Manfred Kaltenbacher

Numerical Simulation of Mechatronic Sensors and Actuators

Finite Elements for Computational Multiphysics

Third Edition

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Preface to the Third Edition

The third edition of this book fully preserves the character of the previous editions to combine physical modeling of mechatronic systems and their numerical simulation using the Finite Element (FE) method. Most of the text and general appearance of the previous editions were retained, while the topics have been strongly extended and the presentation improved. Thereby, the third edition contains the following main extensions:

- Finite elements of higher order
- Flexible discretization towards non-conforming methods
- Computational fluid dynamics and coupled fluid-solid-interaction (FSI)
- Perfectly matched layer (PML) technique in time domain
- Comprehensive discussion of aeroacoustics with latest numerical schemes
- Advanced numerical schemes for piezoelectricity including macro- and micromechanical models.

We have enhanced the basic chapter concerning the Finite Element (FE) method by now providing the FE basis functions and integration points for all geometric elements (quadrilateral, triangle, tetrahedron, hexahedron, wedge and pyramid). Furthermore, we discuss in detail p-FEM (finite elements of higher order) both for nodal (see Sect. 2.9.1) and for edge finite elements (see Sect. 6.7.6) and also extend the scope to spectral elements (see Sect. 2.9.2). In addition, we provide a detailed discussion of non-conforming techniques, both the classical Mortar method and Nitsche type mortaring (see Sect. 2.10).

As a new physical field, we now have also included the flow field (see Chap. 4), which allows us to derive linear acoustics by using a perturbation ansatz on the conservation of mass and momentum. The FE discretization of the Navier-Stokes equations leads to a stabilized FE formulation, more precisely we apply a Streamline Upwind Petrov Galerkin/Pressure Stabilized Petrov Galerkin (SUPG/PSPG) formulation. Furthermore, in Chap. 7 we discuss the fluid–solid interaction (FSI), and now have the capability to present aeracoustics by a more general approach. Thereby, we have extended the discussion of computational aeroacoustics by the classical vortex sound theory and by perturbation approaches, which allows a decomposition of flow and acoustic quantities within the flow region (see Chap. 9).

Towards computational acoustics, we have extended our discussion for open domain problems by a new formulation for the time domain perfectly matched layer (PML) technique (see Sect. 5.5.2) and a mixed FE ansatz to solve the acoustic conservation equations in a quite efficient way using spectral elements (see Sect. 5. 4.2). Furthermore, we present latest piezoelectric models for precisely describing the polarization process (micro-mechanical model, see Sect. 12.4.2) and the whole operation range for actuators (hysteresis operator-based model, see Sect. 12.5.2).

Finally, we present new industrial applications: (1) cofired piezoceramic multilayer actuators simulated with both the micro-mechanical and the hysteresis operator-based model (see Sect. 14.4); (2) simulation of the human phonation (see Sect. 14.7); (3) flow induced sound of obstacles in cross flow, edge tone and airframe noise (see Sect. 14.8).

All presented numerical schemes have been implemented in our in-house research code CFS++ (Coupled Field Simulation) and most of the algorithms have also found their way to the commercial software NACS (see http://www.simetris.eu).

Acknowledgments

The author wishes to acknowledge the many contributions that colleagues and collaborators have made to this third edition. First of all I would like to express my gratitude to all members of my research group at Vienna University of Technology. Amongst many, I wish to specially thank for contributions: Andreas Hüppe to computational acoustics and aeroacoustics as well as spectral finite elements; Stefan Zörner to fluid dynamics and human phonation; Simon Triebenbacher to non-conforming grid techniques and computational aeroacoustics; Andreas Hauck to finite elements of higher order. Furthermore, I would like to thank my former Ph.D. student Gerhard Link (now at CD-adapco Germany) for his contribution towards fluid dynamics and fluid-solid-interactions, and Stefan Becker and his team (Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany) for the fruitful cooperation on aeroacoustics. Much is owned by many discussions with my wife Barbara Kaltenbacher (Alpen-Adria-Universität Klagenfurt, Austria) on the modeling of piezoelectricity and the development of stable perfectly matched layers in time domain. For the exciting cooperation on flexible discretization within our common DFG (German Science Foundation)—FWF (The Austrian Science Foundation) project *Numerical Simulation of Acoustics-Acoustics- and Structural Mechanics-Acoustics-Couplings on Nonmatching Grids* I would like to thank Barbara Wohlmuth (Technische Universität München, Germany). Furthermore, many thanks to Michael Döllinger (Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany) for the fruitful cooperation within the DFG-FWF project *Physical basics of the human voice*. In this context, also special thanks to Petr Sildof (Technical University of Liberec, Czech Republic) for the cooperation within

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Preface to the Second Edition

The second edition of this book fully preserves the character of the first edition to combine the detailed physical modeling of mechatronic systems and their precise numerical simulation using the Finite Element (FE) method. Most of the text and general appearance of the previous edition were retained, while the coverage was extended and the presentation improved.

Starting with Chap. 2, which discusses the theoretical basics and computer implementation of the FE method, we have added a section describing the FE method for one-dimensional cases, especially to provide a easier understanding of this important numerical method for solving partial differential equations. In addition, we provide a section about a priori error estimates. In Chap. 3, which deals with mechanical fields, we now additionally discuss locking effects as occurring in the numerical computation of thin structures, and describe two well established methods (method of incompatible modes and of enhanced assumed strain) as well as a recently newly developed scheme based on balanced reduced and selective integration. The physical discussion of acoustic sound generation and propagation (see Chap. 5 has been strongly improved, including now also a description of plane and spherical waves as well as a section about quantitative measures of sound. The treatment of open domain problems has been extended and include a recently developed Perfectly Matched Layer (PML) technique, which allows to limit the computational domain to within a fraction of the wavelength without any spurious reflections.

Recently developed flexible discretization techniques based on the framework of mortar FE methods for the numerical solution of coupled wave propagation problems allow for the use of different fine meshes within each computational subdomain. This technique has been applied to pure wave propagation problems (see Sect. 5.4.3) as well as coupled mechanical-acoustic field problems (see Sect. 8.3.2), where the computational grids of the mechanical region and the acoustic region can be independently generated and therefore do not match at the interface. Furthermore, we have investigated in the piezoelectric effect and provide in Chap. 9 an extended discussion on the modeling and numerical computation of nonlinear effects including hysteresis.

In the last three years, we have established a research group on computational aeroacoustics to study the complex phenomenon of flow induced noise. Therewith, the totally new Chap. 10 contains a description of computational aeroacoustics with a main focus on a recently developed FE method for efficiently solving Lighthill's acoustic analogy.

Within Chap. 12, which deals with industrial applications, we have rewritten Sect. 12.5 to discuss latest computational results on micromachined capacitive ultrasound transducers, and have added a section on high power ultrasound sources as used for lithotripsy as well as a section on noise generation by turbulent flows.

Most of the formulations described in this book have been implemented in the software NACS (see http://www.simetris.eu/).

Acknowledgment

The author wishes to acknowledge the many contributions that colleagues and collaborators have made to this second edition. First of all I would like to express my gratitude to the members of the Department of Sensor Technology and its head Prof. Reinhard Lerch for the pleasant and stimulating working atmosphere. Amongst many, I wish to specially thank M.Sc. Max Escobar, M.Sc. Andreas Hauck, M.Sc. Gerhard Link, Dipl.-Ing. Thomas Hegewald and Dipl.-Ing. Luwig Bahr for fruitful discussions and proof reading. Much is owned by many intensive discussions with my wife Prof. Barbara Kaltenbacher with whom I work on hysteresis models and parameter identification for electromagnetics and piezoelectrics. Special thanks are dedicated to Dr. Stefan Becker and his co-workers M.Sc. Irfan Ali and Dr. Frank Schäfer for the contribution on computational aeroacoustics and the intensive cooperation within the current research project *Fluid-Structure-Noise* founded by the Bavarian science foundation BFS. Furthermore, the author would like to thank Dr. Bernd Flemisch and Prof. Barbara Wohlmuth for the fruitful cooperation on nonmatching grids. A common research project on *Numerical Simulation of Acoustic-Acoustic- and Mechanical-Acoustic-Couplings on Nonmatching Grids* founded by German Research Foundation DFG has just started. Moreover, the author wants to acknowledge the excellent working environment at the Johann Radon Institute for Computational and Applied Mathematics in Linz, Austria, where the author stayed for one semester in 2005/06 as an invited lecturer for coupled field problems within a special semester on computational mechanics. Special thank is dedicated to Prof. Ulrich Langer, who organized this event, and who did a great job in bringing together different researchers from all over the world. During this time, I also started the cooperation with Prof. Dietrich Braess on enhanced softening techniques to avoid locking in thin mechanical structures, to whom I would like to express my gratitude for revealing new and interesting perspectives to me.

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Preface to the First Edition

The focus of this book is concerned with the modeling and precise numerical simulation of mechatronic sensors and actuators. These sensors, actuators, and sensor-actuator systems are based on the mutual interaction of the mechanical field with a magnetic, an electrostatic, or an electromagnetic field. In many cases, the transducer is immersed in an acoustic fluid and the solid–fluid coupling has to be taken into account. Examples are piezoelectric stack actuators for common-rail injection systems, micromachined electrostatic gyro sensors used in stabilizing systems of automobiles or ultrasonic imaging systems for medical diagnostics.

The modeling of mechatronic sensors and actuators leads to so-called *multifield* problems, which are described by a system of nonlinear partial differential equations. Such systems cannot be solved analytically and thus a numerical calculation scheme has to be applied. The schemes discussed in this book are based on the *finite element* (FE) method, which is capable of efficiently solving the partial differential equations. The complexity of the simulation of multifield problems consists of the simultaneous computation of the involved single fields as well as in the coupling terms, which introduce additional nonlinearities. Examples are moving conductive (electrically charged) body within a magnetic (an electric) field, electromagnetic and/or electrostatic forces.

The goal of this book is to present a comprehensive survey of the main physical phenomena of multifield problems and, in addition, to discuss calculation schemes for the efficient solution of coupled partial differential equations applying the FE method. We will concentrate on electromagnetic, mechanical, and acoustic fields with the following mutual interactions:

- Coupling *Electric Field—Mechanical Field*
  This coupling is either based on the piezoelectric effect or results from the force on an electrically charged structure in an electric field (electrostatic force).

- Coupling *Magnetic Field—Mechanical Field*
  This coupling is two-fold. First, we have the electromotive force (emf), which describes the generation of an electric field (electric voltage respectively current) when a conductor is moved in a magnetic field, and secondly, the electromagnetic force.
- Coupling *Mechanical Field—Acoustic Field*
  Very often a transducer is surrounded by a fluid or a gaseous medium in which an acoustic wave is launched (actuator) or is impinging from an outside source towards the receiving transducer.

In Chap. 2, we give an introduction to the finite element (FE) method. Starting from the strong form of a general partial differential equation, we describe all the steps concerning spatial as well as time discretization to arrive at an algebraic system of equations. Both nodal and edge finite elements are introduced. Special emphasis is put on an explanation of all the important steps necessary for the computer implementation.

A detailed discussion on electromagnetic, mechanical, and acoustic fields including their numerical computation using the FE method can be found in Chap. 3–5. Each of these chapters starts with the description of the relevant physical equations and quantities characterizing the according physical field. Special care is taken with the constitutive laws and the resultant nonlinearities relevant for mechatronic sensors and actuators. In addition, the numerical computation using the FE method is studied for the linear as well as the nonlinear case. In Chap. 4, where the electromagnetic field is discussed, we explain the difficulties arising at interfaces of jumping material parameters (electric conductivity and magnetic permeability), and introduce two correct formulations adequate for the FE method. At the end of each of these chapters, we present an example for the numerical simulation of a practical device.

In Chap. 6, we study the interaction between electrostatic and mechanical fields and concentrate on micromechanical applications. After the derivation of a general expression for the electrostatic force, applying the principle of virtual work, we focus on the numerical calculation scheme. The simulation of a simple electrostatic driven bar will demonstrate the complexity of such problems, and will show the necessity of taking into account mechanical nonlinearities.

The physical modeling and numerical solution of magnetomechanical systems is presented in Chap. 7. In this chapter, we first discuss the correct physical description of moving and/or deforming bodies in a magnetic field. Later, we derive a general expression for the electromagnetic force, again (as for the electrostatic force) by using the principle of virtual work. The discussion on numerical computation will contain a calculation scheme for the efficient solution of magnetomechanical systems and, in addition, electric circuit coupling as arise for voltage-driven coils. Especially for the latter case, we give a very comprehensive description of its numerical computation.

Chapter 8 deals with coupled mechanical-acoustic systems and explains the physical coupling terms and the numerical computation of such systems. The simulation of the sound emission of a car engine will illustrate different approaches concerning time-discretization schemes and solvers for the algebraic system.

A special coupling between the mechanical and electrostatic field occurs in piezoelectric systems, which are studied in Chap. 9. After explaining the piezoelectric effect and its physical modeling, we concentrate on the efficient numerical computation of such systems. Whereas for sensor applications a linear model can be usually used, in many actuator applications nonlinear effects play a crucial role, which we here account for by applying an appropriate hysteresis model.

Since the efficiency of the solution (both with respect to elapsed CPU time and computer memory resources) is of great importance, Chap. 10 deals with geometric and algebraic multigrid solvers. These methods achieve an optimal complexity, that is, the computational effort as well as memory requirement grows only linearly with the problem size. We present new especially adapted multigrid solvers for Maxwell's equation in the eddy current case and demonstrate their efficiency by means of TEAM (Testing Electromagnetic Analysis Methods) workshop problem 20 established by the Compumag Society [318].

After these rigorous derivations of methods for coupled field computation, Chap. 11 demonstrates the applicability to real-life problems arising in industry. This includes the following topics:

- Analysis and optimization of car loudspeakers
- Acoustic emission of electrical power transformers
- Simulation-based improvements of electromagnetic valves
- Piezoelectric stack actuators such as used, e.g., in common-rail diesel injection systems
- Ultrasonic imaging system based on capacitive micromachined ultrasound transducers

The appendices provide an introduction to vector analysis, functional spaces, and the solutions of nonlinear equations.

The structure of this book has been designed in such a way that in each of Chaps. 3–9 we first discuss the physical modeling of the corresponding single or coupled field, then the numerical simulation, followed by a simple computational example. If the reader has no previous knowledge of vector analysis, she/he should start with the first section of the Appendix. Chapter 2 can be omitted if the reader is only interested in the physical modeling of mechatronic sensors and actuators. The three chapters concerning the single field problems (mechanical, electromagnetic, acoustic) are written independently, so that the reader can start with any of them. Clearly, for the coupled field problems, the reader should have a knowledge of the involved physical fields or should have read the corresponding preceding chapters. Chapter 10 presents the latest topics on multigrid methods for electromagnetic fields and requires some knowledge on this topic. For a basic introduction of multigrid methods we refer to classical books [47, 256, 258]. Chapter 11 demonstrates the use of numerical simulation for industrial applications. For each of them, we first discuss the problem to be solved, followed by an analysis study applying numerical simulation to allow a better understanding of the different physical effects. For most applications, we also demonstrate measurements of the CAE-optimized prototype.

Most in this book described formulations for solving multifield problems have been implemented in the software CAPA (see http://www.wissoft.de).

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Notation

Mathematical Symbols

$\boldsymbol{e}$	Unit vector
$\boldsymbol{n}$	Unit normal vector
$\boldsymbol{t}$	Unit tangential vector
$\mathbb{R}$	Set of real numbers
r, x	Position vector
$\int_C \, \mathrm{d}s$	Contour integral
$\oint_C \, \mathrm{d}s$	Closed contour integral
$\int_\Gamma \, \mathrm{d}\boldsymbol{\Gamma}$	Surface integral
$\oint_\Gamma \, \mathrm{d}\boldsymbol{\Gamma}$	Closed surface integral
$\int_\Omega \, \mathrm{d}\Omega$	Volume integral
∇	Nabla operator
curl, $\nabla\times$	Curl
div, $\nabla\cdot$	Divergence
grad, ∇	Gradient
$\partial/\partial x$	Partial derivative
$\partial/\partial\mathbf{n}$	Partial derivative in normal direction
$\mathrm{d}/\mathrm{d}x$	Total derivative
$\Vert\Vert$	Norm
$\vert\;\vert$	Semi-norm
$[\;]$	Tensor notation
$[\boldsymbol{I}]$	Identity tensor

Finite Element Method

$\underline{u}, \underline{a}$, etc.	Nodal vectors of displacement, acceleration, etc
$\boldsymbol{C}$	Damping matrix
F_e	Mapping of element
$\boldsymbol{K}$	Stiffness matrix
$\boldsymbol{M}$	Mass matrix
n_{n}	Number of nodes
n_{en}	Number of nodes per finite element
n_{e}	Number of elements
n_{eq}	Number of equations
n_{d}	Space dimension
N_i	FE basis function for node i
$\mathcal{J}$	Jacobi matrix
$\lvert\mathcal{J}\rvert$	Jacobi determinant
x, y, z	Global coordinates
$\bar{\Omega}$	Whole simulation domain
Ω	Simulation domain without boundary
$\hat{\Omega}$	Domain of reference element
Γ	Boundary of simulation
Γ_{e}	Dirichlet boundary
Γ_{n}	Neumann boundary
γ_{P}	Integration parameter (parabolic PDE)
$\beta_{\mathrm{H}}, \gamma_{\mathrm{H}}$	Integration parameters (hyperbolic PDE)
ξ, η, ζ	Local coordinates

Mechanics

$\mathbf{a}$	Acceleration
$[\boldsymbol{c}]$	Tensor of mechanical modulus
c_{L}	Velocity of longitudinal wave
c_{T}	Velocity of shear wave
E_{m}	Elasticity module
$\boldsymbol{f}_{\mathrm{V}}$	Volume force
$[\boldsymbol{F}_{\mathrm{d}}]$	Deformation gradient
$[\boldsymbol{H}_{\mathrm{d}}]$	Displacement gradient
G	Shear modulus
m	Mass
P_{mech}	Mechanical power
$\boldsymbol{S}$	Linear strains (Voigt notation)
$[\boldsymbol{S}]$	Tensor of linear strains
$\boldsymbol{T}$	Second Piola-Kirchhoff stress (Voigt notation)
$[\boldsymbol{T}]$	Second Piola-Kirchhoff stress tensor

$\boldsymbol{u}$	Mechanical displacement
$\boldsymbol{v}$	Velocity
$\boldsymbol{V}$	Green-Lagrangian strain (Voigt notation)
$[\boldsymbol{V}]$	Green-Lagrangian strain tensor
$\alpha_{\mathrm{M}}, \alpha_{\mathrm{K}}$	Damping coefficients
ρ	Density
ν_{p}	Poisson ratio
$\boldsymbol{\sigma}$	Cauchy stress (Voigt notation)
$[\boldsymbol{\sigma}]$	Cauchy stress tensor
μ_L, λ_L	Lamé-parameters

Flow

e	Inner energy
Eu	Euler number
Fr	Froude number
$\boldsymbol{I}$	Momentum
$\boldsymbol{I}_{\mathrm{m}}$	Molecular momentum
Ma	Mach number
q_{h}	Heat production per volume
$\boldsymbol{q}_{\mathrm{T}}$	Heat flux
p	Flow pressure
P	Kinematic pressure
Re	Reynolds number
s	Entropy
St	Strouhal number
$\boldsymbol{v}$	Flow velocity
$[\epsilon]$	Strain rate
λ_{f}	Bulk viscosity
μ_{f}	Dynamic viscosity
ν_{f}	Kinematic viscosity
$[\boldsymbol{\pi}]$	Momentum flux tensor
ρ	Density
$[\boldsymbol{\sigma}_{\mathrm{f}}]$	Fluid stress tensor
$[\boldsymbol{\tau}]$	Viscous stress tensor
$\boldsymbol{\omega}$	Vorticity

Acoustics

b	Diffusivity of sound
B/A	Parameter of nonlinearity
c	Speed of sound
c_{p}	Specific heat by constant pressure

c_Ω	Specific heat by constant volume
e	Inner energy
I_a	Sound-field intensity
k	Wave number
K_s	Adiabatic bulk modulus
L_{p_a}, SPL	Sound-pressure level
L_{I_a}	Sound-intensity level
L_{P_a}	Sound-power level
p_a	Acoustic pressure
p_0	Mean pressure
P_a	Acoustic power
q_h	Heat production per volume
s	Entropy
$\boldsymbol{v}_a$	Acoustic particle velocity
w_a	Acoustic energy density
x_s	Shock formation distance
Z_a	Acoustic impedance
ρ_a	Acoustic density
ρ_0	Mean density
κ	Adiabatic exponent
λ	Wavelength
λ_f	Bulk viscosity
μ_f	Dynamic viscosity
ψ	Scalar acoustic potential

Electromagnetics

$\boldsymbol{A}$	Magnetic vector potential
$\boldsymbol{B}$	Magnetic flux density
$\boldsymbol{D}$	Electric flux density
$\boldsymbol{E}$	Electric field intensity
$\boldsymbol{F}_{el}$	Electric force
$\boldsymbol{F}_{mag}$	Magnetic force
$\boldsymbol{H}$	Magnetic field intensity
I, i	Electric current
$\boldsymbol{J}$	Current density
$\boldsymbol{J}_i$	Impressed current density
$\boldsymbol{M}$	Magnetization
q_e	Electric charge density
Q_e	Total electric charge
$\boldsymbol{P}$	Electric polarization
R	Ohmic resistor
u_{ind}	Induced voltage
V_e	Scalar electric potential

w_{el}	Electric energy density
W_{el}	Total electric energy
w_{mag}	Magnetic energy density
W_{mag}	Total magnetic energy
δ	Skin depth
ρ_{e}	Specific electric resistance
γ	Electrical conductivity
ε	Electric permittivity
μ	Magnetic permeability
ν	Magnetic reluctivity
σ_{e}	Electric surface charge
ϕ	Magnetic flux
ψ_{m}	Reduced magnetic scalar potential
Ψ	Total magnetic flux