

Sixth Edition

Pijush K. Kundu, Ira M. Cohen, and David R. Dowling with contributions by Grétar Tryggvason



# FLUID MECHANICS

### SIXTH EDITION

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Academic Press is an imprint of Elsevier 125, London Wall, EC2Y 5AS 525 B Street, Suite 1800, San Diego, CA 92101-4495, USA 225 Wyman Street, Waltham, MA 02451, USA The Boulevard, Langford Lane, Kidlington, Oxford OX5 1GB, UK

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ISBN: 978-0-12-405935-1

Library of Congress Cataloging-in-Publication Data
A catalog record for this book is available from the Library of Congress

### British Library Cataloguing in Publication Data

A catalogue record for this book is available from the British Library

For information on all Academic Press publications visit our website at http://store.elsevier.com/

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# FLUID MECHANICS

SIXTH EDITION

# Dedication

This textbook is dedicated to my wife and family whose patience during this undertaking has been a source of strength and consolation, and to the many fine instructors and students with whom I have interacted who have all in some way highlighted the allure of this subject for me.

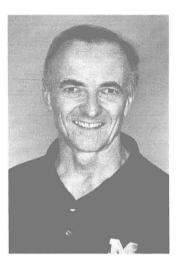
# About the Authors



Pijush K. Kundu, 1941-1994, was born in Calcutta, India. He earned a BS degree in Mechanical Engineering from Calcutta University in 1963 and an MS degree in Engineering from Roorkee University in 1965. After a few years as a lecturer at the Indian Institute of Technology in Delhi, he came to the United States and earned a PhD at Pennsylvania State University in 1972. He then followed a lifelong interest in oceanography and held research and teaching positions at Oregon State University and the University de Oriente in Venezuela, finally settling at the Oceanographic Center of Nova Southeastern University, where he spent most of his career contributing to the understanding of coastal dynamics, mixed-layer physics, internal waves, and Indian Ocean dynamics. He authored the first edition of this textbook, which he dedicated to his mother, wife, daughter, and son.



Ira M. Cohen, 1937-2007, earned a BS degree from Polytechnic University in 1958 and a PhD from Princeton in 1963, both in aeronautical engineering. He taught at Brown University for three years prior to joining the faculty at the University of Pennsylvania in 1966. There he became a world-renowned scholar in the areas of continuum plasmas, electrostatic probe theories and plasma diagnostics, dynamics and heat transfer of lightly ionized gases, low current arc plasmas, laminar shear layer theory, and matched asymptotics in fluid mechanics. He served as Chair of the Department of Mechanical Engineering and Applied Mechanics from 1992 to 1997. During his 41 years as a faculty member, he distinguished himself through his integrity, candor, sense of humor, pursuit of physical fitness, unrivaled dedication to academics, fierce defense of high scholarly standards, and passionate commitment to teaching.



David R. Dowling, 1960—, grew up in southern California where early experiences with fluid mechanics included swimming, surfing, sailing, flying model aircraft, and trying to throw a curve ball. At the California Institute of Technology, he earned BS ('82), MS ('83), and PhD ('88) degrees in Applied Physics and Aeronautics. In 1992, after a year at Boeing Aerospace & Electronics and three at the Applied Physics Laboratory of the University of Washington, he joined the

faculty in the Department of Mechanical Engineering at the University of Michigan, where he has since taught and conducted research in fluid mechanics and acoustics. He is a fellow of the American Physical Society — Division of Fluid Dynamics, the American Society of Mechanical Engineers, and the Acoustical Society of America. Prof. Dowling is an avid swimmer, is married, and has seven children.

### Preface

After the fifth edition of this textbook appeared in print in September of 2011 and I had the chance to use it for instruction, a wide variety of external and self-generated critical commentary was collected to begin the planning for this sixth edition. First of all, I would like the thank all of the book's readers and reviewers worldwide who provided commentary, noted deficiencies, recommended changes, and identified errors. I have done my best to correct the errors and balance your many fine suggestions against the available time for revisions and the desire to keep the printed text approximately the same length while effectively presenting this subject to students at the advancedundergraduate or beginning-graduate level. To this end, I hope this book's readership continues to send suggestions, constructive criticism, and notification of needed corrections for this 6th Edition of Fluid Mechanics.

Fluid mechanics is a traditional field with a long history. Therefore, a textbook such as this should serve as a compendium of established results that is accessible to modern scientists, engineers, mathematicians, and others seeking fluid mechanics knowledge. Thus, the changes made in the revision were undertaken in the hope of progressing toward this goal. In the collected commentary about the 5th Edition, the most common recommendation for the 6th Edition was the inclusion of more examples and more exercises. Thus, over 100 new examples and 110 new exercises, plus nearly 100 new figures, have been added. From a pedagogical standpoint, the new examples may have the

most value since they allowed succinct and self-contained expansion of the book's content. While the sophistication and length of the new examples varies widely, all are intended to illustrate how the various concepts and equations can be applied in circumstances that hopefully appeal to the book's readers. An equally, or perhaps more, important change from the 5th Edition is the completely new chapter on computational fluid dynamics (CFD) authored by Prof. Grétar Tryggvason of the University of Notre Dame (Viola D. Hank Professor and Chair of the Department of Aerospace and Mechanical Engineering, and Editor-in-Chief of the Journal of Computational Physics). This new CFD chapter includes sample MATLAB<sup>TM</sup> codes and 20 exercises. Plus, it has been moved forward in the chapter ordering from tenth to sixth to facilitate instruction using numerical examples and approaches for the topics covered in Chapters 7 to 15. To accommodate all the new examples and the new CFD chapter, the final chapter of the 5th Edition on biofluid mechanics has been moved to the book's companion website (go to http:// store.elsevier.com/9780124059351, under the "Resources" tab at the bottom of the page). Otherwise, the organization, topics, and mathematical level of the 5th Edition have been retained, so instructors who have made prior use of this text should easily be able to adopt the 6th Edition.

There have been a number of other changes as well. Elementary kinetic theory has been added to Chapter 1. Several paragraphs on non-Newtonian constitutive relationships and xiv Preface

flow phenomena have been added to Chapter 4, and the discussions of boundary conditions and dynamic similarity therein have been revised and expanded. A description of flow in a circular tube with an oscillating pressure gradient has been added to Chapter 9 and a tabulation of the Blasius boundary layer profile has been added to Chapter 10. New materials on internal and external rough-wall turbulent flows, and Reynolds-stress closure models have been added to Chapter 12. The presentation of equations in Chapter 13 has been revised in the hope of achieving better cohesion within the chapter. The acoustics section of Chapter 15 has been revised to highlight acoustic source terms, and a section on unsteady one-dimensional gas dynamics has been added to this chapter, too. In addition, some notation changes have been made: the comma notation for derivatives has been dropped, and the total stress tensor, viscous stress tensor, and wall shear stress are now denoted by  $T_{ii}$ ,  $\tau_{ii}$ , and  $\tau_w$ , respectively. Unfortunately, (my) time constraints have pushed the requested addition of new sections on micro-fluid mechanics, wind turbines, and drag reduction technologies off to the 7th Edition

Prior users of the text will no doubt notice that the *Multi-media Fluid Mechanics* DVD from Cambridge University Press is no longer co-packaged with this text. However, a cross listing of chapter sections with the DVD's outline is now provided on the textbook's companion website (see http://store.elsevier.com/9780124059351). Other resources can be found there, too, such as: the errata sheets for the *5th* and *6th Editions*, and (as mentioned above) the sixteenth chapter on biofluid mechanics. Plus, for instructors, solutions for all 500+ exercises are available (requires registration at http://textbooks.elsevier.com/9780124059351).

And finally, responsible stewardship and presentation of this material is my primary goal. Thus, I welcome the opportunity to correct any errors you find, to hear your opinion of how this book might be improved, and to include topics and exercises you might suggest; just contact me at drd@umich.edu.

David R. Dowling, Ann Arbor, Michigan, August 2014

# Acknowledgments

The current version of this textbook has benefited from the commentary, suggestions, and corrections provided by the reviewers of the revision proposal, and the many careful readers of the fifth edition of this textbook who took the time to contact me. I would also like to recognize and thank my technical mentors, Prof. Hans W. Liepmann

(undergraduate advisor), Prof. Paul E. Dimotakis (graduate advisor), and Prof. Darrell R. Jackson (post-doctoral advisor), and my friends and colleagues who have contributed to the development of this text by discussing ideas and sharing their expertise, humor, and devotion to science and engineering.

# Nomenclature

# NOTATION (RELEVANT EQUATION NUMBERS APPEAR IN PARENTHESES)

 $\bar{f}=$  principle-axis version of f, background or quiescent-fluid value of f, or average or ensemble average of f, Darcy friction factor (12.101, 12.102)

 $\hat{f} = \text{complex amplitude of } f$ 

 $\tilde{f}$  = full field value of f

f' = derivative of f with respect to its argument, or perturbation of f from its reference state

 $f^* =$ complex conjugate of f, or the value of f at the sonic condition

 $f^+=$  the dimensionless, law-of-the-wall value of f

 $f_{\xi} = \partial f/\partial \xi$  (6.105)

 $f_{\rm cr} = {\rm critical\ value\ of}\ f$ 

 $f_{av}$  = average value of f

 $f_{CL}$  = centerline value of f

 $f_j$  = the j<sup>th</sup> component of the vector  $\mathbf{f}$ , f at location j (6.14)

f)<sup>n</sup><sub>i</sub>, f<sup>n</sup><sub>i</sub> = f at time n at horizontal x-location f (6.13)

 $f_{ij}$  = the i-j component of the second order tensor f

 $f_{i,j'}^n f_{i,j}^n = f$  at time n at horizontal x-location i and vertical y-location j (6.52, Fig. 6.10)

 $f_R$  = rough-wall value of f

 $f_S = \text{smooth-wall value of } f$ 

 $f_0$  = reference, surface, or stagnation value of f

 $f_{\infty}$  = reference value of f or value of f far away from the point of interest

 $\Delta f = \text{change in } f$ 

### SYMBOLS (RELEVANT EQUATION NUMBERS APPEAR IN PARENTHESES)

 $\alpha$  = contact angle (Fig. 1.8), thermal expansion coefficient (1.26), angle of rotation, iteration number (6.57), angle of attack (Fig. 14.6)

a = triangular area, cylinder radius,
 sphere radius, amplitude

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**a** = generic vector, Lagrangian acceleration (3.1)

A = generic second-order (or higher) tensor

A, A = a constant, an amplitude, area, surface, surface of a material volume, planform area of a wing

 $A^* = \text{control surface}$ , sonic throat area

 $A_o = \text{Avogadro's number}$ 

 $A_0$  = reference area

 $A_{ij}$  = representative second-order tensor

 $\beta$  = angle of rotation, coefficient of density change due to salinity or other constituent, convergence acceleration parameter (6.57), variation of the Coriolis frequency with latitude (13.10), camber parameter (Fig. 14.13)

**b** = generic vector, control surface velocity (Fig. 3.20)

B, B = a constant, Bernoulli function (4.70), log-law intercept parameter (12.88)

**B**,  $B_{ij}$  = generic second-order (or higher) tensor

Bo = Bond number (4.118)

*c* = speed of sound (1.25, 15.1h), phase speed (8.4), chord length (14.2, Figs. 14.2, 14.6)

c = phase velocity vector (8.8)

 $c_g$ ,  $c_g$  = group velocity magnitude (8.67) and vector (8.141)

 $\chi = \text{scalar stream function (Fig. 4.1)}$ 

°C = degrees centigrade

*C* = a generic constant, hypotenuse length, closed contour

Ca = Capillary number (4.119)

 $C_f$  = skin friction coefficient (10.15, 10.32)

 $C_p$  = pressure (coefficient) (4.106, 7.32)

 $c_p$  = specific heat capacity at constant pressure (1.20)

 $C_D$  = coefficient of drag (4.107, 10.33)

 $C_L$  = coefficient of lift (4.108)

 $c_{\rm v} = {\rm specific \ heat \ capacity \ at \ constant}$  volume (1.21)

 $C_{ij}$  = matrix of direction cosines between original and rotated coordinate system axes (2.5)

 $C_{\pm}$  = Characteristic curves along which the  $I_{\pm}$  invariants are constant (15.57)

d = diameter, distance, fluid layer depth

d = dipole strength vector (7.28),
 displacement vector

 $\delta =$  Dirac delta function (B.4.1), similarity-variable length scale (9.32), boundary-layer thickness, generic length scale, small increment, flow deflection angle (15.64)

 $\overline{\delta}=$  average boundary-layer thickness

 $\delta^*$  = boundary-layer displacement thickness (10.16)

 $\delta_{ij}$  = Kronecker delta function (2.16)

 $\delta_{99} = 99\%$  layer thickness

D = distance, drag force, diffusion coefficient (6.10)

D = drag force vector (Example 14.1)

 $D_i = \text{lift-induced drag (14.15)}$ 

D/Dt = material derivative (3.4), (3.5), or (B.1.4)

 $D_T$  = turbulent diffusivity of particles (12.156)

D = generalized field derivative (2.31)

 $\varepsilon$  = roughness height, kinetic energy dissipation rate (4.58), a small distance, fineness ratio h/L (9.14), downwash angle (14.14)

 $\overline{\varepsilon}$  = average dissipation rate of the turbulent kinetic energy (12.47)

 $\bar{\epsilon}_T$  = average dissipation rate of the variance of temperature fluctuations (12.141)

 $\varepsilon_{ijk}$  = alternating tensor (2.18)

e = internal energy per unit mass (1.16)

 $\mathbf{e}_i = \text{unit vector in the } i\text{-direction (2.1)}$ 

 $\bar{e}$  = average kinetic energy of turbulent fluctuations (12.47)

Ec = Eckert number (4.115)

 $E_k$  = kinetic energy per unit horizontal area (8.39)

 $E_p$  = potential energy per unit horizontal area (8.41)

E = numerical error (6.21), average energy per unit horizontal area (8.42), Ekman number (13.18)

 $\overline{E} = \text{kinetic energy of the average flow}$  (12.46)

EF = time average energy flux per unit length of wave crest (8.43)

f = generic function, Maxwell distribution function (1.1) and (1.4), Helmholtz free energy per unit mass, longitudinal correlation coefficient (12.38), Coriolis frequency (13.6), dimensionless friction parameter (15.45)

 $\bar{f}$  = Darcy friction factor (12.101, 12.102)

 $f_i$  = unsteady body force distribution (15.5)

 $\phi$  = velocity potential (7.10), an angle

f = surface force vector per unit area (2.15, 4.13)

F= force magnitude, generic flow field property, generic flux, generic or profile function

 $F_f$  = perimeter friction force (15.25)

F = force vector, average wave energy flux vector (8.157)

 $\Phi = \text{body force potential (4.18), undetermined spectrum function (12.53)}$ 

 $F_D$ ,  $\overline{F}_D$  = drag force (4.107), average drag force

 $F_L = \text{lift force (4.108)}$ 

Fr = Froude number (4.104)

 $\gamma$  = ratio of specific heats (1.30), velocity gradient, vortex sheet strength, generic dependent-field variable

 $\dot{\gamma}$  = shear rate

g = body force per unit mass (4.13)

*g* = acceleration of gravity, undetermined function, transverse correlation coefficient (12.38)

g' = reduced gravity (8.116)

 $\Gamma$  = vertical temperature gradient or lapse rate, circulation (3.18)

 $\Gamma_a$  = adiabatic vertical temperature gradient (1.36)

 $\Gamma_a$  = circulation due to the absolute vorticity (5.29)

G =gravitational constant, profile function

 $G_n$  = Fourier series coefficient

G = center of mass, center of vorticity

h = enthalpy per unit mass (1.19), height, gap height, viscous layer thickness

 $\hbar = \text{Planck's constant}$ 

 $\eta$  = free surface shape, waveform, similarity variable (9.25) or (9.32), Kolmogorov microscale (12.50)

 $\eta_T$  = Batchelor microscale (12.143)

H = atmospheric scale height, water depth, step function, shape factor (10.46), profile function

i =an index, imaginary root

I = incident light intensity, bending moment of inertia

 $I_{\pm}$  = Invariants along the  $C_{\pm}$  characteristics (15.55)

j = an index

J = Jacobian of a transformation (6.110), momentum flux per unit span (10.58)

 $J_s$  = jet momentum flux per unit span (12.62)

 $J_i$  = Bessel function of order i

 $J_m$  = diffusive mass flux vector (1.7)

 $\varphi$  = a function, azimuthal angle in cylindrical and spherical coordinates (Fig. 3.3)

k= thermal conductivity (1.8), an index, wave number (6.12) or (8.2), wave number component

 $\kappa$  = thermal diffusivity, von Karman constant (12.88)

 $\kappa_{\rm s} = {\rm diffusivity} \ {\rm of} \ {\rm salt}$ 

 $\kappa_T$  = turbulent thermal diffusivity (12.116)

 $\kappa_m$  = mass diffusivity of a passive scalar in Fick's law (1.7)

 $\kappa_{mT}$  = turbulent mass diffusivity (12.117)

 $k_{\rm B} = {\rm Boltzmann's\ constant\ } (1.27)$ 

 $k_s$  = sand grain roughness height

Kn = Knudsen number

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K = a generic constant, magnitude of the wave number vector (8.6), lift curve slope (14.16)

K = degrees Kelvin

K = wave number vector (8.5)

*l* = molecular mean free path (1.6), spanwise dimension, generic length scale, wave number component (8.5, 8.6), shear correlation in Thwaites method (10.45), length scale in turbulent flow

 $l_T = \text{mixing length (12.119)}$ 

L, L = generic length dimension, generic length scale, lift force

 $L_M$  = Monin-Obukhov length scale (12.138)

 $\lambda$  = wavelength (8.1, 8.7), laminar boundary-layer correlation parameter (10.44)

 $\lambda_m$  = wavelength of the minimum phase speed

 $\lambda_t$  = temporal Taylor microscale (12.19)

 $\lambda_f$ ,  $\lambda_g$  = longitudinal and lateral spatial Taylor microscales (12.39)

 $\Lambda =$  lubrication-flow bearing number (9.16), Rossby radius of deformation, wing aspect ratio (14.1)

 $\Lambda_f, \Lambda_g = \text{longitudinal and lateral integral spatial scales (12.39)}$ 

 $\Lambda_t$  = integral time scale (12.18)

 $\mu$  = dynamic or shear viscosity (1.9), Mach angle (15.60)

 $\mu_v = \text{bulk viscosity (4.36)}$ 

m = molecular mass (1.1), generic mass, an index, moment order (12.1), wave number component (8.5, 8.6)

M, M = generic mass dimension, mass, Mach number (4.111), apparent or added mass (7.108)

 $M_w = \text{molecular weight}$ 

n = molecular density (1.1), an index, generic integer number, power law exponent (4.37)

n = normal unit vector

 $n_s$  = index of refraction

N= number of molecules (1.27), Brunt-Väisälä or buoyancy frequency (1.35, 8.126), number

 $N_{ij}$  = pressure rate of strain tensor (12.131)

 $\nu =$  kinematic viscosity (1.10), cyclic frequency, Prandtl-Meyer function (15.67)

 $\nu_T$  = turbulent kinematic viscosity (12.115)

O = origin

p = pressure

 $p = t \times n$ , third unit vector

 $p_{atm} = atmospheric pressure$ 

 $p_i$  = inside pressure

 $p_o$  = outside pressure

 $p_0$  = reference pressure at z = 0

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### NOMENCLATURE

 $p_{\infty}=$  reference pressure far upstream or far away

 $\overline{p}$  = average or quiescent pressure in a stratified fluid

P =average pressure

 $\Pi$  = wake strength parameter (12.95)

Pr = Prandtl number (4.116)

q = heat added to a system (1.16), volume flux per unit span, unsteady volume source (15.4), dimensionless heat addition parameter (15.45)

q,  $q_i$  = heat flux (1.8)

 $\dot{q}$  = generic acoustic source (15.8)

 $q_s$  = two-dimensional point source or sink strength in ideal flow (7.13)

 $q_n =$  generic parameter in dimensional analysis (1.42)

Q = volume flux in two or three dimensions, heat added per unit mass (15.21)

 $\theta$  = potential temperature (1.37), unit of temperature, angle in polar coordinates, momentum thickness (10.17), local phase, an angle

 $\rho = \text{mass density } (1.7)$ 

 $ho_{\mathrm{s}}=$  static density profile in stratified environment

 $\rho_m = \text{mass density of a mixture}$ 

 $\overline{\rho}=$  average or quiescent density in a stratified fluid

 $\rho_{\theta}$  = potential density (1.39)

r = matrix rank, distance from the origin, distance from the axis

r = particle trajectory (3.1), (3.8)

R = distance from the cylindrical axis, radius of curvature, gas constant (1.29), generic nonlinearity parameter

 $R_{\Delta}$  = dimensionless grid-resolution (6.42)

 $R_u$  = universal gas constant (1.28)

 $R_i$  = radius of curvature in direction i (1.11)

**R**,  $R_{ij}$  = rotation tensor (3.13), correlation tensor (12.12), (12.23)

Ra = Rayleigh number (11.21)

Re = Reynolds number (4.103)

Ri = Richardson number, gradient Richardson number (11.66, 12.136)

Rf = flux Richardson number (12.135)

Ro = Rossby number (13.13)

 $\sigma =$  surface tension (1.11), interfacial tension, vortex core size (3.28, 3.29), temporal growth rate (11.1), oblique shock angle (Fig. 15.21)

s = entropy (1.22), arc length, salinity, wingspan (14.1), dimensionless arc length

 $\sigma_i$  = standard deviation of molecular velocities (1.3)

S = salinity, scattered light intensity, an area, entropy

NOMENCLATURE XXIII

 $S_e$  = one-dimensional temporal longitudinal energy spectrum (12.20)

 $S_{11}$  = one-dimensional spatial longitudinal energy spectrum (12.45)

 $S_T$  = one-dimensional temperature fluctuation spectrum (12.142), (12.143)

S, S<sub>ij</sub> = strain rate tensor (3.12), symmetric tensor

St = Strouhal number (4.102)

t = time

t = tangent vector

T, T = temperature (1.1), generic time dimension, period

T,  $T_{ii}$  = stress tensor (2.15), (Fig. 2.4)

Ta = Taylor number (11.52)

 $T_o$  = free stream temperature

 $T_w$  = wall temperature

 $\tau$  = shear stress (1.9), time lag

 $\tau$ ,  $\tau_{ii}$  = viscous stress tensor (4.27)

 $\tau_w$  = wall or surface shear stress

 $v = \text{specific volume} = 1/\rho$ 

u = horizontal component of fluid velocity (1.9)

**u** = generic vector, average molecular velocity vector (1.1), fluid velocity vector (3.1)

 $u_i$  = fluid velocity components, fluctuating velocity components

 $u_* = friction velocity (12.81)$ 

U = generic uniform velocity vector

 $U_i$  = ensemble average velocity components

*U* = generic velocity, average stream-wise velocity

 $\Delta U$  = characteristic velocity difference

 $U_e$  = local free-stream flow speed above a boundary layer (10.11), flow speed at the effective angle of attack

 $U_{CL}$  = centerline velocity (12.56)

 $U_{\infty} =$  flow speed far upstream or far away

v = molecular speed (1.4), component of fluid velocity along the y axis

 $\mathbf{v} = \text{molecular}$  velocity vector (1.1), generic vector

V = volume, material volume, average stream-normal velocity, average velocity, complex velocity

 $V^* = \text{control volume}$ 

w = vertical component of fluid velocity, complex potential (7.42), downwash velocity (14.13)

*W* = thermodynamic work per unit mass, wake function

 $\dot{W}$  = rate of energy input from the average flow (12.49)

We = Weber number (4.117)

 $\omega = \text{temporal frequency (8.2)}$