

FLIGHT STABILITY AND AUTOMATIC CONTROL

SECOND EDITION



ROBERT C. NELSON

Flight Stability and Automatic Control

SECOND EDITION

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PREFACE

An understanding of flight stability and control played an important role in the ultimate success of the earliest aircraft designs. In later years the design of automatic controls ushered in the rapid development of commercial and military aircraft. Today, both military and civilian aircraft rely heavily on automatic control systems to provide artificial stabilization and autopilots to aid pilots in navigating and landing their aircraft in adverse weather conditions. The goal of this book is to present an integrated treatment of the basic elements of aircraft stability, flight control, and autopilot design.

NEW TO THIS EDITION

In the second edition, I have attempted to improve the first six chapters from the first edition. These chapters cover the topics of static stability, flight control, aircraft dynamics and flying qualities. This is accomplished by including **more worked-out example problems, additional problems** at the end of each chapter, and new material to provide additional insight on the subject. The major change in the text is the addition of an **expanded section on automatic control theory** and its application to flight control system design.

CONTENTS

This book is intended as a textbook for a course in aircraft flight dynamics for senior undergraduate or first year graduate students. The material presented includes static stability, aircraft equations of motion, dynamic stability, flying or handling qualities, automatic control theory, and application of control theory to the synthesis of automatic flight control systems. Chapter 1 reviews some basic concepts of aerodynamics, properties of the atmosphere, several of the primary flight instruments, and nomenclature. In Chapter 2 the concepts of airplane static stability and control are presented. The design features that can be incorporated into an aircraft design to provide static stability and sufficient control power are discussed. The rigid body aircraft equations of motion are developed along with techniques to model the aerodynamic forces and moments acting on the airplane in Chapter 3. The aerodynamic forces and moments are modeled using the concept of aerodynamic stability derivatives. Methods for estimating the derivatives are presented in Chapter 3 along with a detailed example calculation of the longitudinal derivatives of a STOL transport. The dynamic characteristics of an airplane for free and forced response are presented in Chapters 4 and 5. Chapter 4 discusses the

longitudinal dynamics while Chapter 5 presents the lateral dynamics. In both chapters the relationship between the rigid body motions and the pilot's opinion of the ease or difficulty of flying the airplane is explained. Handling or flying qualities are those control and dynamic characteristics that govern how well a pilot can fly a particular control task. Chapter 6 discusses the solution of the equations of motion for either arbitrary control input or atmospheric disturbances. Chapters 7–10 include the major changes incorporated into the second edition of this book. Chapter 7 provides a review of classical control concepts and discusses control system synthesis and design. The root locus method is used to design control systems to meet given time and frequency domain performance specifications. Classical control techniques are used to design automatic control systems for various flight applications in Chapter 8. Automatic control systems are presented that can be used to maintain an airplane's bank angle, pitch orientation, altitude, and speed. In addition a qualitative description of a fully automated landing system is presented. In Chapter 9, the concepts of modern control theory and design techniques are reviewed. By using state feedback design, it is theoretically possible for the designer to locate the roots of the closed loop system so that any desired performance can be achieved. The practical constraints of arbitrary root placement are discussed along with the necessary requirements to successfully implement state feedback control. Finally in Chapter 10 modern control design methods are applied to the design of aircraft automatic flight control systems.

LEARNING TOOLS

To help in understanding the concepts presented in the text I have included a number of worked-out example problems throughout the book, and at the end of each chapter one will find a problem set. Some of the example problems and selected problems at the end of later chapters require computer solutions. Commercially available computer aided design software is used for selected example problems and assigned problems. Problems that require the use of a computer are clearly identified in the problem sets. A major feature of the textbook is that the material is introduced by way of simple exercises. For example, dynamic stability is presented first by restricted single degree of freedom motions. This approach permits the reader to gain some experience in the mathematical representation and physical understanding of aircraft response before the more complicated multiple degree of freedom motions are analyzed. A similar approach is used in developing the control system designs. For example, a roll autopilot to maintain a wings level attitude is modeled using the simplest mathematical formulation to represent the aircraft and control system elements. Following this approach the students can be introduced to the design process without undue mathematical complexity. Several appendices have also been included to provide additional data on airplane aerodynamic, mass, and geometric characteristics as well as review material of some of the mathematical and analysis techniques used in the text.

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CHAPTER 1

Introduction

"For some years I have been afflicted with the belief that flight is possible to man."

Wilbur Wright, May 13, 1900

1.1

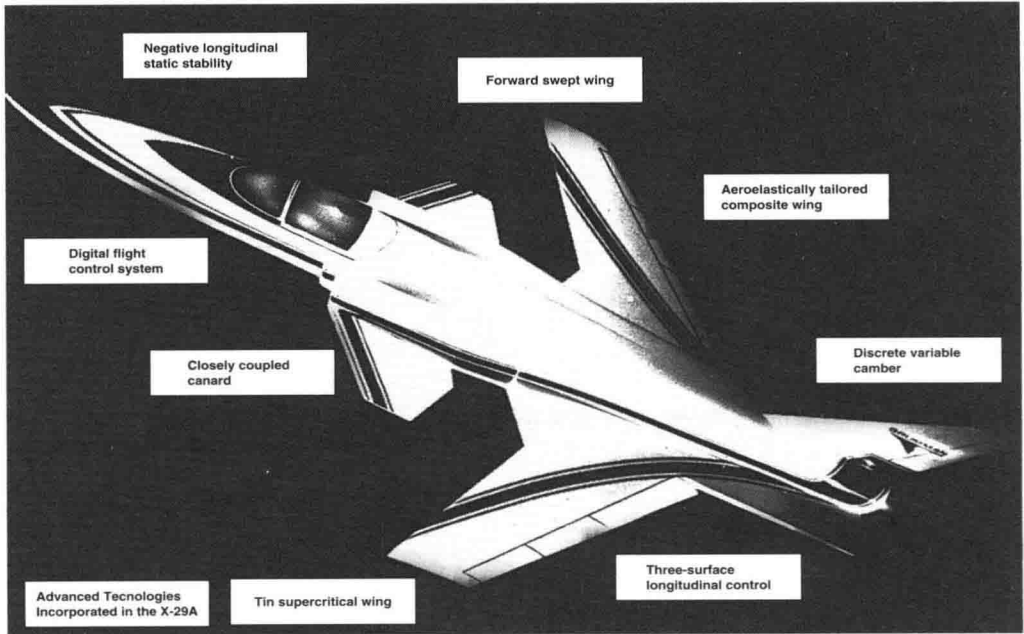
ATMOSPHERIC FLIGHT MECHANICS

Atmospheric flight mechanics is a broad heading that encompasses three major disciplines; namely, performance, flight dynamics, and aeroelasticity. In the past each of these subjects was treated independently of the others. However, because of the structural flexibility of modern airplanes, the interplay among the disciplines no longer can be ignored. For example, if the flight loads cause significant structural deformation of the aircraft, one can expect changes in the airplane's aerodynamic and stability characteristics that will influence its performance and dynamic behavior.

Airplane performance deals with the determination of performance characteristics such as range, endurance, rate of climb, and takeoff and landing distance as well as flight path optimization. To evaluate these performance characteristics, one normally treats the airplane as a point mass acted on by gravity, lift, drag, and thrust. The accuracy of the performance calculations depends on how accurately the lift, drag, and thrust can be determined.

Flight dynamics is concerned with the motion of an airplane due to internally or externally generated disturbances. We particularly are interested in the vehicle's stability and control capabilities. To describe adequately the rigid-body motion of an airplane one needs to consider the complete equations of motion with six degrees of freedom. Again, this will require accurate estimates of the aerodynamic forces and moments acting on the airplane.

The final subject included under the heading of atmospheric flight mechanics is aeroelasticity. Aeroelasticity deals with both static and dynamic aeroelastic phenomena. Basically, aeroelasticity is concerned with phenomena associated with interactions between inertial, elastic, and aerodynamic forces. Problems that arise for a flexible aircraft include control reversal, wing divergence, and control surface flutter, to name just a few.

**FIGURE 1.1**

Advanced technologies incorporated in the X-29A aircraft.

This book is divided into three parts: The first part deals with the properties of the atmosphere, static stability and control concepts, development of aircraft equations of motion, and aerodynamic modeling of the airplane; the second part examines aircraft motions due to control inputs or atmospheric disturbances; the third part is devoted to aircraft autopilots. Although no specific chapters are devoted entirely to performance or aeroelasticity, an effort is made to show the reader, at least in a qualitative way, how performance specifications and aeroelastic phenomena influence aircraft stability and control characteristics.

The interplay among the three disciplines that make up atmospheric flight mechanics is best illustrated by the experimental high-performance airplane shown in Figure 1.1. The X-29A aircraft incorporates the latest advanced technologies in controls, structures, and aerodynamics. These technologies will provide substantial performance improvements over more conventional fighter designs. Such a design could not be developed without paying close attention to the interplay among performance, aeroelasticity, stability, and control. In fact, the evolution of this radical design was developed using trade-off studies between the various disciplines to justify the expected performance improvements.

The forces and moments acting on an airplane depend on the properties of the atmosphere through which it is flying. In the following sections we will review some basic concepts of fluid mechanics that will help us appreciate the atmospheric properties essential to our understanding of airplane flight mechanics. In addition we will discuss some of the important aircraft instruments that provide flight information to the pilot.

1.2 BASIC DEFINITIONS

The aerodynamic forces and moments generated on an airplane are due to its geometric shape, attitude to the flow, airspeed, and the properties of the ambient air mass through which it is flying. Air is a fluid and as such possesses certain fluid properties. The properties we are interested in are the pressure, temperature, density, viscosity, and speed of sound of air at the flight altitude.

1.2.1 Fluid

A fluid can be thought of as any substance that flows. To have such a property, the fluid must deform continuously when acted on by a shearing force. A shear force is a force tangent to the surface of the fluid element. No shear stresses are present in the fluid when it is at rest. A fluid can transmit forces normal to any chosen direction. The normal force and the normal stress are the pressure force and pressure, respectively.

Both liquids and gases can be considered fluids. Liquids under most conditions do not change their weight per unit of volume appreciably and can be considered incompressible for most engineering applications. Gases, on the other hand, change their weight or mass per unit of volume appreciably under the influences of pressure or temperature and therefore must be considered compressible.

1.2.2 Pressure

Pressure is the normal force per unit area acting on the fluid. The average pressure is calculated by dividing the normal force to the surface by the surface area:

$$P = \frac{F}{A} \quad (1.1)$$

The static pressure in the atmosphere is nothing more than the weight per unit of area of the air above the elevation being considered. The ratio of the pressure P at altitude to sea-level standard pressure P_0 is given the symbol δ :

$$\delta = \frac{P}{P_0} \quad (1.2)$$

The relationship between pressure, density ρ , and temperature T is given by the equation of state

$$P = \rho RT \quad (1.3)$$

where R is a constant, the magnitude depending on the gas being considered. For air, R has a value 287 J/(kg $^\circ$ K) or 1718 ft 2 /(s 2 R). Atmospheric air follows the

equation of state provided that the temperature is not too high and that air can be treated as a continuum.

1.2.3 Temperature

In aeronautics the temperature of air is an extremely important parameter in that it affects the properties of air such as density and viscosity. Temperature is an abstract concept but can be thought of as a measure of the motion of molecular particles within a substance. The concept of temperature also serves as a means of determining the direction in which heat energy will flow when two objects of different temperatures come into contact. Heat energy will flow from the higher temperature object to that at lower temperature.

As we will show later the temperature of the atmosphere varies significantly with altitude. The ratio of the ambient temperature at altitude, T , to a sea-level standard value, T_0 is denoted by the symbol θ :

$$\theta = \frac{T}{T_0} \quad (1.4)$$

where the temperatures are measured using the absolute Kelvin or Rankine scales.

1.2.4 Density

The density of a substance is defined as the mass per unit of volume:

$$\rho = \frac{\text{Mass}}{\text{Unit of volume}} \quad (1.5)$$

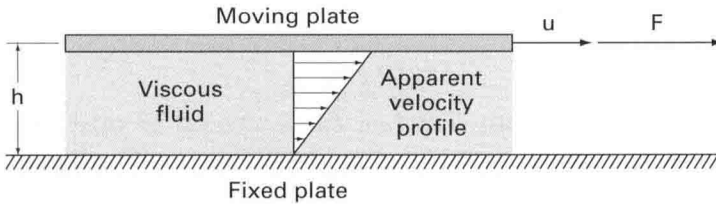
From the equation of state, it can be seen that the density of a gas is directly proportional to the pressure and inversely proportional to the absolute temperature. The ratio of ambient air density ρ to standard sea-level air density ρ_0 occurs in many aeronautical formulas and is given the designation σ :

$$\sigma = \rho/\rho_0 \quad (1.6)$$

1.2.5 Viscosity

Viscosity can be thought of as the internal friction of a fluid. Both liquids and gases possess viscosity, with liquids being much more viscous than gases. As an aid in visualizing the concept of viscosity, consider the following simple experiment. Consider the motion of the fluid between two parallel plates separated by the distance h . If one plate is held fixed while the other plate is being pulled with a constant velocity u , then the velocity distribution of the fluid between the plates will be linear as shown in Figure 1.2.

To produce the constant velocity motion of the upper plate, a tangential force must be applied to the plate. The magnitude of the force must be equal to the

**FIGURE 1.2**

Shear stress between two plates.

friction forces in the fluid. It has been established from experiments that the force per unit of area of the plate is proportional to the velocity of the moving plate and inversely proportional to the distance between the plates. Expressed mathematically we have

$$\tau \propto \frac{u}{h} \quad (1.7)$$

where τ is the force per unit area, which is called the shear stress.

A more general form of Equation (1.7) can be written by replacing u/h with the derivative du/dy . The proportionality factor is denoted by μ , the coefficient of absolute viscosity, which is obtained experimentally.

$$\tau = \mu \frac{du}{dy} \quad (1.8)$$

Equation (1.8) is known as Newton's law of friction.

For gases, the absolute viscosity depends only on the temperature, with increasing temperature causing an increase in viscosity. To estimate the change in viscosity with the temperature, several empirical formulations commonly are used. The simplest formula is Rayleigh's, which is

$$\frac{\mu_1}{\mu_0} = \left(\frac{T_1}{T_0} \right)^{3/4} \quad (1.9)$$

where the temperatures are on the absolute scale and the subscript 0 denotes the reference condition.

An alternate expression for calculating the variation of absolute viscosity with temperature was developed by Sutherland. The empirical formula developed by Sutherland is valid provided the pressure is greater than 0.1 atmosphere and is

$$\frac{\mu_1}{\mu_0} = \left(\frac{T_1}{T_0} \right)^{3/2} \frac{T_0 + S_1}{T_1 + S_1} \quad (1.10)$$

where S_1 is a constant. When the temperatures are expressed in the Rankine scale, $S_1 = 198^\circ\text{R}$; when the temperatures are expressed in the Kelvin scale, $S_1 = 110^\circ\text{K}$.

The ratio of the absolute viscosity to the density of the fluid is a parameter that appears frequently and has been identified with the symbol ν ; it is called the