The Theory and Applications of Instanton Calculations

MANU PARANJAPE

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ON MATHEMATICAL PHYSICS

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ABOUT THIS BOOK

Instantons, or pseudoparticles, are solutions to the equations of motion in classical field theories on a Euclidean spacetime. Instantons are found everywhere in quantum theories as they have many applications in quantum tunnelling. Diverse physical phenomena may be described through quantum tunnelling, for example: the Josephson effect, the decay of meta-stable nuclear states, band formation in tight binding models of crystalline solids, the structure of the gauge theory vacuum, confinement in 2+1 dimensions, and the decay of superheated or supercooled phases. Drawing inspiration from Sidney Coleman's Erice lectures, this volume provides an accessible, detailed introduction to instanton methods, with many applications, making it a valuable resource for graduate students in many areas of physics, from condensed matter, particle and nuclear physics, to string theory.

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Université de Montréal

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MANU PARANJAPE has been a professor at the Université de Montréal for the past 30 years. In this time he has worked on quantum field theory, the Skyrme model, non-commutative geometry, quantum spin tunnelling and conformal gravity. Whilst working on induced fermion numbers, he discovered induced angular momentum on flux tube solitons, and more recently he discovered the existence of negative-mass bubbles in de Sitter space, which merited a prize in the Gravity Research Foundation essay competition.

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Preface

This book is based on a graduate course taught four times, once in French at the Université de Montréal and then three times in English at the Institut für Theoretische Physik, in Innsbruck, Austria, at the Center for Quantum Spacetime, Department of Physics, Sogang University, Seoul, Korea, and most recently, a part of it at the African Institute for Mathematical Sciences (AIMS), Cape Town, South Africa.

The course covered the contents of the magnificent Erice lectures of Coleman [31], "The Uses of Instantons", in addition to several chapters based on independent research papers. However, it might be more properly entitled, "The Uses of Instantons for Dummies". I met Sidney Coleman a few times, more than 30 years ago, and although I am sure that he was less impressed with the meetings than I was and probably relegated them to the dustbin of the memory, my debt to him is enormous. Without his lecture notes I cannot imagine how I would ever have been able to understand what the uses of instantons actually were. However, in his lecture notes, one finds that he also thanks and expresses gratitude to a multitude of eminent and great theoretical physicists of the era, indeed thanking them for "patiently explaining large portions of the subject" to him. Unfortunately, we cannot all be so lucky. Coleman's lecture notes are a work of art; it is clear when one reads them that one is enjoying a master impressionist painter's review of a subject, a review that transmits, as he says, the "awe and joy" of the beauty of the "wonderful things brought back from far places". But then the hard work begins.

Hence, through diligent, fastidious and brute force work, I have been able, I hope, to produce what I believe is a well-rounded, detailed monograph, essentially explaining in a manner accessible to first- and second-year graduate students the beauty and the depth of what is contained in Coleman's lectures and in some elaborations of the whole field itself.

I am indebted to many, but I will thank explicitly Luc Vinet for impelling me to first give this course when I started out at the Université de Montréal; Gebhard Grübl for the opportunity to teach the course at the Universität Innsbruck in Innsbruck, Austria; Bum-Hoon Lee for the same honour at Sogang University in Seoul, Korea; and Fritz Hahne for the opportunity to give the lectures at the African Institute for Mathematical Sciences, Cape Town, South Africa. I thank the many students who took my course and suggested corrections to my lectures. I thank Nick Manton, Chris Dobson, and Duncan Dormor, respectively, Fellow, Master and President of St John's College, University of Cambridge in 2015, for making available to me the many assets of the College that made it possible to work uninterrupted and in a pleasant ambiance on this book, during my stay as an Overseas Visiting Scholar. I also thank my many colleagues and friends who have helped me through discussions and advice; these include Ian Affleck, Richard MacKenzie, Éric Dupuis, Jacques Hurtubise, Keshav Dasgupta and Gordon Semenoff.

I especially thank my wife Suneeti Phadke, who started the typing of my lectures in TeX and effectively typed more than half the book while caring for a six-month-old baby. This was no easy feat for someone with a background in Russian literature, devoid of the intricacies of mathematical typesetting. This book would not have come to fruition had it not been for her monumental efforts.

I also thank my children Kiran and Meghana, whose very existence makes it a joy and a wonder to be alive.

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