mode of operation and the resulting deformation is used as mechanical excitation. Conversely, active materials may generate electrical signals when they are subjected to mechanical loading and deformation. This is the sensor mode of operation. By embedding such sensors and actuators in structures such as helicopter rotor blades, the two modes of operation (i.e., sensing and actuating) are combined. A control strategy then uses the sensor output, processes it, and provides necessary input to the actuators in order to minimize harmful effects such as noise and vibration.

In the linear range of response, small deformations are linearly related to the imposed loads on a structure; so, doubling a load results in doubling deformation. However, beyond a certain level of deformation, the linear relationship between loading and deformation ends and transforms to a geometrically nonlinear relation. In this nonlinear region, the superposition principle is no longer valid. Therefore, many of the conventional methods that are used for solving differential equations, such as splitting the general solution of a non-homogeneous equation into a homogeneous (natural or transient) and a particular (forced or steady-state) part, are no longer applicable. Solving such problems requires the use of alternative methods such as the perturbation methods.

Various methods have been used for analyzing the mechanical behavior of structures. Among them, the finite element method (FEM) has been successful in solving problems with complicated geometry and without the need to accept many simplifying assumptions. Application of the FEM, however, requires modeling of the whole structure and calculation of large stiffness and inertia matrices.

An alternative solution technique is the variational asymptotic method (VAM). This method splits the solution of the three-dimensional (3-D) problem into two major parts. The first is a two-dimensional (2-D) analysis that develops the cross-sectional stiffness and inertia matrices as well as the warping functions. These results can then be used in a 3-D simulation of structures without the need to repeat the 2-D analysis. The second is a geometrically nonlinear one-dimensional (1-D) analysis of the beam-like structure along its longitudinal direction. Combining these two solutions provides the complete 3-D response of the structure. Since VAM eliminates the need to recalculate the cross-sectional properties, it is a more efficient solution method compared to the 3-D FEM.

Using VAM and the corresponding cross-sectional and 1-D solutions, this book covers the elastic response of isotropic and composite beams and rotor blades in geometrically linear and nonlinear statics, as well as nonlinear dynamics situations. The effects of aerodynamic loading, damping, and embedded actuators are also discussed.

This book is intended as a thorough study of nonlinear elasticity of slender beams and is targeted to researchers, graduate students, and practicing engineers in the fields of structural dynamics, aerospace structures, and mechanical engineering. It broadens readers' understanding of the nonlinear static and dynamic response of composite beams, required in many applications such as helicopter rotor blades and wind turbines, through comprehensive and step-by-step analysis. It provides graduated analyses of phenomena beginning with the fundamental (static, linear, isotropic, passive, and clamped) progressing through the complex (dynamic, nonlinear, composite, with actuators, and articulated), and it models both clamped and hinged rotating beams and blades as well as analyzing beams and blades with embedded active fiber composites.

The presented static solution can be used independently or to provide the initial conditions that are needed for performing a dynamic analysis. The considered dynamic problems include the analysis of accelerating clamped (hingeless) and articulated (hinged) rotating blades. Independent numerical solutions for the transient and the steady-state responses are presented, and as a verification, it is illustrated that the transient solution converges to the steady-state solution obtained by the shooting method. Other key topics include calculating the effect of perturbing the steady-state solution, coupled nonlinear flap-lag dynamics of a rotating articulated blade with hinge offset and aerodynamic damping, and static and nonlinear dynamic responses of composite beams with embedded piezocomposite actuators. The results obtained in each section are verified or justified.

The book starts with an introduction in Chap. 1, which is then followed in Chap. 2 by a review of the VAM and the equations of motion. These equations apply to beams made of arbitrary materials and cross sections. In the rest of the book, the equations of motion are used for solving a set of progressively complex problems involving the dynamics of rotating blades.

Chapter 3 is dedicated to the linear static analysis of isotropic and composite beams, and it is followed by Chap. 4 that presents the nonlinear static analysis of such structures. In Chap. 4, foreshortening which is an inherently nonlinear phenomenon is used for the verification of the results.

Chapter 5 is on the transient nonlinear dynamics of a clamped (hingeless) blade that rotates at variable speed. The rotor blade starts its motion from rest and after an acceleration interval converges to a steady-state condition. In order to solve this problem, an explicit (direct) integration algorithm is developed that utilizes the finite difference and the perturbation methods. A computer program that uses this algorithm solves the transient form of the nonlinear differential equations of motion and provides the elasto-dynamic response of the rotating composite blade. Using this method, the steady-state behavior can also be obtained. However, it is only possible after the whole transient response of the blade is calculated.

In Chap. 6, an alternative method for obtaining the steady-state behavior of a rotating blade is presented. This method does not require calculating the transient response in advance. Instead, it solves a boundary value problem that is based on the steady-state form of the nonlinear differential equations of a beam. This problem is then converted to a series of initial value problems which are solved by iterating an implicit (indirect) integration method. In each iteration, the estimations for the unknown initial conditions are improved by the use of the Newton–Raphson algorithm and the shooting method. The solution is repeated and its convergence is checked at every step. When a convergence criterion is satisfied, the correct solution of the boundary value problem and the steady-state response of the blade are obtained. The calculated response includes the steady-state values of forces, moments, velocities, and angular velocities along the blade. These results compare very well with the solution obtained in Chap. 5 as the transient solution discussed in Chap. 5 converges to the steady-state solution of Chap. 6. Having calculated the steady-state response, the effect of imposing input perturbations on the blade (when it is already in its steady-state condition) is analyzed. Small perturbations are considered; therefore, the solution is valid only near the steady-state response.

The rotating blades considered in Chaps. 5 and 6 are all clamped (hingeless). In Chap. 7, the dynamics of rotating articulated (hinged) blades, both rigid and elastic, is discussed. It starts with an introduction on the extended Euler equations of motion and continues by using these equations to calculate the coupled nonlinear flap-lag rigid body dynamics of articulated blades. The rigid body dynamics at the root of the blade is used to provide the boundary conditions for the case of the elastic rotating articulated blade. These boundary conditions together with the solution method developed in Chap. 5 are implemented to calculate the nonlinear dynamic response of an accelerating articulated blade. The solution is shown to be in good agreement with approximate formulas for the axial force and with the implemented boundary conditions.

Embedded actuators are used in rotating blades to control their shape in order to reduce noise and vibrations or to gain other satisfactory performance such as higher lift forces. In Chap. 8, the effect of inclusion of embedded piezocomposite actuators in a composite beam structure is analyzed. Both geometrically nonlinear static and dynamic cases are considered and the response sensitivity to the performance of actuators oriented at various directions along the blade is evaluated. Specifically, the steady-state force and moment components generated in the rotating blade are calculated. Such results can be used to control the elasto-dynamic response of rotating blades.

This book is based on my second Ph.D. thesis in mechanical engineering that I worked on in Carleton University in Ottawa. However, in this book, I also use a few solution techniques that I had developed in my earlier research for my first Ph.D. in mechanical engineering in Sharif University of Technology in Tehran. That research dealt with the dynamics of structures subjected to moving loads.

There have been a number of people that have contributed to this book in one way or another. First, I sincerely thank Professor Fred Nitzsche, for introducing Professor Hodges' fascinating book, Hodges (2006), to me. This is clearly the best resource for understanding the VAM. Also during the research that led to this book, I have had the privilege of receiving plenty of valuable hints and suggestions from Professors Dewey H. Hodges of Georgia Institute of Technology, Wenbin Yu of Purdue University, Carlos E.S. Cesnik of University of Michigan, and Rafael Palacios of Imperial College for which I am grateful. The constructive comments received from four anonymous reviewers of an earlier draft of this book are also thankfully appreciated.

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Mehrdaad Ghorashi, Ph.D., P.E.

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Nomenclature

A	Cross-sectional area of the undeformed beam in the x_2–x_3 plane
B	Deformed reference frame (B_1 is perpendicular to the plane of deformed cross section of the beam and other two base vectors on this plane)
b	Undeformed reference frame (b_1 is perpendicular to the plane of undeformed cross section of the beam and other two base vectors on this plane)
C	Finite rotation tensor
D	Material matrix (Chap. 2)
D	Vector of electric displacements (Chap. 8)
d_{ijk}	Piezoelectric moduli in tensor form (i, j, k = 1, 2, 3)
E_i	Electric field (Chap. 8)
E_i	Moduli of elasticity (i = 1, 2, 3)
e	Hinge offset ratio (offset as a fraction of the rotor radius, R)
e_{ijk}	Permutation symbol (i, j, k = 1, 2, 3)
e_1	$[1 \quad 0 \quad 0]^T$
F_i	Elements of the column matrix of internal forces (i = 1, 2, 3)
f	Applied forces vector per unit length
G_{ij}	Shear moduli (i, j = 1, 2, 3)
g	Determinant of the metric tensor in curvilinear coordinates (Chap. 2)
g	Current boundary value at the tip (Chap. 6)
H	Sectional angular momenta vector
i_2, i_3	Cross-sectional mass moment of inertia
i_{23}	Cross-sectional product of inertia
K	Deformed beam curvature vector $= k + \kappa$
k	Undeformed beam curvature vector
L	Length of the beam
L	Lift force per unit length (Chap. 7)
M_i	Elements of the column matrix of internal moments (i = 1, 2, 3)
m	Applied moments vector per unit length

N	Number of nodes
P	Sectional linear momenta vector
R	Rotor radius
S	Cross-sectional stiffness matrix
$S(x_2, x_3)$	Matrix of the FEM shape functions
t	Time
U	Strain energy per unit length
u_i	Displacement field in the b_i (i = 1, 2, 3) reference frame
V	Vector of nodal warping displacements (Chap. 2)
V	Velocity vector field in the B_i (i = 1, 2, 3) reference frame
v	Velocity field in the b_i (i = 1, 2, 3) reference frame
w_i	Warping displacement components (i = 1, 2, 3)
x_i	Global system of coordinates (i = 1, 2, 3)
x_1	Axis along the beam
x_2, x_3	Cross-sectional axes
$\bar{x}_2$ and $\bar{x}_3$	Offsets from the reference line of the cross-sectional mass center (coordinates of the cross-sectional centroid with respect to the shear center of the cross section)
x_G	Spanwise position of the mass center
α	Magnitude of the rotation used in the Rodrigues parameters (Chap. 2)
α	Assumed initial conditions at the root (Chap. 6)
β	Boundary conditions at the tip (Chap. 6)
β	Flap angle (Chap. 7)
Γ	Strain tensor = $\lfloor \Gamma_{11} \quad 2\Gamma_{12} \quad 2\Gamma_{13} \quad \Gamma_{22} \quad 2\Gamma_{23} \quad \Gamma_{33} \rfloor^T$
γ	$[\gamma_{11} \quad 2\gamma_{12} \quad 2\gamma_{13}]^T$ (Chap. 2)
γ	Lock number (Chap. 7)
$\bar{\gamma}_{11}$	Extension of the reference line (the bar indicates that transversal shear deformation has been neglected: $\bar{\gamma}_{12} = \bar{\gamma}_{13} = 0$)
Δ	3 × 3 identity matrix
$\delta\bar{q}$	Virtual displacement vector (the bar indicates that it need not be the variation of a functional)
$\delta\bar{\psi}$	Virtual rotation vector (the bar indicates that it need not be the variation of a functional)
$\bar{\varepsilon}$	Generalized strain in the classical theory = $[\bar{\gamma}_{11} \quad \bar{\kappa}_1 \quad \bar{\kappa}_2 \quad \bar{\kappa}_3]^T$ (the bar indicates that transversal shear deformation has been neglected: $\bar{\gamma}_{12} = \bar{\gamma}_{13} = 0$)
ε^Γ	Matrix of dielectric permittivity at constant strain
Θ	Rodrigues parameters = $[\theta_1 \quad \theta_2 \quad \theta_3]^T$; $\theta_i = 2e_i \tan(\alpha/2)$
κ_1	Elastic twist
κ_i	Elastic bending curvatures (i = 2, 3)
μ	Mass per unit length
ν_{ij}	Poisson's ratios (i, j = 1, 2, 3)
ρ	Mass density
σ_{ij}	Stress tensor components = $\lfloor \sigma_{11} \quad \sigma_{12} \quad \sigma_{13} \quad \sigma_{22} \quad \sigma_{23} \quad \sigma_{33} \rfloor^T$

ψ	Kernel matrix
Ω	Angular velocity vector in the B_i (i = 1, 2, 3) reference frame
ω	Angular velocity vector in the b_i (i = 1, 2, 3) reference frame
ξ	Lead-lag angle
$\left(\breve{\bullet}\right)$	Perturbations in space
$\left(\hat{\bullet}\right)$	Perturbations in time
$(\bullet)'$	$\frac{\partial(\bullet)}{\partial x_1}$ = derivative w.r.t. the axis along the undeformed reference line
$(\dot{\bullet})$	Derivative w.r.t. time, t
$(\delta\bar{\bullet})$	The bar indicates that it need not be the variation of a functional
$(\tilde{\bullet})_{ij}$	$-e_{ijk}(\bullet)_k$: The cross product operator (transformation from a vector to its dual skew-symmetric matrix)

For two vectors a and b,

$$\tilde{a}b = \underbrace{\begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}}_{\tilde{a}} \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix} = a \times b$$

$\langle\bullet\rangle$	$\int_A (\bullet)\mathrm{d}x_2\mathrm{d}x_3$
$\langle\langle\bullet\rangle\rangle$	$\left\langle(\bullet)\sqrt{g}\right\rangle = \int_A (\bullet)\sqrt{g}\mathrm{d}x_2\mathrm{d}x_3,\ \sqrt{g} = 1 - x_2k_3 - x_3k_2$

Contents

Chapter 1
Introduction

1.1 Preliminary Remarks

The analysis of structures that have one dimension much larger than the other two can be done by modeling them as beams. A beam model, also called a one-dimensional (1-D) model, provides the advantage of simplicity of analysis and faster solution. Such a modeling is widely used in applications like helicopter rotor blades and wind turbine blades. However, one should always assure that the simpler 1-D model is able to provide a satisfactory picture of the truly three-dimensional (3-D) real-life problem.

In many engineering applications where isotropic materials are used and the member under consideration has simple geometry and undergoes small deformations, a classical beam theory can be used to provide an adequate solution for the 3-D problem. However, there are severe limitations for the use of classical beam theories in today's engineering applications.

The first limitation is the replacement of isotropic materials by composite materials in many applications including helicopter rotor blades. A major reason for the growing use of composites as the materials of choice is that they provide much higher strength-to-weight ratios compared to isotropic materials. The second limitation is that structural members in many engineering applications have complex geometry that includes initial curvature.

The third limitation is due to the small deformations assumption that is a major simplifying assumption in classical beam theories. The main impact of this assumption is that all formulations become linear, and therefore, one may use the superposition principle for solving problems. However, many lightweight and thin-walled structural members undergo large deflections (even though at small strain) when subjected to service loads. An example is the aeroelastic analysis of high-altitude, long-endurance (HALE) aircraft that features high aspect ratio flexible wings that requires the analysis of structural geometrical nonlinearities and dynamic stall (Jian and Jinwu 2009). As a result, the nonlinear behavior of such

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