Applied Linear Algebra in Action

Contributors: Alexey A. Kryukov, et al.

About the Book

Linear algebra is the branch of mathematics concerning vector spaces and linear mappings between such spaces. It includes the study of lines, planes, and sub spaces, but is also concerned with properties common to all vector spaces. The set of points with coordinates that satisfy a linear equation forms a hyperplane in an n-dimensional space. The conditions under which a set of n hyperplanes intersect in a single point is an important focus of study in linear algebra. Such an investigation is initially motivated by a system of linear equations containing several unknowns. Such equations are naturally represented using the formalism of matrices and vectors. Linear algebra is central to both pure and applied mathematics. For instance, abstract algebra arises by relaxing the axioms of a vector space, leading to a number of generalizations. Functional analysis studies the infinite-dimensional version of the theory of vector spaces. Combined with calculus, linear algebra facilitates the solution of linear systems of differential equations. Techniques from linear algebra are also used in analytic geometry, engineering, physics, natural sciences, computer science, computer animation, advanced facial recognition algorithms and the social sciences (particularly in economics). Because linear algebra is such a well-developed theory, nonlinear mathematical models are sometimes approximated by linear models.

Applied Linear Algebra in Action envisages on the Matrices, Gaussian elimination, vector spaces, LU-decomposition, orthogonality, Gram-Schmidt process, determinants, inner products, eigenvalue problems, diagonalization of symmetric matrices, applications to differential equations and Markov processes. The book is a compilation of research articles written by renowned experts in their respective fields. It will be of valuable for post graduate students, researchers as well as practitioners also.

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Preface

Linear algebra is the branch of mathematics concerning vector spaces and linear mappings between such spaces. It includes the study of lines, planes, and subspaces, but is also concerned with properties common to all vector spaces. The set of points with coordinates that satisfy a linear equation forms a hyperplane in an n-dimensional space. The conditions under which a set of n hyperplanes intersect in a single point is an important focus of study in linear algebra. Such an investigation is initially motivated by a system of linear equations containing several unknowns. Such equations are naturally represented using the formalism of matrices and vectors. Linear algebra is central to both pure and applied mathematics. For instance, abstract algebra arises by relaxing the axioms of a vector space, leading to a number of generalizations. Functional analysis studies the infinite-dimensional version of the theory of vector spaces. Combined with calculus, linear algebra facilitates the solution of linear systems of differential equations. Techniques from linear algebra are also used in analytic geometry, engineering, physics, natural sciences, computer science, computer animation, advanced facial recognition algorithms and the social sciences (particularly in economics). Because linear algebra is such a well-developed theory, nonlinear mathematical models are sometimes approximated by linear models.

Applied Linear Algebra in Action envisages on the Matrices, Gaussian elimination, vector spaces, LU-decomposition, orthogonality, Gram-Schmidt process, determinants, inner products, eigenvalue problems, diagonalization of symmetric matrices, applications to differential equations and Markov processes. The book is a compilation of research articles written by renowned experts in their respective fields. It will be of valuable for post graduate students, researchers as well as practitioners also.

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1

LINEAR ALGEBRA AND DIFFERENTIAL GEOMETRY ON ABSTRACT HILBERT SPACE

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INTRODUCTION

Various integral transforms are known to be extremely useful in analysis and applications. One example is the Fourier transform in analysis and applied problems; another example is the Segal-Bargmann transform [1] in quantum theory. An important common property of integral transforms is that they relate various spaces of functions and various operators on these spaces and allow one to "transplant" a problem from one space to another. Because of that, the problem at hand may become easier to solve. A somewhat similar situation arises when working with tensor equations in the finite-dimensional setting. Namely, by an appropriate choice of coordinates one can significantly simplify a given tensor equation. Although the analogy is obvious, an infinite-dimensional setting offers a significantly larger variety of situations. In particular, by using the Segal-Bargmann transform, one can relate problems on spaces of ordinary or even generalized functions to problems on spaces of holomorphic functions.

In the paper we attempt to build a systematic approach to functional transformations on Hilbert spaces based on the above mentioned analogy between integral transforms and changes of coordinates on an n-dimensional manifold. Initial results in this direction were announced in [5, 6]. Various applications of the formalism to quantum theory, especially to the problem of emergence of the classical space-time, were considered in [5, 7, 8]. The goal here is to approach the subject in a more formal mathematical way and to justify the previously obtained results. The reader is referred to [9] for an introduction to Hilbert spaces and applications.

We begin by describing a method of building various Hilbert spaces of functions including spaces of smooth and generalized functions. Then a simple formalism based on isomorphisms of these spaces is constructed. The formalism allows one to "move" in a systematic way between Hilbert spaces of functions and at the same time to formulate problems in a way independent of any particular functional realization. The formalism is then applied to building linear algebra on Hilbert spaces, which deals with the ordinary and the generalized functions on an equal footing. After discussing various special transformations preserving properties of differential operators, we concentrate on differential geometry of Hilbert spaces of functions. Namely, using the developed formalism we find a natural isometric embedding of finite dimensional Riemannian manifolds into Hilbert spaces of functions. Finally,

the formalism is applied to construct a Riemannian metric on the unit sphere in a Hilbert space in such a way that solutions of Schrodinger equation ¨ are geodesics on the sphere.

HILBERT SPACES OF C^∞-FUNCTIONS AND THEIR DUALS

We first discuss a general method of constructing various Hilbert spaces of smooth and generalized functions. Consider the convolution

$$(f * \rho)(x) = \int f(y) e^{-(x-y)^2} dy \tag{2.1}$$

of a function $f \in L_2(R)$ and the Gaussian function $\rho(x) = e^{-x2}$. It is the standard result that such a convolution is in $C^\infty \cap L_2(R)$ and

$$D^p(f * \rho)(x) = (f * D^p \rho)(x), \tag{2.2}$$

for any order p of the derivative D.

Theorem 2.1. The linear operator $\rho: L_2(R) \to L_2(R)$ defined by

$$\rho f = f * \rho \tag{2.3}$$

is a bounded invertible operator.

Proof. The operator ρ is bounded because $\|\rho * f\|_{L_2} \le \|\rho\|_{L_1} \|f\|_{L_2}$. To check that it is invertible, assume that $(\rho * f)(x) = 0$ and differentiate both sides of this equation an arbitrary number of times p. We then conclude that all Fourier coefficients c_p of f in the orthonormal in $L_2(R)$ basis of Hermite functions vanish.

The operator ρ induces the Hilbert metric on the image $H = \rho(L_2(R))$ by

$$(\varphi, \psi)_H = (\rho^{-1}\varphi, \rho^{-1}\psi)_{L_2}. \tag{2.4}$$

In particular, the space H with this metric is Hilbert.

Theorem 2.2. The embedding $H \subset L_2(R)$ is continuous and H is dense in $L_2(R)$.

Proof. We have

$$\|\varphi\|_{L_2} = \|\rho f\|_{L_2} \le \|\rho\|_{L_1} \|f\|_{L_2} = M\|\varphi\|_H, \tag{2.5}$$

where M is a constant. In addition, the functions $e^{-(x-a)^2}$, $a \in R$, being in H, form a complete system in $L_2(R)$. Therefore, H is dense in $L_2(R)$.

Provided we identify those linear functionals on H and $L_2(R)$ which are equal on H, we then have

$$L_2^*(R) \subset H^*. \tag{2.6}$$

Indeed, any functional continuous on L_2 will be continuous on H.

Theorem 2.3. When using the standard norm

$$\|f\|_{H^*} = \sup_{\|\varphi\|_H \le 1} |(f, \varphi)| \tag{2.7}$$

on H^* and similarly on L_2^*, we have a continuous embedding $L_2^*(R) \hookrightarrow H^*$.

Proof. Because $\|\varphi\|_{L_2} \le M\|\varphi\|_H$, for any $f \in L_2^*$ we have

$$\|f\|_{H^*} \le N\|f\|_{L_2^*}. \tag{2.8}$$

That is, we have a continuous embedding $L_2^*(R) \hookrightarrow H^*$.

By Riesz theorem, the norm on H^* is induced by the inner product

$$(f, g)_{H^*} = (\varphi, \psi)_H, \tag{2.9}$$

where $f = (\varphi, \cdot)_H = \hat{G}\varphi$ and $g = (\psi, \cdot)_H = \hat{G}\psi$ and $\hat{G}: H \to H^*$ is the Riesz isomorphism. Similarly for L_2^*

On the other hand, since $\rho : L_2 \to H$ is an isomorphism of Hilbert spaces, the adjoint operator $\rho^* : H^* \to L_2^*$, defined for any $f \in H^*, \varphi \in L_2$ by

$$(\rho^* f, \varphi) = (f, \rho\varphi), \tag{2.10}$$

is continuous and invertible (with $(\rho^*)^{-1} = (\rho^{-1})^*$)

Theorem 2.4. The operator ρ^* is an isomorphism of Hilbert spaces H^* and L_2^* with the above inner product (2.9). The metric on H^* can be defined by the isomorphism $\hat{G}^{-1} : H^* \to H,\ \hat{G}^{-1} = \rho\rho^*$.

Proof. For any f, g ∈ H^* the inner product (f, g)H^* induced by ρ^* is given by

$$(f,g)_{H^*} = (\rho^* f, \rho^* g)_{L_2^*} = (f, \rho\rho^* g), \tag{2.11}$$

where $\rho\rho^* : H^* \to H$ defines the metric on H^*. On the other hand, if $f = (\varphi, \cdot)_H$ and g = (ψ,·)H, then $f = \hat{G}\varphi$, and $g = \hat{G}\psi$, where $\hat{G} : H \to H^*$ is the Riesz isomorphism.

Therefore, metric (2.9) on H∗ can be written as follows:

$$(f,g)_{H^*} = (\varphi,\psi)_H = (\hat{G}\varphi,\psi) = (\hat{G}\hat{G}^{-1}f, \hat{G}^{-1}g) = (f, \hat{G}^{-1}g). \tag{2.12}$$

From (2.4) we conclude that $\hat{G}^{-1} = \rho\rho^*$. Also, because the inner products (2.11) and (2.12) coincide, ρ^* is an isomorphism of Hilbert spaces H^* and L_2^*.

Because the kernel of ρ is given by $\rho(x,y) = e^{-(x-y)^2}$, the kernel $k(x,y)$ of $\hat{G}^{-1}$, computed up to a constant factor, is equal to

$$k(x,y) = e^{-(x-y)^2/2}. \tag{2.13}$$

Therefore, the inner product on H^* can be written as

$$(\varphi,\psi)_{H^*} = \int e^{-(x-y)^2/2}\varphi(x)\psi(y)dx\,dy. \tag{2.14}$$

Note that the integral symbol here denotes the action of the bilinear functional G^{-1} on $H^* \times H^*$. Only in special cases does this action coincide with Lebesgue integration with respect to x, y.

Theorem 2.5. The space H^* contains the pointwise evaluation functional (the deltafunction) $\delta_a : \varphi \to \varphi(a)$.

Proof. The sequence

$$f_L = \frac{L}{\sqrt{\pi}} e^{-L^2x^2} \tag{2.15}$$

is fundamental in H^* and so it converges in H^* Also, the sequence f_L is a deltaconverging sequence, so that $(f_L,\varphi) \to (\delta_a,\varphi)$ as $L \to \infty$ for any ϕ ∈ H. In particular, f_L strongly converges to the evaluation functional δ_a in H^*.

Formally,

$$(\delta_a,\delta_b)_{H^*} = \int e^{-(x-y)^2/2}\delta(x-a)\delta(y-b)dx\,dy = e^{-(a-b)^2/2}. \tag{2.16}$$

Note that the inner product on $L_2(R)$ can be written in the form

$$(\varphi,\psi)_{L_2} = \int \delta(x-y)\varphi(x)\psi(y)dx\,dy, \tag{2.17}$$

where $\int \delta(x-y)\psi(y)dy$ denotes the convolution $(\delta * \psi)(x) = \psi(x)$. The main difference between metrics (2.14) and (2.17) is that the metric on $L_2(R)$ is "diagonal," while the metric on H^* is not.

Consider now another Hilbert space obtained in a similar fashion. For this, note that since $H \hookrightarrow L_2(R)$, the Fourier transform σ on H is defined. Moreover, σ induces a Hilbert metric on the image $\tilde{H} = \sigma(H)$ by

$$(\sigma(\varphi),\sigma(\psi))_{\tilde{H}} = (\varphi,\psi)_H, \tag{2.18}$$

for any φ, ψ in H.

We can then define the Fourier transform on H^* in the standard way by

$$(\sigma(f),\sigma(\varphi)) = 2\pi(f,\varphi), \tag{2.19}$$

for any f ∈ H^* and $\varphi \in H$. The image $\sigma(H^*)$ is the Hilbert space $\tilde{H}^*$ dual to $\tilde{H}$. The metric on this space is induced by the map $\sigma\rho : L_2 \to \tilde{H}$:

$$(f,g)_{\tilde{H}^*} = ((\sigma\rho)^* f, (\sigma\rho)^* g)_{L_2^*}. \tag{2.20}$$

In other words, the metric on $\tilde{H}^*$ is given by $\sigma\rho\rho^*\sigma^* : \tilde{H}^* \to \tilde{H}$. The kernel of this metric is then equal to

$$\tilde{k}(x,y) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \delta(x-y), \tag{2.21}$$

where δ(x − y) is the kernel of the operator δ∗ of convolution with the delta function. Note that because of the weight $e^{-x^2/2}$, the space $\tilde{H}^*$ contains the plane wave functions e^{ikx}.

So, we have the Hilbert space H of C^∞ (in fact, analytic) functions, its dual H^* containing singular generalized functions, in particular, δ, $D^p\delta$. We also have the Fourier transformed space $\tilde{H}$ and its dual $\tilde{H}^*$, which contains, in particular, the plane wave function e^{ipx}.

The method used to construct the above spaces of generalized functions is very general. All we need is a linear injective map ρ on a space L_2 of square-integrable functions. By changing ρ, we change the metric $\hat{G}$ on the Hilbert space $H = \rho(L_2)$ and therefore the metric $\hat{G}^{-1}$ on the dual space H^*. Roughly speaking, by "smoothing" the kernel of the metric $\hat{G}^{-1}$, we extend the class of (generalized) functions for which the norm defined by this metric is finite. The same is true if we "improve" the behavior of the metric for large |x|. Conversely, by "spoiling the metric" we make the corresponding Hilbert space "poor" in terms of a variety of the elements of the space. This is further illustrated by the following theorem.

Theorem 2.6. If ρ is the map on $L_2(R)$ with the kernel $\rho(x,y) = e^{-(x-y)^2-x^2}$, then the Hilbert space $\rho(L_2(R))$ with the induced metric is continuously embedded into the Schwartz space W of C^∞ rapidly decreasing functions on R.

Proof. From Theorem 2.1 we immediately conclude that the map ρ is injective and the image $\rho(L_2(R))$ consists of C^∞ functions. Moreover, we have

$$\left| \int f(y) e^{-(x-y)^2-x^2} dy \right| \le M \|f\|_{L_2} e^{-x^2}, \tag{2.22}$$

where M is a constant. In particular, the functions in $\rho(L_2(R))$ decrease faster than any power of 1/|x|. Now, the topology on W may be defined by the countable system of norms

$$\|\varphi\|_p = \sup_{x\in R; k,q\le p} |x^k \varphi^{(q)}(x)|, \tag{2.23}$$

where k, q, p are nonnegative integers and $\varphi^{(q)}$ is the derivative of φ of order q. For any $\varphi \in H$, by the Schwarz inequality, we have

$$\begin{aligned} |x^k \varphi^{(q)}(x)| &= \left| \int x^k f(y) \frac{d^q}{dx^q} e^{-(x-y)^2-x^2} dy \right| \\ &\le \left(\int x^{2k} P_q^2(x) e^{-2(x-y)^2-2x^2} dy \right)^{1/2} \|f\|_{L_2} = M_{k,q} \|f\|_{L_2}, \end{aligned} \tag{2.24}$$

where $\rho f = \varphi$, P_q is a polynomial of degree q and $M_{k,q}$ are constants depending only on k, q. This proves that topology on H is stronger than topology on W, that is, H ⊂ W is a continuous embedding.

It follows from the theorem that $W^* \subset H^*$ as a set. In fact, any functional continuous on W will be continuous on H. Moreover, by choosing the strong topology on W^* one can show that the embedding of W^* into H^* is continuous. Notice also that one could choose the weak topology on W^* instead as the weak and strong topologies on W^* are equivalent [2].

Remarks. (1) The considered Hilbert spaces of generalized functions could have also been obtained by completing spaces of ordinary functions with respect to the discussed inner products. More generally, in many cases the expression

$$(f,g)_k = \int k(x,y) f(x) g(y) dx\, dy, \tag{2.25}$$

where k is a continuous function on a region D in R^{2n}, defines an inner product on $L_2(R^n)$. Then the completion with respect to the corresponding metric yields a Hilbert space containing singular generalized functions.

(2) The isomorphism ρ defined on $L_2(R)$ by $(\rho f)(x) = \int e^{-(x-y)^2} f(y) dy$ is closely related to the Segal and Bargmann transform $C_t : L_2(R^d) \to H(C^d)$, where $H(C^d)$ is the space of holomorphic functions on C^d, given by

$$(C_t f)(z) = \int (2\pi t)^{-d/2} e^{-(z-y)^2/2t} f(y) dy. \tag{2.26}$$

The theorem by Segal and Bargmann [1] (see also [4]) says the following.

Theorem 2.7. For each t the map C_t is an isomorphism of $L_2(R^d)$ onto $HL_2(C^d, \nu_t)$. Here HL2(Cd,νt) denotes the space of holomorphic functions that are square-integrable with respect to the measure $\nu_t(z)dz$, where dz is the 2d-dimensional Lebesgue measure on Cd and the density ν_t is given by

$$\nu_t(x+iy) = (\pi t)^{-d/2} e^{-y^2/t}, \quad x, y \in R^d. \tag{2.27}$$

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