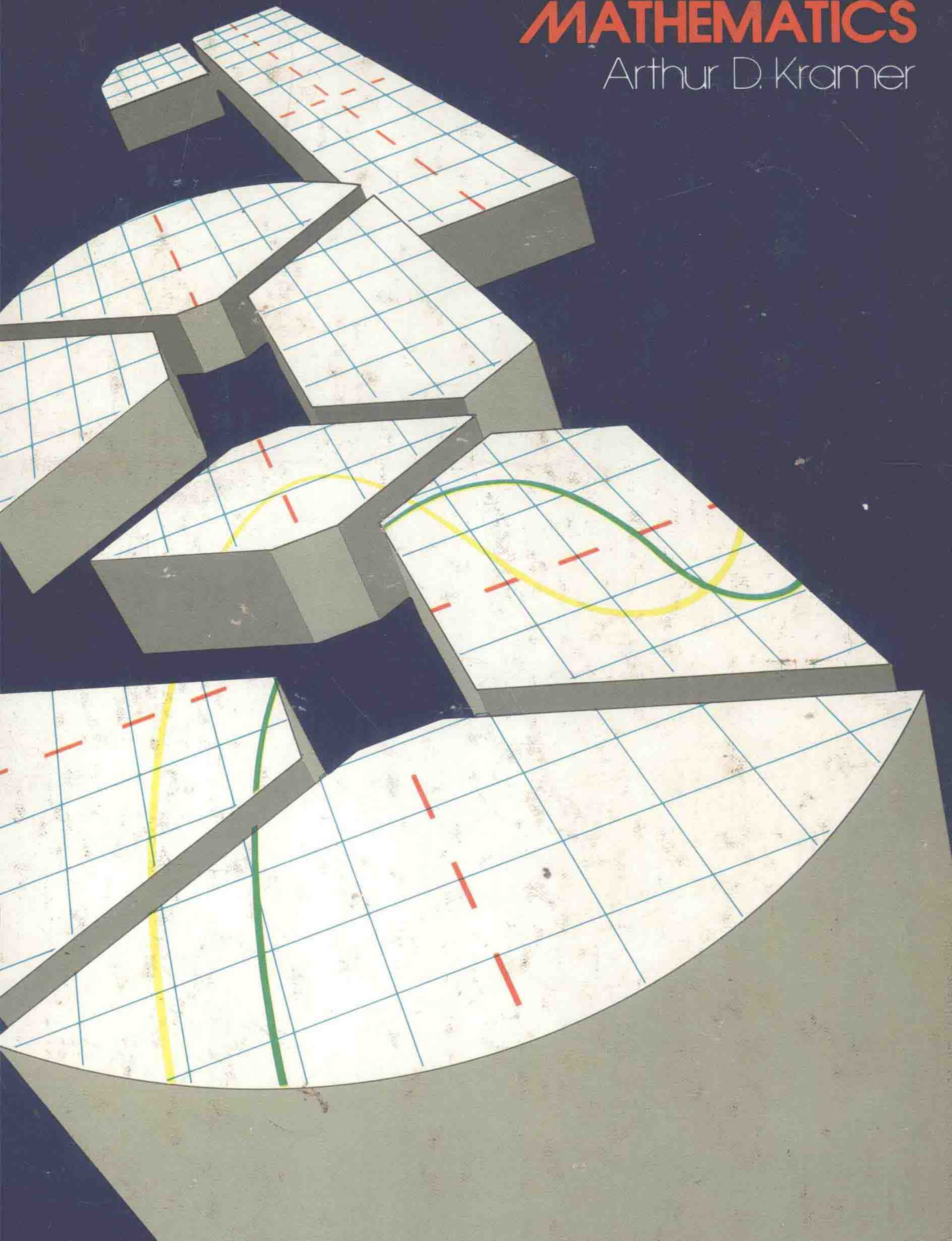
FUNDAMENTALS OF TECHNICAL MATHEMATICS

Arthur D. Kramer



Fundamentals of Technical Mathematics

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Preface

Fundamentals of Technical Mathematics is designed for students who are preparing for technical or scientific careers. While it is desirable to have some background in elementary algebra, it is not essential. Chapter 2 is a thorough treatment of elementary algebra that can be used as a review or as an introduction. The text covers those areas of mathematics necessary for the beginning student up to and including the pre-calculus-level student. The approach of this text stresses a working knowledge of mathematics and an application of mathematical ideas to technical and practical areas.

Students can best grasp and use mathematics by working out many exercises and problems. A major feature of *Fundamentals of Technical Mathematics* is that it contains over 800 worked-out examples and over 4,000 exercises, almost all of which are meaningful applications taken from various scientific, technical, and practical areas. The applications do not require any prior knowledge of a specific subject and serve to develop an understanding of where and how mathematics is used in many fields. Many of the exercises relate back to the worked-out examples and help to reinforce the ideas learned in the text. The odd-numbered exercises are generally similar to the following even-numbered exercises with the answers to all odd-numbered exercises given in Appendix C, so they may be used for self-study. Answers to even-numbered exercises are available to instructors in the *Instructor's Manual and Key*. Every chapter is followed by a series of review questions which combine several of the ideas covered in that chapter. There are photographs relating to the subject matter and numerous detailed illustrations are used throughout the text to clarify ideas.

The hand-held calculator is now a fact of everyday life especially in science and technology. To familiarize students with the use of the calculator, special calculator sections are presented at the end of those chapters which introduce calculator functions: Chapters 1, 2, 3, 8, and 10. These sections can be studied separately or integrated into the chapter material throughout the text. In addition to these special sections many examples which lend themselves to calculator usage are included in the regular exercises and are marked with the symbol . The calculator is clearly not a substitute for understanding and applying mathematical skills. On the contrary, it allows you more time to master ideas by reducing time spent on tedious calculations. I have designed the calculator exercises with just this in mind; they reinforce understanding and strengthen the ability to estimate results and determine errors.

The chapters and chapter sections have been arranged in a carefully thought-out logical order so that ideas flow smoothly from one topic to another. Chapters 1, 2, and 3 review arithmetic, algebra, and geometry respectively, and introduce the basic arithmetic and trigonometric calculator functions. Depending on the background of the student, various topics in these chapters can be studied or used as a reference. Metric units introduced in Chapter 3 are becoming increasingly important and are used throughout the text along with U.S. Customary units. Many of the chapters after Chapter 3 can be studied independently as there are cross references to aid in their understanding. The exceptions are Chapter 13, which requires Chapter 6 as a prerequisite, and chapters 9, 11, and 15, which depend on the material in Chapter 8. Chapter 16 contains material on linear programming which is finding more and more application in business and industry with the increased use of the computer. Chapter 17, which contains an introduction to computers and BASIC, includes applications not usually found in introductory BASIC texts and can be a useful instructional tool in many scientific and technical areas. The chapter sequence is therefore flexible, and can be arranged if necessary, to suit various student needs.

I wish to thank Pamela, above all, for her continued support and endurance during the writing of this book. Thanks also to my colleagues Joel Greenstein of New York City Technical College, Milton Eisner of Mount Vernon College, and Peter Philiou of Wentworth Institute for their helpful comments and thorough reviews of the manuscript. Special thanks are owed to Nancy Warren for her careful checking of the answers. Last, but not least, thanks to my many students over the years whose experiences with the subject have helped me to better understand their needs and have proven to be an invaluable guide in writing *Fundamentals of Technical Mathematics*.

Arthur D. Kramer

Contents

1 Arithmetic and the Hand-Held Calculator

A thorough understanding of arithmetic is essential for a good grasp of basic mathematics, including algebra and geometry. This chapter is primarily designed to help you strengthen and reinforce your ability in arithmetic. Sections 1-1 to 1-3 review the basic arithmetic operations ($+$, $-$, $\times$, $\div$), and Sec. 1-4 introduces the concepts of powers and roots. Section 1-5 then discusses these same operations on the calculator. Its purpose is to help you learn arithmetic concepts well enough to interpret calculator results, identify errors, and make the necessary corrections. Section 1-5 can be studied before, during, or after Secs. 1-1 to 1-4. Section 1-6 explains the ideas of precision and accuracy, which are important in scientific and technical applications and in interpreting calculator results. Sections 1-5, 1-6, and the last section, 1-7 (Review Questions), contain many useful exercises designed to develop your skill with the calculator and, more important, your skill with arithmetic. Later chapters (2, 3, 8, and 10) introduce other calculator functions: sin, cos, log, and so on as the need arises. These functions are presented within the appropriate chapter and are explained in more detail in a calculator section at the end of the chapter.

1–1 Laws of Arithmetic

There are two basic laws of arithmetic that apply to the operations of addition and multiplication. The first law states that it makes no difference in what order you add or multiply two numbers. For example, $2 + 5 = 5 + 2$ and $3 \times 4 = 4 \times 3$. This is called the **commutative law**:

$$A + B = B + A \quad \text{and} \quad A \times B = B \times A \tag{1-1}$$

The second law states that if three numbers are to be added or multiplied together, it makes no difference whether you start the operations with the first and second numbers or with the second and third. For example, in addition, $(2 + 3) + 5 = 2 + (3 + 5)$ or $5 + 5 = 2 + 8$. In multiplication, $(3 \times 4) \times 5 = 3 \times (4 \times 5)$ or $12 \times 5 = 3 \times 20$. This is called the **associative law**:

$$(A + B) + C = A + (B + C)$$

$$\text{and} \quad (A \times B) \times C = A \times (B \times C) \tag{1-2}$$

When we apply these two laws together, it follows that three or more numbers can be added in any order or multiplied in any order. For example, $2 + 3 + 4$ (or $2 \times 3 \times 4$) can be added (or multiplied) in any one of six different ways:

$2 + 3 + 4,\ 2 + 4 + 3,\ 3 + 2 + 4,\ 3 + 4 + 2,\ 4 + 2 + 3,\ 4 + 3 + 2$

Another important law of arithmetic which combines multiplication and addition is the **distributive law**. This law says that multiplication distributes over addition:

$$A \times (B + C) = A \times B + A \times C \tag{1-3}$$

The distributive law is important in algebra. It is used in Chapter 2.

The **order of operations** in arithmetic, when there are no parentheses, is multiplication or division first, addition or subtraction second. Most scientific calculators are programmed to perform the operations in this order. It is called algebraic logic or the algebraic operating system.

Example 1-1

Calculate the following: $5 \times 21 - 36 + 4 \div 2$

SOLUTION

Multiply and divide first: $105 - 36 + 2$

Then subtract and add: $69 + 2 = 71$

You can test if your calculator uses algebraic logic as follows. Enter this example exactly as it appears above and see if you get 71 when you press the equals key. See Sec. 1-5 for further discussion on calculator operation.

Example 1-2

Calculate the following: $5 \times 21 - (36 + 4) \div 2$

SOLUTION

Do the operation in parentheses first:

$$5 \times 21 - 40 \div 2 = 105 - 20 = 85$$

See Example 1-20 for the calculator solution of Example 1-2.

Example 1-3

Calculate the following:

$$\frac{5 \times 21 \times 12}{15 \times 7 \times 3}$$

SOLUTION

You can multiply and divide in any order. One way is to multiply across the top and bottom, and then divide:

$$\frac{5 \times 21 \times 12}{15 \times 7 \times 3} = \frac{1260}{315} = 4$$

An easier way is to divide common factors in the top and bottom first:

$$\frac{\overset{1}{\cancel{5}} \times \overset{\overset{1}{\cancel{3}}}{\cancel{21}} \times \overset{4}{\cancel{12}}}{\underset{\underset{1}{\cancel{3}}}{\cancel{15}} \times \underset{1}{\cancel{7}} \times \underset{1}{\cancel{3}}} = 4$$

See Example 1-21 for the calculator solution of Example 1-3.

Exercise 1-1 In numbers 1 to 16 test your understanding of arithmetic by mentally calculating the result. Check by doing the problem by hand. (You can check further with the calculator.)

1. $6 + 5 + 7 + 3 + 5 + 4$

2. $8 + 2 - 3 + 9 - 1$

3. $5 \times 2 \times 3 \times 4$

4. $12 \div 3 \div 2 \div 2$

5. $(800 + 20) \div 20$

6. $10 \div (5 + 40) \div 9$

7. $8 + 13 \times 2 - 4$

8. $7 - 6 \div 3 + 8 \div 4$

9. $5 + (8 - 1) \times 6 \div 2$

10. $(5 - 1) \div 2 + 3 \times 4$

11. $\dfrac{9 \times 4}{3 \times 6} + \dfrac{18}{6}$

12. $\dfrac{8 \times 9}{4} - \dfrac{15}{3}$

13. $\dfrac{12 \times 15}{5 \times 3 \times 2}$

14. $\dfrac{8 \times 7 \times 6}{4 \times 28}$

15. $\dfrac{(3 + 5) \times 2}{13 - 11}$

16. $\dfrac{6 + 8 \times (4 - 1)}{4 - 1 \times 2}$

Solve the following applied problems by hand. (You can check with the calculator.)

17. One car travels 228 miles on 12 gallons of gasoline and a second car travels 336 miles on 16 gallons. How many more miles per gallon does the car with the better gas mileage get?

18. An experienced electronics technician earns \$330 for a 40-hour (h) week and a chemical engineer earns \$287 for a 35-h week. Who earns more per hour and how much more?

19. A Mariner space probe traveling at an average speed of 6000 miles per hour (mi/h) takes 400 days to reach Mars. What is the total distance traveled by the space probe?

20. A bus route is 22 kilometers (km) long. It takes the bus 50 minutes to complete the route in one direction and 70 minutes to complete it in the other direction. What is the average rate of speed of the bus in kilometers per hour for the total trip back and forth? (Average rate = distance/time.)

21. The formula $N(N + 1)/2$ can be used to calculate the sum of the first N numbers. Check the formula for the first 12 numbers by *(a)* adding 1 through 12 directly; *(b)* letting $N = 12$ in the formula and calculating the result.

22. A number is divisible by 9 if the sum of its digits is divisible by 9. Otherwise it is not. Test this with *(a)* the eight-digit serial number on a dollar or another bill; *(b)* your nine-digit social security number.

1–2 Fractions

Calculations with fractions, decimals, and percents lead to mistakes because of a misunderstanding of the concepts involved. The calculator can prevent some of these mistakes, but it is not a substitute for clear understanding. The following examples review the basic arithmetic of fractions. Each example is designed to be done by hand. (You can check the results with the calculator.)

Example 1-4

Simplify (reduce to lowest terms):

$$\frac{28}{42}$$

SOLUTION

Divide out any common factors (divisors) in the top and bottom:

$$\frac{28}{42} = \frac{\overset{1}{\cancel{2}} \times 2 \times \overset{1}{\cancel{7}}}{\underset{1}{\cancel{2}} \times 3 \times \underset{1}{\cancel{7}}} = \frac{2}{3}$$

It is not necessary to show all the factors. This is done to clearly illustrate the procedure. The numbers 28/42 and 2/3 are called **equivalent fractions**. A fraction can be changed to an equivalent fraction by dividing out common factors or multiplying the top and the bottom by the same factor. For example,

$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} \text{ and so on}$$

Example 1-5

Calculate each of the following:

1. $\frac{3}{8} \times \frac{2}{9}$

SOLUTION

To multiply fractions, first divide out common factors that occur in any numerator and denominator. Then multiply across the numerators and denominators:

$$\frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{8}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{9}}} = \frac{1}{12}$$

2. $\frac{5}{12} \div \frac{15}{16}$

SOLUTION

To divide fractions, invert the divisor, that is, the fraction after the division sign, and multiply:

$$\frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{12}}} \times \frac{\overset{4}{\cancel{16}}}{\underset{3}{\cancel{15}}} = \frac{4}{9}$$

3. $4 \times \frac{3}{14} \times \frac{5}{9} = \frac{\overset{2}{\cancel{4}}}{1} \times \frac{\overset{1}{\cancel{3}}}{\underset{7}{\cancel{14}}} \times \frac{5}{\underset{3}{\cancel{9}}} = \frac{10}{21}$

4. $\frac{5}{4} \times 8 \div \frac{1}{4} = \frac{5}{\underset{1}{\cancel{4}}} \times \frac{8}{1} \times \frac{\overset{1}{\cancel{4}}}{1} = 40$

Notice in (3) and (4) that a whole number can be written with a denominator of 1. To multiply a fraction by a whole number, multiply the numerator by the whole number.

Example 1-6

Combine:

$$\frac{2}{3} + \frac{5}{6}$$

SOLUTION

To combine fractions, that is, add or subtract, first change each fraction to an equivalent fraction so that the denominators are the same. The easiest denominator to use is the lowest common denominator (lcd) which is the smallest number that each denominator divides into. Then combine the numerators over the lcd:

$$\frac{2(2)}{3(2)} + \frac{5}{6} = \frac{4}{6} + \frac{5}{6} = \frac{4 + 5}{6} = \frac{9}{6} = \frac{3}{2}$$

The lcd = 6, and only the first fraction needs to be changed. Note that the result 9/6 can be reduced to 3/2. Parentheses or a dot · also indicate multiplication. They are used in algebra to avoid confusing the multiplication sign × with the letter x.

Example 1-7

Combine:

$$\frac{7}{15} + \frac{5}{12} - \frac{1}{6}$$

SOLUTION

The lcd is 60. This can be found by taking multiples of the largest denominator—15, 30, and so on—until each denominator divides into a multiple. Another way is to factor each denominator:

$$\frac{7}{(3)(5)} + \frac{5}{(2)(2)(3)} - \frac{1}{(2)(3)}$$

and make up the lcd so that it contains all the factors that appear in each denominator: (2)(2)(3)(5) = 60. The solution is then

$$\frac{7(4)}{15(4)} + \frac{5(5)}{12(5)} - \frac{1(10)}{6(10)} = \frac{28 + 25 - 10}{60} = \frac{43}{60}$$

Example 1-8

Calculate:

$$\frac{13}{8} - \frac{7}{5} \times \frac{15}{14} + 2 \div \frac{8}{15}$$

SOLUTION

Invert the last fraction and change to multiplication. Then divide common factors and multiply:

$$\frac{13}{8} - \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{3}{\cancel{15}}}{\underset{2}{\cancel{14}}} + \overset{1}{\cancel{2}} \times \frac{15}{\underset{4}{\cancel{8}}} = \frac{13}{8} - \frac{3}{2} + \frac{15}{4}$$

Now combine over the lcd, 8:

$$\frac{13}{8} - \frac{3(4)}{2(4)} + \frac{15(2)}{4(2)} = \frac{13 - 12 + 30}{8} = \frac{31}{8}$$

See Example 1-22 for the calculator solution of Example 1-8.

Exercise 1-2 Simplify each fraction (reduce to lowest terms).

1. $\frac{6}{10}$

2. $\frac{12}{36}$

3. $\frac{28}{35}$

4. $\frac{27}{54}$

5. $\frac{39}{52}$

6. $\frac{34}{51}$

Calculate each exercise by hand. (You can check with the calculator.)

7. $\frac{5}{9} \times \frac{6}{25}$

8. $\frac{2}{21} \times \frac{7}{16}$

9. $\frac{8}{9} \div \frac{2}{3}$

10. $\frac{3}{11} \div \frac{1}{22}$

11. $\frac{3}{5} \times \frac{15}{7} \times \frac{14}{9}$

12. $6 \times \frac{4}{5} \div \frac{8}{15}$

13. $\frac{3}{17} \div \left(\frac{1}{34} \times \frac{1}{2}\right)$

14. $\left(\frac{9}{8} \div \frac{3}{4}\right) \div \frac{3}{2}$

15. $\frac{3}{8} + \frac{1}{4}$

16. $\frac{4}{15} + \frac{5}{6}$

17. $\frac{3}{4} - \frac{1}{2} + \frac{7}{10}$

18. $\frac{1}{6} - \frac{2}{3} + \frac{11}{20}$

19. $2 + \frac{7}{8} + \frac{2}{3}$

20. $\frac{5}{2} + \frac{5}{3} + \frac{5}{6}$

21. $\frac{16}{9} \times \frac{1}{2} + \frac{1}{4}$

22. $\frac{1}{6} + \frac{3}{8} \div \frac{1}{4}$

23. $3 \times \frac{1}{6} + \frac{7}{2} - \frac{4}{5} \div 8$

24. $\frac{3}{100} + \frac{7}{10} \times \frac{2}{35} - \frac{1}{50}$

25. $\left(\frac{1}{2} + \frac{1}{3}\right) \times \left(8 \div \frac{4}{3}\right)$

26. $\left(1 + \frac{3}{8}\right) \div \left(1 - \frac{3}{8}\right)$

27. A $30,000 inheritance is distributed as follows: half to the spouse, two-thirds of what is left to the children, and the remainder to charity. How much money is given to charity?

28. A bookcase is to be 8 ft $3\frac{1}{2}$ in high and to contain six equally spaced shelves and a top, each $\frac{1}{2}$ in thick (seven pieces total). How many feet and inches apart should each shelf be?

29. Calculate the resistance of a series-parallel circuit given by

$$R = \frac{1}{1/12 + 1/4} + 3$$

30. Calculate the focal length of a lens given by

$$f = \frac{1/10 \times 3/20}{(3/2 - 1)(1/10 + 3/20)}$$

1–3 Decimals and Percent

Decimals

Decimals represent fractions whose denominators are powers of 10: 10, 100, 1000, and so on. The number of decimal places equals the number of zeros in the denominator:

$$0.3 = \frac{3}{10}, \quad 0.21 = \frac{21}{100}, \quad 0.067 = \frac{67}{1000}$$

and so on.

Example 1-9

Calculate each of the following:

1. $6.23 + 17.87 + 0.15$

SOLUTION

Add or subtract decimals in the same way as whole numbers, lining up the columns:

$$\begin{array}{r} 6.23 \\ 17.87 \\ 0.15 \\ \hline 24.25 \end{array}$$

2. 1.3×0.05

SOLUTION

To multiply decimals, add the decimal places in each number to determine the number of decimal places in the answer:

$$1.3 \times 0.05 = 0.065$$

(Decimal places: one + two = three)

3. $\dfrac{13.2}{0.12}$

SOLUTION

To divide decimals, move the decimal point in the numerator and denominator to the right as many places as there are in the denominator:

$$\frac{13.2}{0.12} = \frac{1320.}{12.} = 110$$

4. $\frac{0.5 \times 0.02}{0.06 - 0.01} = \frac{0.010}{0.05} = \frac{1.0}{5} = 0.2$

Study Example (4), which combines subtraction, multiplication, and division of decimals. Note that moving the decimal point in the numerator and denominator to the right is the same as multiplying the top and bottom by 10, 100, 1000, etc., and does not change the value of the fraction. In (3) and (4) the top and bottom are multiplied by 100.

Percent

Percents are fractions with denominators of 100. They are the same as decimals written to two places. To change from a percent to a decimal, move the decimal point two places to the left, and vice versa.

Example 1-10

Express each as a fraction, decimal, and percent.

1. $\frac{53}{100} = 0.53 = 53\%$

2. $\frac{1}{10} = 0.10 = 10\%$

3. $\frac{2}{50} = 0.04 = 4\%$

4. $\frac{27}{1000} = 0.027 = 2.7\%$

5. $\frac{7}{4} = 1.75 = 175\%$

6. $\frac{1}{3} = 0.333\ldots = 33\frac{1}{3}\%$

Example 1-11

The current in a circuit is 3.50 amperes (A). The voltage is increased by 4 percent, causing the current to increase by the same percentage.

1. What is the increase in the current?

SOLUTION

$$3.50\ (4\%) = 3.50\ (0.04) = 0.14\ \text{A}$$