

APPLIED STRUCTURAL AND MECHANICAL VIBRATIONS

Theory and Methods • Second Edition

Paolo L. Gatti

$$p_i = p_i^{(0)} - \frac{1}{2} p_i^{(0)T} M_1 p_i^{(0)} + \sum_{\substack{r=1 \\ (r \neq i)}}^n \left(\frac{p_r^{(0)T} (K_1 - \lambda_i^{(0)} M_1) p_i^{(0)}}{\lambda_i^{(0)} - \lambda_r^{(0)}} \right) p_r^{(0)}$$



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Although physics may one day tell us that time does not even exist at a fundamental level, I wish to dedicate this book to

My present and future: my wife Simonetta
Our (my wife's and my) future: our daughter Greta J.
My past: my parents Paolina and Remo and
my grandmother Maria Margherita

Paolo L.J. Gatti

Maybe life to me has been pretty fair
But there's no point to things if you don't share,
Like a breeze you can feel it from inside
If your windows are open wide

Excerpt from the song *Diamond Nights*
(words and music by Paolo L. Gatti)

Preface to the second edition

In light of the very positive feedback to the first edition, the main structure of the book remains, for the most part, the same. All the material, however, has been scrupulously revised and updated, and hardly any page has been left untouched. Besides correcting the errors (at least the ones that I have found), there are a few major changes and many minor changes. In particular, some new material which corrects certain omissions in the first edition has been included, Chapters 2, 3, 11 and 12 and Appendix A have been almost entirely rewritten and some significant changes have been made in Chapter 8. In addition to all this, many minor changes pervade the entire text.

A new feature in style and presentation of the text is the introduction of numbered remarks in every chapter. These remarks vary in nature; while some are short digressions or incidental comments that can be skipped at a first reading, most of them are complements, extensions and/or clarifying points that bear a strict relation to the subject being discussed or that anticipate some major topic to be considered in more detail in other parts of the book. Separating the remarks from the main text, in the author's opinion, enhances clarity and does not interrupt the flow of ideas.

In this edition, however, there is less on electronic measuring instrumentation because Dr. V. Ferrari – who, in the first edition, wrote the three chapters on the subject – could not, for personal reasons and not without regret, give his contribution to this new edition.

As for the first edition, the author will be glad to hear from readers and will be grateful for the notification of errors and suggestions for improvement.

Paolo L. Gatti
Milan

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The MathWorks, Inc.
3 Apple Hill Drive
Natick, MA 01760-2098 USA
Tel: 508 647 7000
Fax: 508-647-7001
E-mail: info@mathworks.com
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Preface to the first edition

This book deals primarily with fundamental aspects of engineering vibrations within the framework of the linear theory. Although it is true that in practical cases it is sometimes not easy to distinguish between linear and nonlinear phenomena, the basic assumption throughout this text is that the principle of superposition holds.

Without claiming completeness, the authors' intention has been to discuss a number of important topics of the subject matter by bringing together, in book form, a central set of ideas, concepts and methods which form the common background of real-world applications in disciplines such as structural dynamics, mechanical, aerospace, automotive and civil engineering, to name a few.

In all, the authors claim no originality for the material presented. However, we feel that a book such as this one can be published at the end of the 1990s because, while it is true that the general theory of linear vibrations is well established (Lord Rayleigh's book *Theory of Sound* is about a century old), this by no means implies that the subject is 'closed' and outside the mainstream of ongoing research. In fact, on the one hand, the general approach to the subject has significantly changed in the last 30 years or so. On the other hand, the increasing complexity of practical problems puts ever higher demands on the professional engineer who, in turn, should acquire a good knowledge in a number of disciplines which are often perceived as distinct and separate fields.

Also, in this regard, it should be considered that the computer revolution of recent years, together with the development of sophisticated algorithms and fully automated testing systems, provides the analyst with computation capabilities that were unimaginable only a few decades ago. This state of affairs – despite the obvious advantages – may simply lead to confusion and/or erroneous results if the phenomena under study and the basic assumptions of the analysis procedures are not clearly understood.

The book is divided into two parts. Part I (Chapters 1 to 12) has been written by Paolo L. Gatti and is concerned with the theory and methods of linear engineering vibrations, presenting the topics in order of increasing

difficulty – from single-degree-of-freedom systems to random vibrations and stochastic processes – and also including a number of worked examples in every chapter. Within this part, the first three chapters consider some basic definitions and concepts to be used throughout the book (Chapter 1), a number of important aspects of a mathematical nature (Chapter 2) and a concise treatment of analytical mechanics (Chapter 3). In a first reading, if the reader is already at ease with Fourier series and Fourier and Laplace transforms, Chapter 2 can be skipped without loss of continuity. However, it is assumed that the reader is familiar with fundamental university calculus, matrix analysis (although Appendix A is dedicated to this topic) and some basic notions of probability and statistics.

Part II (Chapters 13 to 15) has been written by Vittorio Ferrari and deals with the measurement of vibrations by means of modern electronic instrumentation. The reason why this practical aspect of the subject has been included as a complement to Part I lies in the importance – which is sometimes overlooked – of performing valid measurements as a fundamental requirement for any further analysis. Ultimately, any method of analysis, no matter how sophisticated, is limited by the quality of the raw measurement data at its input, and there is no way to fix a set of poor measurements. The quality of measurement data, in turn, depends to a large extent on how properly the available instrumentation is used to set up a measuring chain in which each significant source of error is recognized and minimized. This is especially important in the professional world where, due to a number of reasons such as limited budgets, strict deadlines in the presentation of results and/or real operating difficulties, the experimenter is seldom given a second chance.

The choice of the topics covered in Part II and the approach used in the exposition reflect the author's intention of focusing the attention on basic concepts and principles, rather than presenting a set of notions or getting too much involved in inessential technological details. The aim and hope is, first, to help the reader – who is only assumed to have a knowledge of basic electronics – in developing an understanding of the essential aspects related to the measurement of vibrations, from the proper choice of transducers and instruments to their correct use, and, second, to provide the experimenter with guidelines and advice on how to accomplish the measurement task.

Finally, it is possible that this book, despite the attention paid to reviewing all the material, will contain errors, omissions, oversights and/or misprints. We will be grateful to readers who spot any of the above or who have any comment for improving the book. Any suggestion will be received and considered.

Paolo L. Gatti
Vittorio Ferrari
Milan

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Paolo L. Gatti

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Vittorio Ferrari

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Author

Paolo L. Gatti was born in 1959. He graduated in nuclear physics from the State University of Milan (Italy) and worked for 12 years for a private engineering company, where he became head of the vibration testing and data acquisition department.

Since 2000, he has worked as an independent consultant in mechanical and structural vibrations, acoustics and statistical analyses of experimental data. In these fields of activity, he is also an accredited technical consultant for the Court of Justice of Milan.

He is also the author of *Probability Theory and Mathematical Statistics for Engineers*, published by Spon Press (Taylor & Francis Group) in 2005.

Contents

<i>Preface to the second edition</i>	xv
<i>Preface to the first edition</i>	xvii
<i>Acknowledgements (second edition)</i>	xix
<i>Acknowledgements (first edition)</i>	xxi
<i>Author</i>	xxiii
1 Review of some fundamentals	1
1.1 Introduction	1
1.2 The role of modelling (linear and nonlinear, discrete and continuous systems, deterministic and random data)	2
1.3 Some definitions and methods	4
1.3.1 The phenomenon of beats	7
1.3.2 Displacement, velocity and acceleration	9
1.3.3 Quantification of vibration level and the decibel scale	10
1.4 Springs, dampers and masses	14
1.5 Summary and comments	19
2 Mathematical preliminaries	21
2.1 Introduction	21
2.2 Fourier series and Fourier transform	21
2.2.1 Periodic functions: Fourier series	22
2.2.2 Nonperiodic functions: Fourier transform	29
2.2.3 Main properties of Fourier transforms	33
2.2.4 Some mathematical facts	36
2.2.5 The bandwidth theorem (uncertainty principle)	39

2.3	<i>Laplace transform</i>	40
2.3.1	<i>Laplace transforms: Basic properties and some examples</i>	42
2.4	<i>Dirac delta and related topics</i>	48
2.4.1	<i>A short introduction to distributions</i>	53
2.5	<i>The notion of Hilbert space</i>	62
2.5.1	<i>Hilbert spaces l^2 and L^2</i>	67
2.5.1.1	<i>The spaces $l^2(C)$ and $l^2(R)$</i>	67
2.5.1.2	<i>The space L^2</i>	68
2.5.2	<i>Sturm–Liouville problems</i>	72
2.5.3	<i>Some generalisations</i>	79
3	Analytical mechanics: An overview	83
3.1	<i>Introduction</i>	83
3.2	<i>Systems of material particles</i>	84
3.2.1	<i>Generalised coordinates, constraints and degrees of freedom</i>	85
3.3	<i>The principle of virtual work and d’Alembert’s principle: Lagrange’s and Hamilton’s equations</i>	87
3.3.1	<i>Hamilton’s equations</i>	91
3.4	<i>Lagrange’s equations: Fundamental properties, some generalisations and complements</i>	93
3.4.1	<i>Invariance in form of LEs and monogenic forces</i>	93
3.4.2	<i>The structure of the kinetic energy and the conservation of energy</i>	94
3.4.3	<i>Elastic forces, viscous forces and Rayleigh dissipation function</i>	97
3.4.4	<i>More coordinates than degrees of freedom: Lagrange’s multipliers</i>	99
3.5	<i>Hamilton’s principle</i>	102
3.5.1	<i>More than one independent variable: The equation of motion of continuous systems</i>	107
3.5.2	<i>The boundary terms in Hamilton’s principle: Natural boundary conditions</i>	109
3.6	<i>Small-amplitude oscillations</i>	112
4	Single degree of freedom systems	119
4.1	<i>Introduction</i>	119
4.2	<i>Harmonic oscillator I: Free vibration</i>	120
4.2.1	<i>Undamped free vibrations</i>	121

4.2.2	<i>Damped free vibration</i>	124
4.2.2.1	Case 1. Critically damped motion: $\zeta = 1$ ($c = c_{cr}$)	125
4.2.2.2	Case 2. Overdamped motion: $\zeta > 1$ ($c > c_{cr}$)	125
4.2.2.3	Case 3. Underdamped motion: $0 < \zeta < 1$ ($c < c_{cr}$)	126
4.2.3	<i>Logarithmic decrement</i>	128
4.2.4	<i>Further analogies</i>	130
4.3	<i>Harmonic oscillator II: Forced vibration</i>	131
4.3.1	<i>Forced vibration: Harmonic excitation</i>	132
4.3.2	<i>Force transmissibility and harmonic motion of the support</i>	137
4.3.3	<i>Resonant response of damped and undamped SDOF systems</i>	140
4.3.4	<i>Some energy considerations</i>	142
4.4	<i>Damping in real systems, equivalent viscous damping</i>	144
4.4.1	<i>Measurement of damping</i>	147
4.4.1.1	<i>Free-vibration decay</i>	147
4.4.1.2	<i>Resonant response</i>	148
4.4.1.3	<i>Half-power bandwidth</i>	149
4.4.1.4	<i>Energy loss per cycle</i>	149
4.4.1.5	<i>Frequency response function</i>	150
4.5	<i>Summary and comments</i>	154
5	More SDOF systems: Shock response, transient response and some approximate methods	157
5.1	<i>Introduction</i>	157
5.2	<i>Time domain: Impulse response function and Duhamel integral</i>	158
5.2.1	<i>Excitation due to support motion</i>	168
5.2.2	<i>Concept of shock and response spectrum</i>	170
5.3	<i>Frequency and Laplace domains: Frequency response function and transfer function</i>	175
5.3.1	<i>Response to periodic excitation</i>	175
5.3.2	<i>Fourier and Laplace transform methods</i>	178
5.3.3	<i>Relationship between the characterising functions in time, frequency and Laplace domains</i>	182
5.4	<i>Generalised SDOF systems</i>	184

5.5	<i>Rayleigh (energy) method and improved Rayleigh method</i>	191
5.6	<i>Summary and comments</i>	198
6	Multiple degrees of freedom (MDOF) systems	201
6.1	<i>Introduction</i>	201
6.2	<i>A simple undamped 2-DOF system: Free vibration</i>	202
6.3	<i>Undamped n-DOF systems: Free vibration</i>	206
6.3.1	<i>Eigenvectors' orthogonality relations and normalisation</i>	209
6.3.2	<i>General solution of the undamped free-vibration problem, degeneracy and normal coordinates</i>	213
6.3.2.1	<i>Eigenvalue degeneracy</i>	215
6.3.2.2	<i>Normal coordinates</i>	216
6.4	<i>Eigenvalues and eigenvectors sensitivity analysis</i>	224
6.4.1	<i>Light damping as a perturbative term</i>	229
6.5	<i>A few considerations on the structure and properties of the matrices M, K and C</i>	231
6.5.1	<i>Mass properties</i>	232
6.5.2	<i>Elastic properties</i>	234
6.5.3	<i>More mass- and stiffness-orthogonality conditions</i>	239
6.6	<i>Unrestrained systems: Rigid-body modes</i>	240
6.7	<i>Damped systems: Proportional and nonproportional damping</i>	246
6.7.1	<i>Proportional damping</i>	246
6.7.2	<i>Nonproportional damping</i>	250
6.8	<i>Generalised and complex eigenvalue problems: Reduction to standard form</i>	253
6.8.1	<i>Undamped systems</i>	253
6.8.2	<i>Viscously damped systems</i>	256
6.9	<i>Summary and comments</i>	260
7	More MDOF systems: Forced vibration and response analysis	263
7.1	<i>Introduction</i>	263
7.2	<i>Mode superposition</i>	264
7.2.1	<i>Mode displacement and mode acceleration methods</i>	269
7.3	<i>Harmonic excitation: Proportional viscous damping</i>	271
7.4	<i>Time-domain and frequency-domain response</i>	274
7.4.1	<i>A few comments on FRFs</i>	276
7.4.2	<i>More on FRFs: Kramers–Kronig relations</i>	278
7.5	<i>Systems with rigid-body modes</i>	281

7.6	<i>The case of nonproportional viscous damping</i>	282
7.6.1	<i>Harmonic excitation and receptance FRF matrix</i>	285
7.7	<i>MDOF systems with hysteretic damping</i>	289
7.8	<i>A few remarks on other solution strategies: Laplace transform and direct integration</i>	291
7.8.1	<i>Laplace transform method</i>	292
7.8.2	<i>Direct integration methods</i>	293
7.9	<i>Frequency response functions of a 2-DOF system</i>	296
7.10	<i>Summary and comments</i>	305
8	Continuous systems	309
8.1	<i>Introduction</i>	309
8.2	<i>The flexible string in transverse motion</i>	310
8.2.1	<i>The initial value problem</i>	313
8.2.2	<i>Sinusoidal waves</i>	314
8.2.3	<i>Some energy considerations</i>	316
8.2.4	<i>The presence of boundaries</i>	318
8.3	<i>Free vibration of a finite string: Standing waves and normal modes</i>	320
8.4	<i>Axial and torsional vibrations of rods</i>	326
8.5	<i>Flexural (bending) vibrations of beams</i>	330
8.5.1	<i>Case 1. Both ends simply supported (pinned–pinned configuration)</i>	332
8.5.2	<i>Case 2. One end clamped and one end free (cantilever configuration)</i>	333
8.5.3	<i>Case 3. Both ends clamped (clamped–clamped configuration)</i>	334
8.5.4	<i>Case 4. Both ends free (free–free configuration)</i>	335
8.5.5	<i>Axial force effects on flexural vibrations</i>	336
8.5.6	<i>The effects of shear deformation and rotary inertia (Timoshenko beam)</i>	339
8.5.6.1	<i>Case 1. Shear deflection alone</i>	343
8.5.6.2	<i>Case 2. Rotary inertia alone</i>	343
8.6	<i>A two-dimensional continuous system: The flexible membrane</i>	344
8.6.1	<i>The circular membrane with fixed edge</i>	346
8.7	<i>The differential eigenvalue problem</i>	349
8.7.1	<i>The differential eigenvalue problem: Some further considerations</i>	355

8.8	<i>Bending vibrations of thin plates</i>	361
8.8.1	<i>Circular plates</i>	364
8.8.2	<i>Rectangular plates</i>	367
8.9	<i>Forced vibration and response analysis: The modal approach</i>	372
8.9.1	<i>Forced response of continuous systems: Some examples</i>	378
8.10	<i>Some final considerations: Alternative form of FRFs and the introduction of damping</i>	388
8.11	<i>Summary and comments</i>	392
9	MDOF and continuous systems: Approximate methods	395
9.1	<i>Introduction</i>	395
9.2	<i>The Rayleigh quotient</i>	396
9.2.1	<i>Courant–Fisher max–min and min–max characterisation of eigenvalues and the eigenvalue separation property</i>	399
9.2.2	<i>Systems with lumped masses: Dunkerley’s formula</i>	405
9.3	<i>The Rayleigh–Ritz method</i>	406
9.3.1	<i>The Rayleigh–Ritz method (and the assumed-modes method) for continuous systems</i>	410
9.3.2	<i>Continuous systems: A few comments on admissible and comparison functions</i>	415
9.4	<i>Summary and comments</i>	419
10	Experimental modal analysis	421
10.1	<i>Introduction</i>	421
10.2	<i>Experimental modal analysis: Overview of the fundamentals</i>	422
10.2.1	<i>FRFs of SDOF systems</i>	424
10.2.2	<i>FRFs of MDOF systems</i>	435
10.3	<i>Modal testing procedures</i>	444
10.3.1	<i>Supporting the structure</i>	445
10.3.2	<i>Excitation systems</i>	446
10.3.3	<i>Measurement of response</i>	449
10.3.4	<i>Excitation functions</i>	450
10.4	<i>A few selected topics in experimental modal analysis</i>	453
10.4.1	<i>Characteristic phase lag theory and Asher’s method</i>	454