

Simulation Techniques in Financial Risk Management

Second Edition

NGAI HANG CHAN
HOI YING WONG

STATISTICS IN PRACTICE



WILEY

SIMULATION TECHNIQUES IN FINANCIAL RISK MANAGEMENT

Second Edition

NGAI HANG CHAN AND HOI YING WONG

The Chinese University of Hong Kong

WILEY

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**SIMULATION
TECHNIQUES IN
FINANCIAL RISK
MANAGEMENT**

Second Edition

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To our families
N.H. Chan and H.Y. Wong

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PREFACE

PREFACE TO THE SECOND EDITION

This book has now been in print for almost 10 years and has seen several printings. During this period, the field of quantitative finance has experienced abrupt changes, some for better and some for worse. But it has been very gratifying to us to have heard from many readers that this book has been helpful to them in dealing with the ever-changing financial landscape. It appears that to some extent at least the original objectives set out in the first edition have been realized. This book can be used either as an introductory text to simulations at the senior undergraduate or as a Master's level course. It can also be used as a complimentary source to the more specialized treatise by Chan and Wong (2013) entitled *Handbook of Financial Risk Management: Simulations and Case Studies*.

This second edition has been thoroughly revised and enhanced. Many of these changes were results of teaching different courses in simulation for financial risk managers over the years. In addition to cleaning up as many errors and misprints as possible, the following specific changes have been incorporated in this revision.

- Many readers suggested more exercises with worked solutions. As a result, we enlarge the problems and answers section in light of these requests.
- Because the use of VBA in Excel has been common in the financial industry, the current edition incorporates this suggestion. We have now replaced all S-Plus codes with VBA codes.
- Due to the advent in IT technology, a new website has been set up for readers to download the VBA computer codes.
<http://www.sta.cuhk.edu.hk/Book/SRMS/>

As long as the website is available, we no longer print computer codes, so that more space can be used for expanded topics.

- Likewise, suggested solutions to exercises at the end of each chapter are now available via online supplementary materials.
- To make the book self-contained, two new chapters, Chapters 1 and 2, have been added. Chapter 1 introduces basic concepts of Excel VBA, and Chapter 2 introduces basic concepts of derivatives.
- Corresponding to Chapter 9 in the first edition, Chapter 11 of this edition is expanded to discuss in detail a one-factor interest rate model and the calibration to yield curves.
- More examples have been added to illustrate the concept of MCMC, in particular the Metropolis–Hastings algorithm.

Finally, we would like to thank colleagues and students alike, who have been giving us suggestions and ideas throughout the years. In particular, we would like to thank the editorial assistance of Dr. Warwick Yuen and Mr. Tom Ng of CUHK and Ms. Sari Friedman and Mr. Jon Gurstelle of Wiley. We also want to express our gratitude to the Research Grants Council of HKSAR for support at various stages of our work on this revision.

NGAI HANG CHAN AND HOI YING WONG

Shatin, Hong Kong

PREFACE TO THE FIRST EDITION

Risk management is an important subject in finance. Despite its popularity, risk management has a broad and diverse definition that varies from individual to individual. One fact remains, however. Every modern risk management method comprises a significant amount of computations. To assess the success of a risk management procedure, one has to rely heavily on simulation methods. A typical example is the pricing and hedging of exotic options in the derivative market. These over-the-counter options experience very thin trading volume, and yet their nonlinear features forbid the use of analytical techniques. As a result, one has to rely on simulations in order to examine their properties. It is therefore not surprising that simulation has become an indispensable tool in the financial and risk management industry today.

Although simulation as a subject has a long history by itself, the same cannot be said about risk management. To fully appreciate the power and usefulness of risk management, one has to acquire a considerable amount of background knowledge across several disciplines: finance, statistics, mathematics, and computer science. It is the synergy of various concepts across these different fields that marks the success of modern risk management. Although many excellent books have been written on the subject of simulation, none has been written from a risk management perspective. It is therefore timely and important to have a text that readily introduces the modern techniques of simulation and risk management to the financial world.

This text aims at introducing simulation techniques for practitioners in the financial and risk management industry at an intermediate level. The only prerequisite is a standard undergraduate course in probability at the level of Hogg and Tanis (2006), say, and some rudimentary exposure to finance. The present volume stems from a set of lecture notes used at the Chinese University of Hong Kong. It aims at striking a balance between theory and applications of risk management and simulations, particularly along the financial sector. The book comprises three parts.

- Part one consists of the first three chapters. After introducing the motivations of simulation in Chapter 1, basic ideas of Wiener processes and Itô's calculus are introduced in Chapters 2 and 3. The reason for this inclusion is that many students have experienced difficulties in this area because they lack the understanding of the theoretical underpinnings of these topics. We try to introduce these topics at an operational level so that readers can immediately appreciate the complexity and importance of stochastic calculus and its relationship with simulations. This will pave the way for a smooth transition to option pricing and Greeks in later chapters. For readers familiar with these topics, this part can be used as a review.
- Chapters 4–6 comprise the second part of the book. This part constitutes the main core of an introductory course in risk management. It covers standard topics in a traditional course in simulation, but at a much higher and succinct level. Technical details are left in the references, but important ideas are explained in a conceptual manner. Examples are also given throughout to illustrate the use of these techniques in risk management. By introducing simulations this way, both students with strong theoretical background and students with strong practical motivations get excited about the subject early on.

- The remaining Chapters 7–10 constitute part 3 of the book. In this part, more advanced and exotic topics of simulations in financial engineering and risk management are introduced. One distinctive feature in these chapters is the inclusion of case studies. Many of these cases have strong practical bearings such as pricing of exotic options, simulations of Greeks in hedging, and the use of Bayesian ideas to assess the impact of jumps. By means of these examples, it is hoped that readers can acquire a first-hand knowledge about the importance of simulations and apply them to their work.

Throughout the book, examples from finance and risk management have been incorporated as much as possible. This is done throughout the text, starting at the early chapter that discusses VaR of Dow to pricing of basket options in a multiasset setting. Almost all of the examples and cases are illustrated with Splus and some with Visual Basics. Readers would be able to reproduce the analysis and learn about either Splus or Visual Basics by replicating some of the empirical work.

Many recent developments in both simulations and risk management, such as Gibbs sampling, the use of heavy-tailed distributions in VaR calculation, and principal components in multiasset settings are discussed and illustrated in detail. Although many of these developments have found applications in the academic literature, they are less understood among practitioners. Inclusion of these topics narrows the gap between academic developments and practical applications.

In summary, this text fills a vacuum in the market of simulations and risk management. By giving both conceptual and practical illustrations, this text not only provides an efficient vehicle for practitioners to apply simulation techniques, but also demonstrates a synergy of these techniques. The examples and discussions in later chapters make recent developments in simulations and risk management more accessible to a larger audience.

Several versions of these lecture notes have been used in a simulation course given at the Chinese University of Hong Kong. We are grateful for many suggestions, comments, and questions from both students and colleagues. In particular, the first author is indebted to Professor John Lehoczy at Carnegie Mellon University, from whom he learned the essence of simulations in computational finance. Part 2 of this book reflects many of the ideas of John and is a reminiscence of his lecture notes at Carnegie Mellon. We would also like to thank Yu-Fung Lam and Ka-Yung Lau for their help in carrying out some of the computational tasks in the examples and for producing the figures in LaTeX, and to Mr. Steve Quigley and Ms. Susanne Steitz, both from Wiley, for their patience and professional assistance in guiding the preparation and production of this book. Financial support from the Research Grant Council of Hong Kong throughout this project is gratefully acknowledged. Last, but not least, we would like to thank our families for their understanding and encouragement in writing this book. Any remaining errors are, of course, our sole responsibility.

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