



ARTHUR W. LEES

Vibration Problems in Machines

Diagnosis and Resolution



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Preface

The understanding of vibration signals from turbo-machinery is an important feature of many industries. It forms the basis of condition monitoring and is of crucial importance in the rapid diagnosis and rectification of faults. The field calls for a variety of skills ranging from instrumentation expertise to mathematical manipulation and signal processing. The focus is on gaining a physical understanding of the processes giving rise to vibration signals. This may be viewed as a process of seeking to infer internal conditions from (sometimes limited) external measured data.

This often requires some form of mathematical model, and the approach taken is to use some basic finite element analysis to study system behaviour. These methods were first applied to rotating machinery several decades ago, but with progress in computing and the advent of packages such as MATLAB®, the ease with which the concepts may be applied has dramatically improved. It is now a relatively straightforward matter to examine proposed design changes – in effect, to perform numerical experiments.

The material in this book stems both from my experiences in the industry and, in more recent years, of teaching and research in academia. Inevitably, my own experiences have, to some degree, influenced my choice of examples. In my various posts, I have had fruitful interactions with many people who have significantly enhanced my perception of problems in machinery. There are too many people involved to thank individually, but I must mention three. Professor Mike Friswell and I have worked together closely for 20 years, and I acknowledge innumerable helpful and illuminating discussions. I also thank Professor John Penny for his help with proofreading and a number of helpful suggestions on the structure of the text. Both of these people were my co-authors, together with Professor Seamus Garvey, of the book I refer to in many places which gives a more complete discussion of rotor dynamics. My deepest thanks go to my wife, Rita: Not only has she given me grammatical advice, but has also helped maintain my sanity throughout the final months of preparing this text.

Arthur W. Lees

Author

Professor Arthur W. Lees, BSc, PhD, DSc, CEng, CPhys, FIMechE, FInstP, LRPS, graduated in physics and remained in Manchester University, Manchester, United Kingdom for three years' research. After completing his PhD, he joined the Central Electricity Generating Board, London, United Kingdom, initially developing finite element codes and later resolving plant problems.

After a sequence of positions, he was appointed head of the Turbine Group for Nuclear Electric Plc, London, Kingdom. He moved to Swansea University in 1995 and has been active in both research and teaching.

He is a regular reviewer of many technical journals and was, until his recent retirement, on the editorial boards of the *Journal of Sound and Vibration* and *Communications in Numerical Methods in Engineering*. His research interests include structural dynamics, rotor dynamics, inverse problems and heat transfer. Professor Lees is a fellow of the Institution of Mechanical Engineers and the Institute of Physics, a chartered engineer and a chartered physicist. He was a member of the Council of the Institute of Physics, 2001–2005. He is now professor emeritus at Swansea University but remains an active researcher.

MATLAB[®]

Modelling can be carried out using the package of MATLAB scripts freely available at www.rotordynamics.info. A further toolbox is under construction for the following studies:

- a. Rigid rotor analysis
- b. Flexible rotor analysis
- c. Single plane balancing
- d. Two plane balancing
- e. Modal balancing
- f. Gear torsional analysis
- g. Transient predictions
- h. Catenary calculations

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1

Introduction

The general field of condition monitoring has received substantial attention over the last few decades and it is worth reflecting on the state of the topic, because, although it has always been practised at some level, the manner in which condition is assessed is constantly under review in the light of recent developments in understanding.

In assessing the condition of a piece of equipment, the operator gathers data such as vibration, operating temperature, noise, performance and electrical parameters where appropriate. At one time, the comparison with normal condition was achieved largely on the basis of staff experience, but the general trend has been towards a more precise quantified approach. This has been required by revised patterns of working and increasing plant complexity, but it is, in essence, the same operation. A fundamental question arises of how one can 'codify' the knowledge of an experienced engineer and focus the knowledge on a specific area of plant. This presents indeed a challenge which has shown significant progress in recent years, although the issue cannot be regarded as completely resolved. Important progress has been made in computational modelling (both finite element analysis (FEA) and computational fluid dynamics (CFD)), artificial neural networks (ANNs), statistical approaches, expert systems and identification methods. All of these have a role to play in assessing the condition of a piece of equipment and their role will be outlined in subsequent chapters. First of all however, the general field of condition monitoring, as applied to rotating machines, is reviewed.

1.1 Monitoring and Diagnosis

Generally, these two terms are linked under the general heading of condition monitoring, but in fact, there are two quite distinct functions. In both areas, the first requirement is to gather and record all salient details of the operation of the piece of equipment but, as will be discussed, the choice as to what details are salient is a far from trivial task. However, that discussion is deferred for the present.

To illustrate this point with a specific example, let us consider a centrifugal pump driven by an electric motor. In such a case, the monitored parameters

would include bearing vibration levels, temperature, water pressure, water flow rate, motor current and voltage. Note that although this is a fairly long list, it is by no means exhaustive. In some circumstances, one may wish to record the rotor vibration (as opposed to that of the bearings) and bearing oil temperature. In fact, even a relatively simple piece of machinery may have a significant number of parameters which may be useful for monitoring purposes and a judicious choice is required to limit the measured set to cost-effective proportions; however, making this choice requires some appreciable physical insight.

Having decided on a set of monitored parameters, some method of recording is the next choice to be made and this ranges from regular spot checks to some form of continuous monitoring, now almost invariably computer based. Whilst the latter represents a more expensive option, it does offer more flexibility in terms of the ways in which data can be manipulated to offer insight into the underlying features of machine operation. Here again decisions are required which demand physical insight into machine operation and the likely failure scenarios.

We now consider some of the ways in which plant data may be analysed and how this may be used to form judgements about plant operation. Clearly, any general trend in the plant data, or indeed a sudden change, suggests that the equipment has changed in some way and, subject to some checks, may require the removal of the plant from service. Note that realistically, all equipment is subject to some random perturbations and so, statistical techniques are needed to form a valid decision such as when to remove plant from service.

To proceed further, we examine the three basic questions that are posed in condition monitoring systems which are

1. Has something gone wrong?
2. What has gone wrong?
3. How long can the plant run safely?

To address the first of these questions, it is often sufficient to adopt a purely statistical analysis of monitored data and seek trends and changes. At the most basic level, no knowledge of the internal operation of the machinery is required. For example a monitoring system may simply plot the overall vibration levels at the bearings (or elsewhere) and not examine the (short-term) time variation. Some of the ways in which data can be examined is discussed in Chapter 2. As discussed in later chapters, a great deal depends on the frequency with which measurements are monitored and provided several measurements are recorded during each rotor revolution, the orbit of the shaft may be traced and this gives some further insight into the machine's behaviour. In many cases, rudimentary measurements will give some indication that something has happened to a machine and straightforward

comparison with records will suffice to answer the first of the three questions. To answer the second, however, generally requires considerably more insight which may be provided by extensive experience, a detailed theoretical analysis or, very often, a combination of these. The third question, as to how long the plant will/can run is more difficult still, and is to a large extent still at the research stage. Nevertheless, an in-depth understanding of the machine's operation is an essential pre-requisite. In later chapters, a discussion is given as to the interpretation of plant data, but first, we give a brief survey of quantities that may be useful in assessing a machine's behaviour.

1.1.1 Monitored Parameters

1.1.1.1 Vibration

On rotating plant, vibration is the most commonly monitored parameter. The reasons for this are twofold: first, it is readily measured with convenient instrumentation and, perhaps more important, vibrations give a comprehensive reflection of the state of a rotating machine. The disadvantage is that much of the information for diagnosis can only be gained after extensive data processing, but standard techniques have now been developed which go some way towards relieving this burden. For basic monitoring, vibration levels can be, and are, used to great effect.

1.1.1.2 Pressure, Flow and Temperature

In the case of pumps, pressure and flow represent the main performance parameters, and hence, it is important to monitor these on a regular basis, although clearly, these will not be expected to vary as rapidly as vibration. There may, of course, be fluctuations in pressure, but flow rate will not track these owing to substantial inertia effects. Chapter 6 has a discussion of some of the ways in which the pressure field within a typical centrifugal pump has a direct influence on the vibration characteristics. The treatment of this is by no means comprehensive, and the interested reader is referred to the work of Childs (1993). The important point to emphasise here is that various pieces of information are inter-related and, taken together, they present a complex picture; it is the role of the diagnostic engineer to form an overall view from this complex pattern.

1.1.1.3 Voltage and Current

Electrical measurements also form an important part of this picture. A very large number of machines are driven by electric motors and the electrical measurements can be used to yield information on the overall machine efficiency, a key indicator of general deterioration. Used at this level, periodic checks would suffice, but more detailed analysis of fluctuations gives added information on both the motor and the auxiliary machine which is being driven.

1.1.1.4 Acoustic Emission

Acoustic emission (AE) is, in one sense, simply vibration at very high frequency, but this definition fails to give due recognition to its important distinction from conventional vibration data. As its name suggests in looking at AE, the engineer is monitoring the acoustic energy emitted by the material as changes (e.g. crack formation and propagation) take place. In effect one is 'listening' to the high-frequency waves generated by the breaking of inter-molecular bonds within a component, rather than the direct consequences of externally applied forces. Until very recent times, AE was used rather as a coarse monitor to count so-called events, as a guide to the presence or otherwise of cracking activity. For many years, it has been used to monitor the integrity of structures, but it is only much more recently (Price et al. 2005, Sikorska and Mba 2008) that it has been applied to rotating machinery. Improving hardware and software has facilitated the examination of the frequency resolution of AE signals, and this too has enhanced our understanding. Measurements are often made in the range of several hundred kHz as this is reasonably easy to obtain and process with modern equipment and, it turns out, yields information of interest. A common misunderstanding is that these frequencies are characteristic of the breaking bonds within the material, but this is not the case: such bonds give frequencies which are several orders of magnitude higher. The more accurate picture is that a breaking bond generates a very short pulse which contains a wide range of frequencies. As the waves travel, those which resonate within some part of the body become predominant and hence, the spectrum of the AE may be used to tell the operator the nature of the body around the fault. It is the essence of AE that it provides a good approach to the identification of highly localised phenomena.

1.1.2 Fault Localisation

The effective use of condition monitoring and condition-based maintenance encompasses a whole hierarchy of inter-related disciplines. At what might be termed level 1, purely statistical approaches can be applied to measured data to detect if there has been some underlying change to the condition of the machine, and some insight into such approaches is given in Chapter 2. This type of analysis can be carried without any knowledge of the machine's construction or operation and this is both a strength and a weakness: it means that the operator can determine if a fault/change has occurred without any assumptions as to the physical processes, but nothing further can be gleaned about the location or nature of the fault.

It may appear at first sight that the requirement to localise a fault is largely academic, but this is far from the case. On large machines, such as turbo-alternators, many days of production can be saved by focussing maintenance work on the appropriate part of the machine. However, to make any

valid progress on localisation requires insight or prior knowledge into the machine's design and dynamic properties. Foremost among the requirements is a knowledge of the machines' natural frequencies (critical speeds) and mode shapes. This knowledge may be developed by operational experience, plant tests and/or validated mathematical models, most commonly finite element (FE) models. More details on the requirements of these models are given in Section 1.2. The essential point at this stage is to emphasize that although condition monitoring is carried out without any reference to models for some simple machines, the potential is greatly extended by the use of models. This is because their key role is to relate the external measurements to the internal operating conditions.

Having established a model, the determination of natural frequencies and mode shapes is a straightforward matter and is discussed in many text books (see, e.g. Friswell et al. 2010, Inman 2008). The mode shapes will often give some clues as to the important locations for particular types of fault. Conclusive location identification may require some further analysis.

1.1.3 Root Cause

The issue of root cause is a theme running throughout this text and embraces the issues of fault localisation and frequency composition of the vibration signals. Chapters 4 and 5 discuss a range of common machine faults and the types of vibration signal they give rise to. The process of fault identification is basically one of recognising the appropriate patterns of response and matching them to fault types. More importantly, the process may be seen as one of gaining insight into the physical processes within the machine.

One might imagine that it would be easy to automate this process, but whilst there has been progress, there is still the need for a human expert on more complex machines. Recent research in this area has focussed on two areas: the development of extended models as discussed in Chapter 6 and the development of expert systems as in Chapter 8. For the most complex machines however, no fully automated system is likely to be available in the foreseeable future.

1.1.4 Remaining Life

Whilst the issues of fault localisation and root cause raise some problems, the identification of remaining life remains the 'Holy Grail of all Condition Monitoring'. It is an extremely complex topic and involves input from a number of disciplines. Whilst in some instances, reasonable predictions can be made, in most cases, these rely substantially on practical experience of the plant involved. Predictions can be made on, for example, crack growth rates, but these rely heavily on materials data which are in some instances subject to substantial error.

1.2 Mathematical Models

Many of the principles throughout this text are explained and illustrated with mathematical models, in some cases following analytic expressions and in others a numerical model. Some readers may find this surprising as in many cases, a machine operator may not have precise data with which to formulate such models: this will be so particularly in specifying fine clearances which will change with time as the machine components wear, but this does not invalidate the need or the role for theoretical models – in many ways, it emphasises the role. The aim of the model is to enhance and extend operational experience: to a degree, the actual numbers are secondary and what really matters is the extent to which the different parameters influence each other. This type of sensitivity is extremely valuable in gaining an understanding of the machine's operation. The sentiment which may surprise some readers is completely aligned with the theme presented here; although mathematical models and computing (and hence numbers) are used extensively throughout the text, the basic motivation is to elicit understanding of the underlying physical processes as opposed to any precise prediction. It is far more important to use models to examine interdependencies of various parameters and to understand the way in which physical changes (or uncertainties) can/will influence measured behaviour. Whilst the onset of problems can often be identified using purely statistical or empirical approaches, a full diagnosis and localisation of the fault will almost invariably require some form of model. This should not be surprising as the model is simply a means of expressing knowledge of the operation of a system.

Such an approach will not always be appropriate. For small items of plant which are readily interchangeable, a suitable approach would be to remove the item from service pending a general overhaul. For high capital plant, however, this approach is rarely feasible and it is in such cases that fault diagnosis, as a means to minimising maintenance/repair time, is imperative: this is where it becomes vital to glean all the information possible from the data available.

The need to gain insight into a machine's operational behaviour raises another interesting contrast to which we will return several times, namely, the distinction between data and information. This distinction is discussed more quantitatively in Chapter 7 in the context of elucidating the influence of foundations on the dynamics of machines, an effect which is particularly important with very large machines such as turbo-generators.

For a given machine, all these parameters will be linked and one of the aims of mathematical modelling must be to elucidate these links with a view to understanding the fluctuations which occur in machines running under nominally steady-state conditions. To some extent, steady state is a