J. C. WILLMOTT

ATOMIC PHYSICS

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John Wiley & Sons Ltd.

LONDON NEW YORK SYDNEY TORONTO

Library of Congress Cataloging in Publication Data:
Willmott, John Charles.
Atomic physics.

(Manchester physics series)
Bibliography: p.
1. Nuclear physics. 2. Quantum theory. I. Title.

QC171.2.W54 539.7 74-9580

ISBN 0 471 94930 2
ISBN 0 471 94931 0 (pbk.)

Printed in Great Britain by J. W. Arrowsmith Ltd.
Winterstoke Road, Bristol

Editors' Preface to the Manchester Physics Series

In devising physics syllabuses for undergraduate courses, the staff of Manchester University Physics Department have experienced great difficulty in finding suitable textbooks to recommend to students; many teachers at other universities apparently share this experience. Most books contain much more material than a student has time to assimilate and are so arranged that it is only rarely possible to select sections or chapters to define a self-contained, balanced syllabus. From this situation grew the idea of the Manchester Physics Series.

The books of the Manchester Physics Series correspond to our lecture courses with about fifty per cent additional material. To achieve this we have been very selective in the choice of topics to be included. The emphasis is on the basic physics together with some instructive, stimulating and useful applications. Since the treatment of particular topics varies greatly between different universities, we have tried to organize the material so that it is possible to select courses of different length and difficulty and to emphasize different applications. For this purpose we have encouraged authors to use flow diagrams showing the logical connection of different chapters and to put some topics into starred sections or subsections. These cover more advanced and alternative material, and are not required for the understanding of later parts of each volume.

Since the books of the Manchester Physics Series were planned as an integrated course, the series gives a balanced account of those parts of physics which it treats. The level of sophistication varies: '*Properties of Matter*' is for the first year, '*Solid State Physics*' for the third. The other volumes are intermediate, allowing considerable flexibility in use. '*Electromagnetism*', '*Optics*', '*Electronics*' and '*Atomic Physics*' start from first year level and progress to material suitable for second or even third year courses. '*Statistical Physics*' is suitable for second or third year. The books have been written in such a way that each volume is self-contained and can be used independently of the others.

Although the series has been written for undergraduates at an English university, it is equally suitable for American university courses beyond the Freshman year. Each author's preface gives detailed information about the prerequisite material for his volume.

In producing a series such as this, a policy decision must be made about units. After the widest possible consultations we decided, jointly with the authors and the publishers, to adopt SI units interpreted liberally, largely following the recommendations of the International Union of Pure and Applied Physics. Electric and magnetic quantities are expressed in SI units. (Other systems are explained in the volume on electricity and magnetism.) We did not outlaw physical units such as the electron-volt. Nor were we pedantic about factors on 10 (is 0.012 kg preferable to 12 g?), about abbreviations (while s or sec may not be equally acceptable to a computer, they should be to a scientist), and about similarly trivial matters.

Preliminary editions of these books have been tried out at Manchester University and circulated widely to teachers at other universities, so that much feedback has been provided. We are extremely grateful to the many students and colleagues, at Manchester and elsewhere, who through criticisms, suggestions and stimulating discussions helped to improve the presentation and approach of the final version of these books. Our particular thanks go to the authors, for all the work they have done, for the many new ideas they have contributed, and for discussing patiently, and frequently accepting, our many suggestions and requests. We would also like to thank the publishers, John Wiley and Sons, who have been most helpful in every way, including the financing of the preliminary editions.

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Author's Preface

Some years ago at Manchester we decided to try to define our physics syllabuses in terms of textbooks, or portions thereof. This exercise brought to light the absence of books at an appropriate level, and of an appropriate length, on certain topics. One of these topics was atomic physics at a level suitable for second year undergraduates in a British university. This textbook is an attempt to fill that gap.

Broadly speaking there are two possible approaches to the teaching of atomic physics and quantum mechanics. The first is to present the material in a roughly historical manner, while the second is to postulate the laws of quantum mechanics, in the same way as one postulates Newton's laws of motion, and to develop the subject from there. It is possible to use the latter technique in teaching classical mechanics because most people are familiar with the ideas of classical mechanics from everyday experience, even if they have not formalized them; the equality of action and reaction does not strike most people as extraordinary. There is nothing in everyday experience to suggest the laws of quantum mechanics. The advantage of proceeding from the postulates is that it makes clear the formal structure of quantum mechanics, but it is the author's view that the postulates of quantum mechanics are so extraordinary to the newcomer and contain so many unfamiliar concepts that it is better to lead the student, the first time, more gently explaining why classical physics will not do and introducing the new ideas as far as possible one at a time.

This book, therefore, leans towards the historical approach. The danger in this approach is a tendency to make it all seem inevitable and leave an impression that one can almost derive the Schrödinger equation. An attempt has been made to overcome this mainly by repeated assertion that derivation of quantum mechanics is not possible. Also, a certain amount of hindsight is employed to bring out points not obvious at the time; for example, the fact that without Planck's constant one could not construct a length with which to set the scale of atomic phenomena, and the inevitable failure of classical physics as a result, was surely not generally appreciated at the time.

The text is intended to form the basis of a course of some thirty lectures, each lasting fifty minutes. Needless to say, it would not be possible to cover the whole of the material in that time and it is assumed some selection will be made and some topics will be left to the student to read for himself. The first two chapters are concerned with the evidence for the atomic nature of matter, leading up to the Rutherford model of the atom. Much of this will be familiar to the student and may be omitted. Chapter 3 is concerned with the universal occurrence of Planck's constant in electromagnetic phenomena. Chapter 4 starts with electron diffraction, considers the nature of the wavefunction required to describe the phenomenon and goes directly to the Uncertainty Principle, it being regarded as important that the student realize that the association of wavelike phenomena with particle motion introduced an uncertainty independent of the exact form taken by the equations of motion. Chapter 5 is concerned with the Schrödinger equation and chapter 6 with angular momentum. After this the only new ideas introduced are the Pauli Exclusion Principle in Chapter 8, and parity in Chapter 10.

Each chapter is followed by a selection of problems. These vary from simple numerical problems intended to familiarize the student with the orders of magnitude involved in atomic physics to problems that go well beyond the scope of the book and are intended to stretch the better student.

An attempt has been made to avoid disposing of difficult material by phrases like '. . . it turns out that' but rather to try and give a plausible physical explanation. This is not always possible, but one of the attractions of physics is that there are simple situations which require the most advanced and penetrating analysis. It is hoped that this book will give its readers a taste for the subject and stimulate them to go on to more advanced texts.

April, 1974. J. C. WILLMOTT
Manchester, England.

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★ Starred sections may be omitted as they are not required later in the book.

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CHAPTER 1

The atomic nature of matter and electricity

1.1 INTRODUCTION

During the nineteenth century a large amount of evidence had been accumulated that gave support to the idea that matter is composed of atoms.

We list below some of the chemical laws which receive a simple explanation on an atomic hypothesis.

(1) The law of constant proportions

When chemical elements are brought together under appropriate conditions to form a specific compound, the proportion in weight of the combining elements is always the same.

(2) The law of multiple proportions

When two elements combine together in different ways, to form different compounds, then the weights of one element that combine with a definite weight of the other always bear a simple ratio to each other.

On the atomic hypothesis these laws are explained as follows:

A quantity of a given chemical element consists of a large number of atoms of that element, each atom having the same weight, the weight being peculiar to that element (Fig. 1.1). When two elements combine to form a compound, the atoms of the elements combine in a simple ratio to form a *molecule* of the compound.

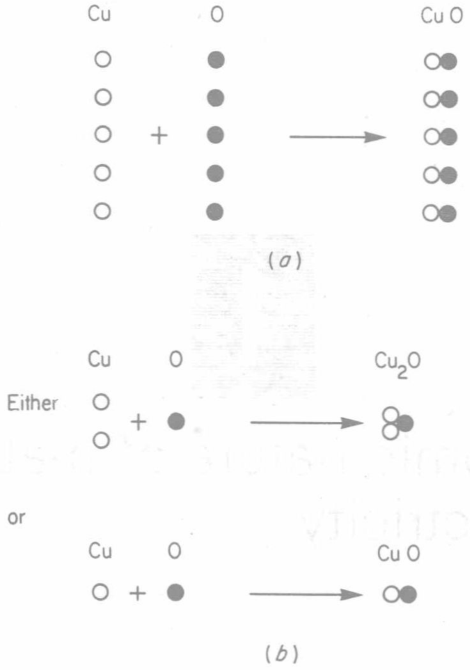

Fig. 1.1. (*a*) The law of constant proportions on the atomic hypothesis.
(*b*) The law of multiple proportions on the atomic hypothesis.

Thus, if we form the compound cupric oxide from copper and oxygen, we find that 63.5 g of copper always combine with 16 g of oxygen. Further, we can also form the compound cuprous oxide in which 63.5 g of copper combine with 8 g of oxygen.

The atomic hypothesis explains this by saying the atomic weights of copper and oxygen are in the ratio 63.5 : 16, and that cupric oxide is CuO and cuprous oxide Cu_2O.

With this simple hypothesis it was found possible to give a quantitative explanation of observed combining weights for the whole of simple inorganic chemistry.

It was found that in the gaseous state not only the weights but the volumes entering into chemical reactions were governed by simple laws.

(3) Gay–Lussac's Law

In every gas formed or decomposed, the volumes of the component and compound gases bear a simple ratio to one another.

In fact, this law only applies when the gases closely follow the perfect gas laws.

Consideration of Gay–Lussac's law with the previous laws leads to the conclusion that the *volume* of a gas is related to the number of particles in it, and we get

(4) Avogadro's Law

Equal volumes of different gases under the same conditions of temperature and pressure contain equal numbers of molecules.

It will be seen that this law implies that the molecular weight in grams of a gas will always occupy a specific volume, at a given temperature and pressure, whatever the gas. At normal temperature and pressure (0 °C and 760 mm of mercury) this volume is *22.4 litres.*

The *number* of molecules in one gram molecular weight is known as Avogadro's number. This number is

$$N_0 = 6.023 \times 10^{23} .$$

Avogadro's number may be measured in a variety of ways, the most accurate of which depends on the measurement of atomic spacings by X-ray diffraction. All of them naturally depend on the assumption of the atomic hypothesis, and the agreement of the various methods lends powerful support to this hypothesis.

1.2 KINETIC THEORY

Further support for the atomic hypothesis was provided by the great success of the kinetic theory of gases, as developed by Maxwell and Boltzmann. Perhaps the most dramatic support for an atomic hypothesis comes from the observation of *Brownian motion.*

Kinetic theory explains the pressure exerted by a gas as being due to the bombardment of the walls of the container by the molecules of the gas. If a small object is suspended in a gas, it too will be bombarded by the molecules, and because the number of molecules is finite, an exact balance will not occur at any one instant, and the object will undergo a random motion as a result. This was first observed by the botanist Brown.

As an example, we may consider a suspended mirror, such as a galvanometer mirror. Because the two halves of the mirror will not undergo bombardment by exactly equal numbers of molecules, it will suffer torsional motion. Kinetic theory not merely says this will happen, but makes assertions about the magnitude of the effect. According to the law of equipartition of energy (see F. Mandl, *Statistical Physics* (Manchester Physics Series), Wiley, London, 1971) every term in the expression for the total energy which enters in quadratic form has a mean energy equal to $\frac{1}{2}kT$ where T is the absolute temperature and k (known as Boltzmann's constant) is R/N_0.

R is the gas constant, and N_0 Avogadro's number, i.e. the number of molecules in one gram molecule.

Applying this to the galvanometer mirror, the expression for the energy will contain a term $\frac{1}{2}c\theta^2$ where θ is the deflection of the mirror and c the torsional constant. Thus we have

$$\tfrac{1}{2}c\overline{\theta^2} = \tfrac{1}{2}kT \tag{1.1}$$

or

$$\overline{\theta^2} = \frac{kT}{c} \tag{1.2}$$

where $\overline{\theta^2}$ is the mean square deflection.

Fig. 1.2 shows the results one obtains. These fluctuations provide impressive support for the atomic hypothesis.

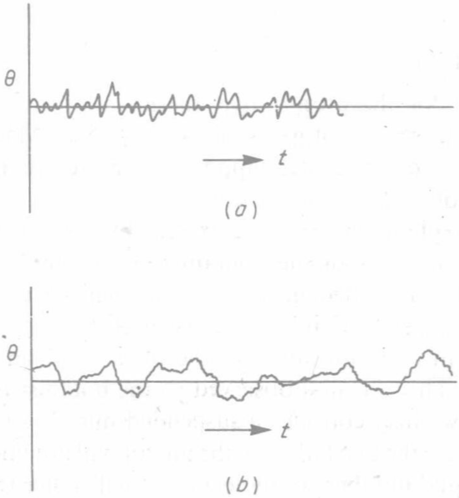

Fig. 1.2. The fluctuations of the angular position of a galvanometer mirror due to its Brownian motion (a) for a lightly damped galvanometer and (b) for a more heavily damped galvanometer. The mean square fluctuation is the same in both cases.