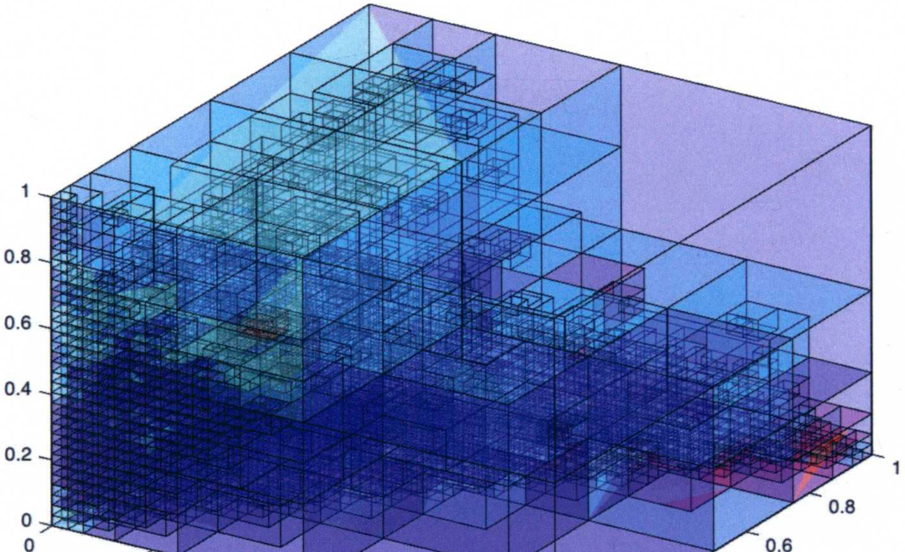
Mathematical Methods of Physics

Karl Barnes



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Mathematical Methods of Physics continues to provide all the mathematical methods that aspiring scientists and engineers are likely to encounter as students and beginning researchers. It provides an accessible account of most of the current, important mathematical tools required in physics these days. It is assumed that the reader has an adequate preparation in general physics and calculus. The book is designed primarily for researcher and practitioner as well as advanced graduate physics majors, but could also be used by students in other subjects, such as engineering, astronomy and mathematics. First chapter reviews the fundamental diffusion theories relevant to the general F2 law, where they are systematically reframed in points of view different from the previous works, adding some new discussions to them. The new findings obtained here will be widely applicable to fundamental problems as a standard theory in various actual diffusion phenomena. The main objective of second chapter is used the extended mapping method and auxiliary equation method to construct the exact solutions for nonlinear evolution equations in the mathematical physics via the variant Boussinesq equations and the coupled KdV equations. The purpose of third chapter is to avoid such contradictions by using new mathematical methods coming from the formal theory of systems of partial differential equations and Lie pseudo groups. The purpose of fourth chapter is to present for the first time an elementary summary of a few recent results obtained through the application of the formal theory of partial differential equations and Lie pseudogroups in order to revisit the mathematical foundations of general relativity. Fifth chapter demonstrate that the use of Bayesian statistics conforms to the Maximum Entropy Principle in information theory and Bayesian approach successfully resolves dilemmas in the uneven probability Monty Hall variant. Sixth chapter presents a numerical method for nonlinear singularly perturbed multi-point boundary value problem. In seventh chapter, an analytical model for multifractal systems is developed by combining and improving the Jake model, Tyler fractal model and Gompertz curve, which allows one to obtain explicit expressions of a multifractal spectrum. Eighth chapter emphasizes on transformation formulas for the first kind of lauricella's function of several variables. In ninth chapter, we will investigate the solution of the nonlinear Zhiber-Shabat equation and last chapter deals with methods for ordinary differential equations.

Karl Barnes hold Ph.D. in Mathematical Analysis. He served as Visiting Professor of Physics more than seven years. He has published several articles and research papers in physics journals and is the co-author of two textbooks.

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Preface

Mathematical Methods of Physics continues to provide all the mathematical methods that aspiring scientists and engineers are likely to encounter as students and beginning researchers. It provides an accessible account of most of the current, important mathematical tools required in physics these days. It is assumed that the reader has an adequate preparation in general physics and calculus. The book is designed primarily for researcher and practitioner as well as advanced graduate physics majors, but could also be used by students in other subjects, such as engineering, astronomy and mathematics. First chapter reviews the fundamental diffusion theories relevant to the general F2 law, where they are systematically reframed in points of view different from the previous works, adding some new discussions to them. The new findings obtained here will be widely applicable to fundamental problems as a standard theory in various actual diffusion phenomena. The main objective of second chapter is used the extended mapping method and auxiliary equation method to construct the exact solutions for nonlinear evolution equations in the mathematical physics via the variant Boussinesq equations and the coupled KdV equations. The purpose of third chapter is to avoid such contradictions by using new mathematical methods coming from the formal theory of systems of partial differential equations and Lie pseudo groups. The purpose of fourth chapter is to present for the first time an elementary summary of a few recent results obtained through the application of the formal theory of partial differential equations and Lie pseudogroups in order to revisit the mathematical foundations of general relativity. Fifth chapter demonstrate that the use of Bayesian statistics conforms to the Maximum Entropy Principle in information theory and Bayesian approach successfully resolves dilemmas in the uneven probability Monty Hall variant. Sixth chapter presents a numerical method for nonlinear singularly perturbed multi-point boundary value problem. In seventh chapter, an analytical model for multifractal systems is developed by combining and improving the Jake model, Tyler fractal model and Gompertz curve, which allows one to obtain explicit expressions of a multifractal spectrum. Eighth chapter emphasizes on transformation formulas for the first kind of lauricella's function of several variables. In ninth chapter, we will investigate the solution of the nonlinear Zhiber-Shabat equation and last chapter deals with methods for ordinary differential equations.

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MATHEMATICAL PHYSICS IN DIFFUSION PROBLEMS

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ABSTRACT

Using the divergence theorem and the coordinate transformation theory for the general Fickian second law, fundamental diffusion problems are investigated. As a result, the new findings are obtained as follows. The unified diffusion theory is reasonably established, including a self-diffusion theory and an N (N ^{3} 2) elements system interdiffusion one. The Fickian first law is incomplete without a constant diffusion flux corresponding to the Brown motion in the localized space. The cause of Kirkendall effect and the nonexistence of intrinsic diffusion concept are theoretically revealed. In the parabolic space, an elegant analytical method of the diffusion equation is mathematically established, including a nonlinear diffusion equation. From the Schrödinger equation and the diffusion equation, the universal expression of diffusivity proportional to the Planck constant is reasonably obtained. The material wave equation proposed by de Broglie is also derived in relation to the Brown motion. The fundamental diffusion theories discussed here will be highly useful as a standard theory for the basic study of actual interdiffusion problems such as an alloy, a compound semiconductor, a multilayer thin film, and a microstructure material.

Keywords: Brown Particle, Boltzmann Factor, Markov Process, Parabolic Law, Error Function

INTRODUCTION

First of all, we state that the basic diffusion equation of the general nonlinear Fickian second law is discussed in accordance with the fundamental mathematical physics in the present work. The extended diffusion equations in detail are not thus discussed. Nevertheless, the new findings, which are extremely dominant in the diffusion study, are reasonably obtained. In the diffusion history, the problems relevant to the coordinate transformation of diffusion equation had not been discussed in accordance with the Gauss divergence theorem until recently. That is just a reason why the new diffusion theories are discussed in the present study. It will be gradually clarified in the text that the coordinate transformation theory is essentially indispensable for the diffusion study. It is obvious that analyzing the extended diffusion equation must be based on the fundamental diffusion theory. The new fundamental findings different from the existing diffusion theories obtained here will thus exert a great influence on the actual diffusion problems in detail, just because of fundamental ones.

A great many phenomena in various science fields are expressed by using the well-known evolution equations. The diffusion equation is one of them and mathematically corresponds to the Markov process in relation to the normal distribution rule [1] . In other words, the motion of diffusion particles corresponds to the well-known Brown movement satisfying the parabolic law [2] [3] . It is widely accepted that the Brown problem is a general term of investigating subjects in various science fields relevant to the Markov process, such as material science, information science, life science, and social science [4] - [9] .

In physics, we can also understand the diffusion equation in accordance with the Gauss divergence theorem [10] . If we apply the divergence theorem to the diffusion problem for a material under the condition of no sink and source of the material, it is found that the material conservation law is valid for the diffusion particles, regardless of a thermodynamic state of material. The diffusion equation is also called "the continuous equation" and is extremely fundamental one in physics. In history, the heat conduction equation, which is mathematically equivalent to the diffusion equation, was proposed by Fourier, regardless of the Markov process and the divergence theorem [11] .

In accordance with the industrial requirement, the solid materials, such as alloys, semiconductors, and multilayer materials, have been widely fabricated. The heat treatment is indispensable for their fabrication processes then. The migration of particles in a material is caused by the heat treatment. In relation to the migration of their particles, the diffusion problems of various solid materials have been thus widely investigated [12] . Therefore, the diffusion problem is a fundamental study subject in the materials science including the cases of liquid and gas states.

In the present work, the fundamental problems of the general Fickian second law where a driving force affects the diffusion system are discussed in accordance with the mathematical theory. The present analytical method is applicable to interdiffusion problems of an N elements system of every material in an arbitrary thermodynamic state. Although the physical validity of the present method is investigated by using the diffusion data concerning the solid metals, the mathematical generality discussed here is still kept.

The heat conduction equation proposed by Fourier in 1822 has been applied to investigating the temperature distribution in materials [11] . In 1827, the so-called Brown motion was found, where the self-diffusion of water was visualized by pollen micro particle motions [2] [3] . In 1855, Fick applied the heat conduction equation to diffusion phenomena as it had been [13] . Nevertheless, the Brown motion had not been

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