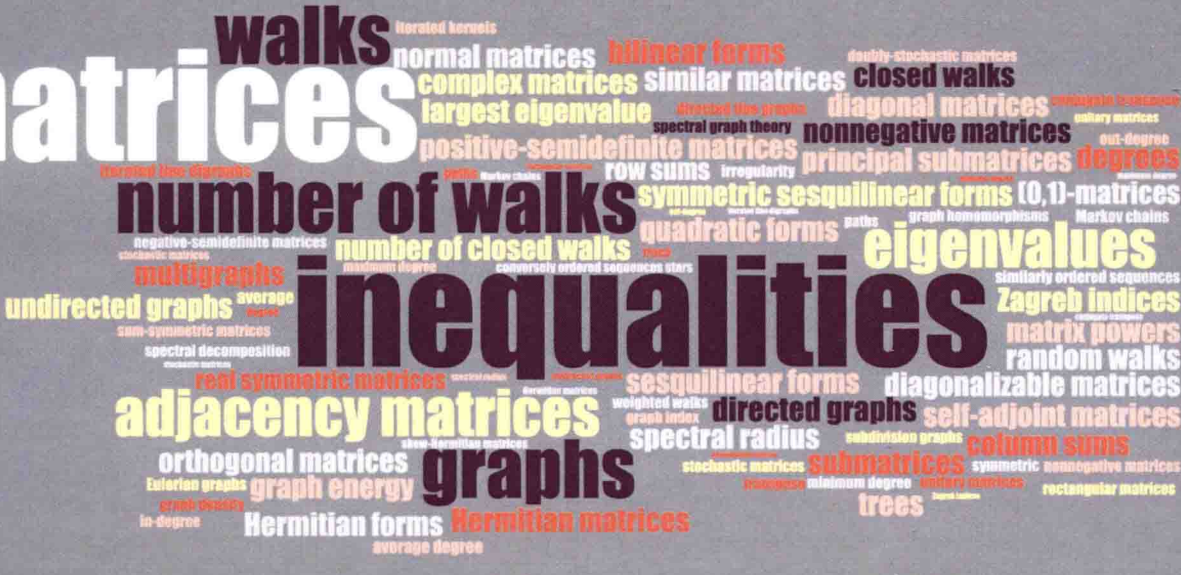

Matrix Inequalities for Iterative Systems

Hanjo Täubig

CRC Press
Taylor & Francis Group
A SCIENCE PUBLISHERS BOOK

Matrix Inequalities for Iterative Systems

Hanjo Täubig

Department of Computer Science
Technische Universität München
Garching, Germany

CRC Press
Taylor & Francis Group
Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **Informa** business
A SCIENCE PUBLISHERS BOOK

CRC Press
Taylor & Francis Group
6000 Broken Sound Parkway NW, Suite 300
Boca Raton, FL 33487-2742

© 2017 by Taylor & Francis Group, LLC
CRC Press is an imprint of Taylor & Francis Group, an Informa business

No claim to original U.S. Government works

Printed on acid-free paper
Version Date: 20160819

International Standard Book Number-13: 978-1-4987-7777-3 (Hardback)

This book contains information obtained from authentic and highly regarded sources. Reasonable efforts have been made to publish reliable data and information, but the author and publisher cannot assume responsibility for the validity of all materials or the consequences of their use. The authors and publishers have attempted to trace the copyright holders of all material reproduced in this publication and apologize to copyright holders if permission to publish in this form has not been obtained. If any copyright material has not been acknowledged please write and let us know so we may rectify in any future reprint.

Except as permitted under U.S. Copyright Law, no part of this book may be reprinted, reproduced, transmitted, or utilized in any form by any electronic, mechanical, or other means, now known or hereafter invented, including photocopying, microfilming, and recording, or in any information storage or retrieval system, without written permission from the publishers.

For permission to photocopy or use material electronically from this work, please access www.copyright.com (http://www.copyright.com/) or contact the Copyright Clearance Center, Inc. (CCC), 222 Rosewood Drive, Danvers, MA 01923, 978-750-8400. CCC is a not-for-profit organization that provides licenses and registration for a variety of users. For organizations that have been granted a photocopy license by the CCC, a separate system of payment has been arranged.

Trademark Notice: Product or corporate names may be trademarks or registered trademarks, and are used only for identification and explanation without intent to infringe.

Library of Congress Cataloging-in-Publication Data

Names: Täubig, Hanjo, 1975- | Hou, Xu (Engineer), editor.
Title: Matrix inequalities for iterative systems / Hanjo Täubig, Department of Computer Science, Technische Universität München, Garching, Germany, Design, Fabrication, Properties, and Applications of Smart and Advanced Materials ; editor, Xu Hou, Harvard University, School of Engineering and Applied Sciences.
Description: Boca Raton, FL : CRC Press, 2017. | "A science publishers book." | Includes bibliographical references and index.
Identifiers: LCCN 2016030679| ISBN 9781498777773 (hardback) | ISBN 9781498777797 (e-book)
Subjects: LCSH: Matrix inequalities.
Classification: LCC QA188 .T36 2017 | DDC 512.9/434--dc23
LC record available at https://lccn.loc.gov/2016030679

Visit the Taylor & Francis Web site at
http://www.taylorandfrancis.com

and the CRC Press Web site at
http://www.crcpress.com

Printed and bound in Great Britain by
TJ International Ltd, Padstow, Cornwall

MIX
Paper from responsible sources
FSC www.fsc.org FSC® C013056

Matrix Inequalities for Iterative Systems

此为试读,需要完整PDF请访问: www.ertongbook.com

To my parents
Brigitte and Klaus Täubig

Acknowledgments

It seems to me shallow and arrogant for any man in these times to claim he is completely self-made, that he owes all his success to his own unaided efforts. Many hands and hearts and minds generally contribute to anyone's notable achievements.

Walt Disney

First of all, I want to thank Prof. Ernst Mayr for the generous support through all the years and for providing me with the freedom to work on whatever seemed to be interesting to me. I am also deeply indebted to all the other people that contributed to the work on the number of walks, especially the members of our "NetLEA" group (consisting of Sven Kosub, Klaus Holzapfel, Moritz Maaß, Alexander Offtermatt-Souza, and me). When we started in spring of 2002 to work on spectral graph theory, we made an attempt to prove our first conjectures. Some of these early ideas are still at the heart of the proofs for the much more general statements that we showed only recently. Numerous important ideas, in particular for the only long and complex proof in this work, were contributed by Jeremias Weihmann. I am thankful for his contributions and for all the fruitful discussions that we had. I am also grateful for all the hints that I got from Raymond Hemmecke, Thomas Kahle, David Reeb, Werner Meixner, and from my brother Holger Täubig. For quite recent interesting remarks and discussions, I am thankful to Tamás Réti. For proofreading a draft of this work, I am indebted to Moritz Fuchs.

It was a great pleasure to meet the real experts at the Conference on Applications of Graph Spectra in Computer Science (Barcelona, 2012) and at the Conference on Algebraic Combinatorics: Spectral Graph Theory, Erdős-Ko-Rado Theorems and Quantum Information Theory, a conference to celebrate the work of Chris Godsil (Waterloo, 2014).

Furthermore, I want to thank all the other people that I had the pleasure to work with, in particular, all the members of the efficient algorithms group at TUM (especially Riko Jacob, Christian Scheideler, Stefan Schmid, and Harald Räcke).

Kind regards go to the members of my ProLehre teaching course and to our coaches Barbara Greese, Hans-Christoph Bartscherer, Pit Forster, and Adi Winteler.

Preface

Our interest for the topic was attracted by a conjecture that was brought up by our colleague Sven Kosub. While we did not make much progress in proving the conjecture at the beginning (because it was wrong), it turned out later, that one of the first ideas for proving a certain special case was still at the heart of the more general results that were obtained only recently by Jeremias Weihmann and the author of this book.

This book is based on the author's habilitation thesis [Täu15a]. The main goal of the habilitation process is to develop and prove the ability to teach. As such, the aim of the book is to present the topic in such a way that the results and their underlying methods of proof can be used by the reader in the easiest possible way. To this end, our work tries to unify the various inequalities for the number of walks in graphs and for the sum of entries of matrix powers in a generalized form. These generalizations reveal the fundamental principles underlying the different results.

To the best of our knowledge all the results claimed to be new, in this work, have never appeared before. During the whole time of working on this subject, we found many examples of results that were (re)discovered more than just once. But in view of the large list of references, we can probably claim that we did a thorough job for finding most of the relevant literature. In any case, the focus of this book is not only on the collection, but also on the systematization of the vast number of results. We have generalized and unified the related results, and put them into a common frame and discussed their relations. In conclusion, we hope to contribute to a deeper understanding of the origin of the different inequalities for matrix powers and the number of walks in graphs. Our main goal is thus a clear presentation of the common principles underlying the tremendous number of different results.

Almost all of the proofs in this book are elementary. While it is normal to be satisfied with an arbitrary complex proof for a certain statement, it must be emphasized that elementary proofs (even for known results) have an additional value because they are instructive and convincing.

Most of the results in this book have been published in the following articles and reports: [HKM+11; HKM+12; Täu12; TW12; TWK+13; TW14; Täu14; Täu15b].

此为试读,需要完整PDF请访问: www.ertongbook.com

Symbol Description

A	matrix with entries a_{ij}, usually square of size $n \times n$	G	graph, usually with vertex set V and edge set E
$A[X, Y]$	submatrix of A induced by row index set X and column index set Y	V	vertex set of a graph
$A[X]$	principal submatrix of A induced by row and column index set X	E	edge set of a graph
A^T	transpose of A	$d_{\text{in}}(v)$	in-degree of vertex v
A^*	conjugate transpose of A	$d_{\text{out}}(v)$	out-degree of vertex v
a_{ij}	matrix entry that resides in row i and column j	Δ	maximum degree
I_n	$n \times n$ identity matrix	$\rho(G)$	edge density of the graph G
sum(A)	sum of all entries of A	$w_k(x, y)$	number of walks of length k starting at vertex x and ending at vertex y
$r_i(A)$, r_i	row sum of the entries in row i of the given matrix A	$s_k(x)$	number of walks of length k starting at vertex x
$c_j(A)$, c_j	column sum of the entries in column j of the given matrix A	$e_k(x)$	number of walks of length k ending at vertex x
tr(A)	trace of matrix A (sum of main diagonal entries)	$w_k(x)$	replacement for $s_k(x)$ and $e_k(x)$ in undirected graphs
$\rho(A)$	spectral radius of A	w_k	total number of walks of length k in a given graph
$\langle x, y\rangle$	(standard) inner product of vectors x and y	cl_k	total number of closed walks of length k in a given graph
$\|\|x\|\|$	(Euclidean) length of vector x	$cl_k(x)$	number of closed walks of length k starting at vertex x
1_n	n-dimensional all-ones vector	v_k	total number of nonreturning walks of length k in a given graph
$\chi(S)$	characteristic vector of set S		

Contents

此为试读,需要完整PDF请访问: www.ertongbook.com