FUNDAMENTALS OF LINEAR CONTROL

A Concise Approach

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Fundamentals of Linear Control

A Concise Approach

Taking a different approach from standard thousand-page reference-style control textbooks, *Fundamentals of Linear Control* provides a concise yet comprehensive introduction to the analysis and design of feedback control systems in fewer than 300 pages.

The text focuses on classical methods for dynamic linear systems in the frequency domain. The treatment is, however, modern and the reader is kept aware of contemporary tools and techniques, such as state-space methods and robust and nonlinear control.

Featuring fully worked design examples, richly illustrated chapters, and an extensive set of homework problems and examples spanning across the text for gradual challenge and perspective, this textbook is an excellent choice for senior-level courses in systems and control or as a complementary reference in introductory graduate-level courses. The text is designed to appeal to a broad audience of engineers and scientists interested in learning the main ideas behind feedback control theory.

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To Beatriz and Victor

Preface

The book you have in your hands grew out of a set of lecture notes scribbled down for MAE 143B, the senior-level undergraduate *Linear Control* class offered by the Department of Mechanical and Aerospace Engineering at the University of California, San Diego.

The focus of the book is on classical methods for analysis and design of feedback systems that take advantage of the powerful and insightful representation of dynamic linear systems in the frequency domain. The required mathematics is introduced or revisited as needed. In this way the text is made mostly self-contained, with accessory work shifted occasionally to homework problems.

Key concepts such as tracking, disturbance rejection, stability, and robustness are introduced early on and revisited throughout the text as the mathematical tools become more sophisticated. Examples illustrate graphical design methods based on the root-locus, Bode, and Nyquist diagrams. Whenever possible, without straying too much from the classical narrative, the reader is made aware of contemporary tools and techniques such as state-space methods, robust control, and nonlinear systems theory.

With so much to cover in the way of insightful engineering *and* relevant mathematics, I tried to steer clear of the curse of the engineering systems and control textbook: becoming a treatise with 1000 pages. The depth of the content exposed in fewer than 300 pages is the result of a compromise between my utopian goal of *at most* 100 pages on the one hand and the usefulness of the work as a reference and, I hope, inspirational textbook on the other. Let me know if you think I failed to deliver on this promise.

I shall be forever indebted to the many students, teaching assistants, and colleagues whose exposure to earlier versions of this work helped shape what I am finally not afraid of calling the *first* edition. Special thanks are due to Professor Reinaldo Palhares, who diligently read the original text and delighted me with an abundance of helpful comments.

I would like to thank Sara Torenson from the UCSD Bookstore, who patiently worked with me to make sure earlier versions were available as readers for UCSD students, and Steven Elliot from Cambridge University Press for his support in getting this work to a larger audience.

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San Diego, California

Overview

This book is designed to be used in a quarter- or semester-long senior-level undergraduate linear control systems class. Readers are assumed to have had some exposure to differential equations and complex numbers (good references are [BD12] and [BC14]), and to have some familiarity with the engineering notion of signals and systems (a standard reference is [Lat04]). It is also assumed that the reader has access to a high-level software program, such as MATLAB, to perform calculations in many of the homework problems. In order to keep the focus on the content, examples in the book do not discuss MATLAB syntax or features. Instead, we provide supplementary MATLAB files which can produce all calculations and figures appearing in the book. These files can be downloaded from http://www.cambridge.org/deOliveira.

Chapters 1 and 2 provide a quick overview of the basic concepts in control, such as feedback, tracking, dynamics, disturbance rejection, integral action, etc. Math is kept at a very basic level and the topics are introduced with the help of familiar examples, such as a simplistic model of a car and a toilet bowl.

Chapter 3 formalizes the concept of a transfer-function for dynamic linear system models. Its first part is a review of the Laplace transform and its application to linear ordinary differential equations. The second part introduces systems concepts such as stability, transient and steady-state response, and the frequency response method. Some topics, e.g. complex integration, the calculus of residues, and norms of signals and systems, are covered in more depth than is usually found in typical introductory courses, and can be safely skipped at first read.

Equipped with the concept of a transfer-function, Chapter 4 formalizes fundamental concepts in feedback analysis, such as tracking, sensitivity, asymptotic and internal stability, disturbance rejection, measurement noise, etc. Homework problems in this chapter expose readers to these concepts and anticipate the more sophisticated analytic methods to be introduced in the following chapters.

Chapter 5 takes a slight detour from classic methods to introduce the reader to state-space models. The focus is on practical questions, such as realization of dynamic systems and controllers, linearization of nonlinear systems, and basic issues that arise when using linear controllers with nonlinear systems. It is from this vantage point that slightly more complex dynamic systems models are introduced, such as a simple pendulum and a pendulum in a cart, as a well as a simplified model of a steering car. The simple pendulum model is used in subsequent chapters as the main illustrative example.

Table I.1 Homework problems classified by theme per chapter

Problem theme	Ch. 1	Ch. 2	Ch. 3	Ch. 4	Ch. 5	Ch. 6	Ch. 7	Ch. 8
DC motor		2.41–2.46	3.95–3.99	4.29–4.37	5.30–5.33	6.30–6.33	7.29–7.31	8.41–8.43
Elevator		2.18–2.26	3.62–3.70	4.23–4.28	5.19–5.22	6.14–6.15	7.15–7.16	8.37
Free-fall		2.4–2.7	3.53		5.8–5.13			
Inclined plane	1.10							
Insulin homeostasis					5.49–5.52	6.38–6.40	7.36–7.37	
Mass–spring–damper		2.27–2.33	3.71–3.79		5.23–5.27	6.16–6.18	7.17–7.19	8.38–8.40
Modulator			3.50					
One-eighth-car model						6.19–6.23	7.20–7.23	
One-quarter-car model						6.24–6.29	7.24–7.28	
OpAmp circuit		2.38–2.40	3.90–3.94		5.28–5.29			
Orbiting satellite					5.41–5.45	6.37	7.35	8.47
Pendulum in a cart					5.6			
Population dynamics					5.46–5.48			
RC circuit		2.34–2.35	3.80–3.84					
Rigid body					5.39–5.40			
RLC circuit		2.36–2.37	3.85–3.89					
Rotating machine		2.10–2.17	3.54–3.61	4.20–4.22	5.15–5.18	6.11–6.13	7.12–7.14	8.35–8.36
Sample-and-hold			3.51–3.52					
Simple pendulum						6.9–6.10		
Sky-diver		2.8–2.9			5.14			
Smith predictor				4.11–4.14				
Steering car					5.7			
Water heater		2.49–2.56	3.100–3.104	4.38–4.43	5.36–5.38	6.34–6.36	7.32–7.34	8.44–8.46
Water tank					5.34–5.35			

Chapter 6 takes the reader back to the classic path with an emphasis on control design. Having flirted with second-order systems many times before in the book, the chapter starts by taking a closer look at the time-response of second-order systems and associated performance metrics, followed by a brief discussion on derivative action and the popular proportional–integral–derivative control. It then introduces the root-locus method and applies it to the design of a controller with integral action to the simple pendulum model introduced in the previous chapter.

Chapter 7 brings a counterpoint to the mostly time-domain point of view of Chapter 6 by focusing on frequency-domain methods for control design. After introducing Bode and polar plots, the central issue of closed-loop stability is addressed with the help of the Nyquist stability criterion. The same controller design problem for the simple pendulum is revisited, this time using frequency-domain tools.

An introductory discussion on performance and robustness is the subject of the final chapter, Chapter 8. Topics include Bode's sensitivity integral, robustness analysis using small gain and the circle criterion, and feedforward control and filtering. Application of some of these more advanced tools is illustrated by certifying the performance of the controllers designed for the simple pendulum beyond the guarantees offered by local linearization.

In a typical quarter schedule, with 20 or 30 lectures, the lightweight Chapters 1 and 2 can be covered rather quickly, serving both as a way to review background material and as a means to motivate the reader for the more demanding content to come. Instructors can choose to spend more or less time on Chapter 3 depending on the prior level of comfort with transfer-functions and frequency response and the desired depth of coverage.

Homework problems at the end of Chapters 1 through 3 introduce a variety of examples from various engineering disciplines that will appear again in the following chapters and can be used as effective tools to review background material.

Chapters 4 through 7 constitute the core material of the book. Chapters 5 and 7, especially, offer many opportunities for instructors to select additional topics for coverage in class or relegate to reading, such as discussions on nonlinear analysis and control, a detailed presentation of the argument principle, and more unorthodox topics such as non-minimum-phase systems and stability analysis of systems with delays.

The more advanced material in Chapter 8 can be covered, time permitting, or may be left just for the more interested reader without compromising a typical undergraduate curriculum.

This book contains a total of almost 400 homework problems that appear at the end of each chapter, with many problems spanning across chapters. Table I.1 on page xiv provides an overview of select problems grouped by their motivating theme. Instructors may choose to follow a few of these problems throughout the class. As mentioned previously, many of the problems require students to use MATLAB or a similar computer program. The supplementary MATLAB files provided with this book are a great resource for readers who need to develop their programming skills to tackle these problems.

Contents

1 Introduction

In controls we make use of the abstract concept of a *system*: we identify a phenomenon or a process, the *system*, and two classes of *signals*, which we label as *inputs* and *outputs*. A signal is something that can be measured or quantified. In this book we use real numbers to quantify signals. The classification of a particular signal as an input means that it can be identified as the *cause* of a particular system behavior, whereas an output signal is seen as the *product* or *consequence* of the behavior. Of course the classification of a phenomenon as a system and the labeling of input and output signals is an abstract construction. A mathematical description of a system and its signals is what constitutes a *model*. The entire abstract construction, and not only the equations that we will later associate with particular signals and systems, is the model.

We often represent the relationship between a system and its input and output signals in the form of a *block-diagram*, such as the ones in Fig. 1.1 through Fig. 1.3. The diagram in Fig. 1.1 indicates that a system, G, produces an output signal, y, in the presence of the input signal, u. Block-diagrams will be used to represent the interconnection of systems and even algorithms. For example, Fig. 1.2 depicts the components and signals in a familiar controlled system, a water heater; the block-diagram in Fig. 1.3 depicts an algorithm for converting temperature in degrees Fahrenheit to degrees Celsius, in which the output of the circle in Fig. 1.3 is the algebraic sum of the incoming signals with signs as indicated near the incoming arrows.

1.1 Models and Experiments

Systems, signals, and models are often associated with concrete or abstract experiments. A model reflects a particular setup in which the outputs appear *correlated* with a prescribed set of inputs. For example, we might attempt to model a car by performing the following experiment: on an unobstructed and level road, we depress the accelerator pedal and let the car travel in a straight line.[1] We keep the pedal excursion constant and let the car reach constant velocity. We record the amount the pedal has been depressed and the car's terminal velocity. The results of this experiment, repeated multiple times with different amounts of pedal excursion, might look like the data shown in Fig. 1.4. In this experiment the signals are

[1] This may bring to memory a bad joke about physicists and spherical cows . . .

Figure 1.1 System represented as a block-diagram; u is the input signal; y is the output signal; y and u are related through $y = G(u)$ or simply $y = Gu$.

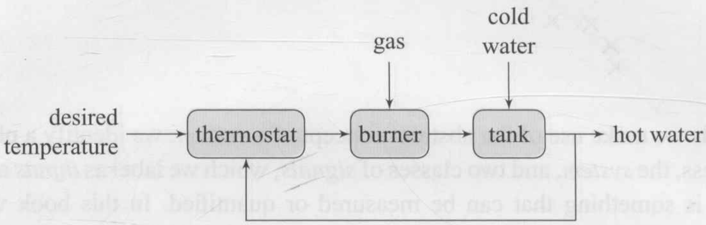

Figure 1.2 Block-diagram of a controlled system: a gas water heater; the blocks thermostat, burner, and tank, represent components or sub-systems; the arrows represent the *flow* of input and output signals.

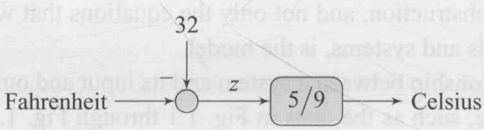

Figure 1.3 Block-diagram of an algorithm to convert temperatures in Fahrenheit to Celsius: Celsius = 5/9(Fahrenheit − 32); the output of the circle block is the algebraic sum of the incoming signals with the indicated sign, i.e. $z =$ Fahrenheit − 32.

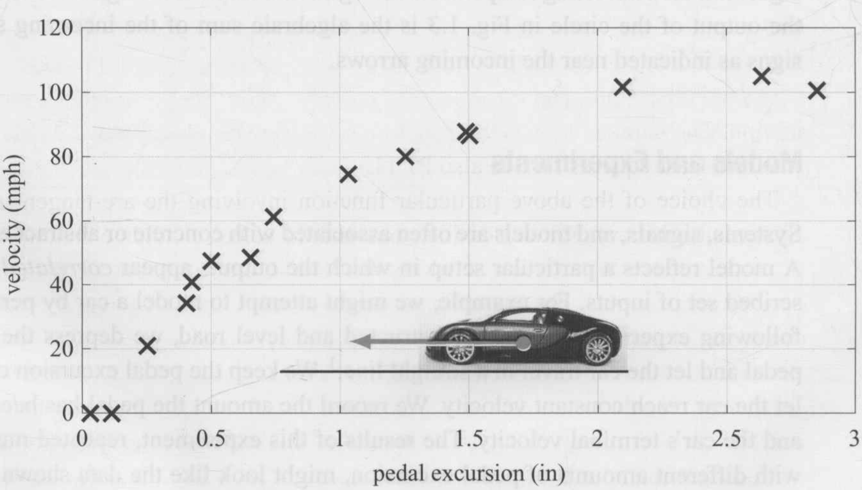

Figure 1.4 Experimental determination of the effect of pressing the gas pedal on the car's terminal velocity; the pedal excursion is the input signal, u, and the car's terminal velocity is the output signal, y.

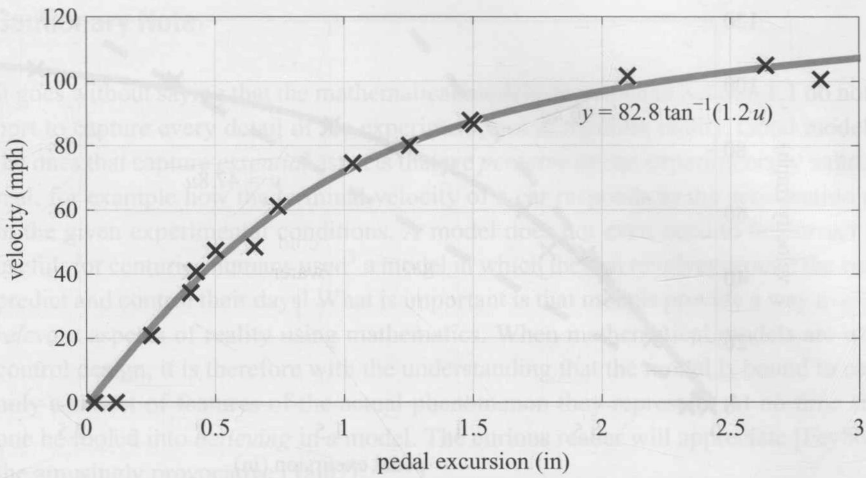

Figure 1.5 Fitting the curve $y = \alpha \tan^{-1}(\beta u)$ to the data from Fig. 1.4.

input: pedal excursion, in cm, inches, etc.;
output: terminal velocity of the car, in m/s, mph, etc.

The **system** is the car *and* the particular conditions of the experiment. The data captures the fact that the car does not move at all for small pedal excursions and that the terminal velocity *saturates* as the pedal reaches the end of its excursion range.

From Fig. 1.4, one might try to *fit* a particular mathematical function to the experimental data[2] in hope of obtaining a *mathematical model*. In doing so, one invariably loses something in the name of a simpler description. Such trade-offs are commonplace in science, and it should be no different in the analysis and design of control systems. Figure 1.5 shows the result of fitting a curve of the form

$$y = \alpha \tan^{-1}(\beta u),$$

where u is the input, pedal excursion in inches, and y is the output, terminal velocity in mph. The parameters $\alpha = 82.8$ and $\beta = 1.2$ shown in Fig. 1.5 were obtained from a standard least-squares fit. See also P1.11.

The choice of the above particular function involving the arc-tangent might seem somewhat arbitrary. When possible, one should select candidate functions from first principles derived from physics or other scientific reasoning, but this does not seem to be easy to do in the case of the experiment we described. Detailed physical modeling of the vehicle would involve knowledge and further modeling of the components of the vehicle, not to mention the many uncertainties brought in by the environment, such as wind, road conditions, temperature, etc. Instead, we make an "educated choice" based on certain physical aspects of the experiment that we believe the model should capture. In this case, from our daily experience with vehicles, we expect that the terminal velocity

[2] All data used to produce the figures in this book is available for download from the website http://www.cambridge.org/deOliveira.

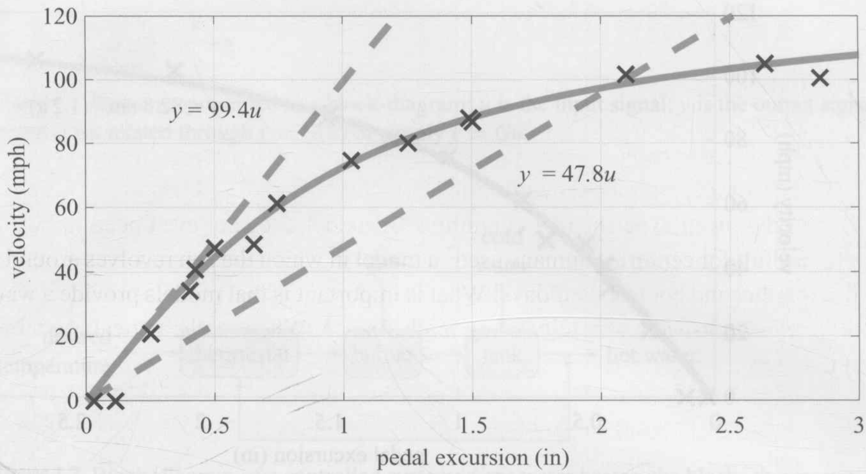

Figure 1.6 Linear mathematical models of the form $y = \gamma u$ for the data in Fig. 1.4 (dashed); the model with $\gamma = 47.8$ was obtained by a least-squares fit; the model with $\gamma = 99.4$ was obtained after linearization of the nonlinear model (solid) obtained in Fig. 1.5; see P1.12 and P1.11.

will eventually *saturate*, either as one reaches full throttle or as a result of limitations on the maximum power that can be delivered by the vehicle's powertrain. We also expect that the function be *monotone*, that is, the more you press the pedal, the larger the terminal velocity will be. Our previous exposure to the properties of the arc-tangent function *and* engineering intuition about the expected outcome of the experiment allowed us to successfully select this function as a suitable candidate for a model.

Other families of functions might suit the data in Fig. 1.5. For example, we could have used *polynomials*, perhaps constrained to pass through the origin and ensure monotonicity. One of the most useful classes of mathematical models one can consider is that of *linear models*, which are, of course, first-order polynomials. One might be tempted to equate linear with simple. Whether or not this might be true in some cases, simplicity is far from a sin. More often than not, the loss of some feature neglected by a linear model is offset by the availability of a much broader set of analytic tools. It is better to *know* when you are wrong than to *believe* you are right. As the title suggests, this book is mostly concerned with linear models. Speaking of linear models, one might propose describing the data in Fig. 1.4 by a linear mathematical model of the form

$$y = \gamma u. \tag{1.1}$$

Figure 1.6 shows two such models (dashed lines). The curve with slope coefficient $\gamma = 47.8$ was obtained by performing a least-squares fit to all data points (see P1.11). The curve with coefficient $\gamma = 99.4$ is a first-order approximation of the nonlinear model calculated in Fig. 1.5 (see P1.12). Clearly, each model has its limitations in describing the experiment. Moreover, one model might be better suited to describe certain aspects of the experiment than the other. Responsibility rests with the engineer or the scientist to select the model, or perhaps set of models, that better fits the problem in hand, a task that at times may resemble an art more than a science.

1.2 Cautionary Note

It goes without saying that the mathematical models described in Section 1.1 do not purport to capture every detail of the experiment, not to mention reality. Good models are the ones that capture *essential* aspects that we *perceive* or can experimentally validate as real, for example how the terminal velocity of a car responds to the acceleration pedal in the given experimental conditions. A model does not even need to be *correct* to be useful: for centuries humans used[3] a model in which the sun revolves around the earth to predict and control their days! What is important is that models provide a way to express *relevant* aspects of reality using mathematics. When mathematical models are used in control design, it is therefore with the understanding that the model is bound to capture only a subset of features of the actual phenomenon they represent. At no time should one be fooled into *believing* in a model. The curious reader will appreciate [Fey86] and the amusingly provocative [Tal07].

With this caveat in mind, it is useful to think of an idealized *true* or *nominal model*, just as is done in physics, against which a particular setup can be *mathematically* evaluated. This nominal model might even be different than the model used by a particular control algorithm, for instance, having more details or being more complex or more accurate. Of course *physical* evaluation of a control system with respect to the underlying natural phenomenon is possible only by means of experimentation which should also include the physical realization of the controller in the form of computer hardware and software, electric circuits, and other necessary mechanical devices. We will discuss in Chapter 5 how certain physical devices can be used to implement the dynamic controllers you will learn to design in this book.

The models discussed so far have been *static*, meaning that the relationship between inputs and outputs is *instantaneous* and is independent of the past history of the system or their signals. Yet the main objective of this book is to work with *dynamic* models, in which the relationship between present inputs and outputs may depend on the present and past history[4] of the signals.

With the goal of introducing the main ideas behind feedback control in a simpler setup, we will continue to work with static models for the remainder of this chapter. In the case of static models, a mathematical *function* or a set of *algebraic equations* will be used to represent such relationships, as done in the models discussed just above in Section 1.1.

Dynamic models will be considered starting in Chapter 2. In this book, signals will be continuous functions of time, and dynamic models will be formulated with the help of *ordinary differential equations*. As one might expect, experimental procedures that can estimate the parameters of dynamic systems need to be much more sophisticated than the ones discussed so far. A simple experimental procedure will be briefly discussed in Section 2.4, but the interested reader is encouraged to consult one of the many excellent works on this subject, e.g. [Lju99].

[3] Apparently 1 in 4 Americans and 1 in 3 Europeans still go by that model [Gro14].
[4] What about the future?