Decision Making, Models and Algorithms

A First Course

SAUL. I. GASS

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Preface

My purpose in writing this book is to make available to undergraduate students a presentation of what I feel has been one of the most exciting advances in mathematics since the invention of the calculus. That is, the development and application of mathematical methods and analysis techniques that enable us to structure and solve important decision problems of business, industry, and government. This class of problems—decision problems—are different from the classical types in many respects. How they are different will become clear as you read the text.

It is my further belief that undergraduate training in mathematics must move away from the current required calculus-based program to a more varied one that stresses the important and modern applied areas contained in the mathematical decision sciences, as practiced by operations researchers and management scientists. There is a need to teach the fundamentals of the decision sciences to engineers, business majors, mathematicians, computer scientists, and just about anyone who wants to obtain an understanding of what mathematics and the decision sciences can do to solve real-world problems. To my mind, the finite mathematics books—as good as they are and as valuable as they have been to students and teachers—do not convey the excitement and structure inherent in decision-making models and the related mathematical content. Neither do the standard texts that explain management science/operations research. I have tried to develop a different approach. I emphasize the decision nature of problems. I bring the students into the problem-solving part by challenging them to use their training and basic knowledge. It is a book that will also challenge the instructors. They will have to teach model formulation and algorithm development, motivate students to study variations of a problem under changing assumptions, and encourage students to use their knowledge and intuition in developing solution techniques..

I believe the presentation and material in this text will assist both instructors and students to meet these important challenges. Thus, this is not a methodology book with which students just learn how to solve problems. Rather, I have attempted to present the material in a manner that would strengthen their abilities to analyze decision situations—situations that call for the selection of one possible solution over others.

Certain principles and ideas have been developed that help us organize our approach to analyzing decision problems. It is these principles and ideas—what I collectively term the *decision framework*—that are emphasized. This framework and the concept of mathematical decision models are described in Part I. In the succeeding Parts II–V, we introduce many problem situations and show how to formulate their mathematical statements. Depending on the resulting mathematical structures, the problems can be solved by obvious or not-so-obvious methods. Whenever possible, the student is asked to invent solution techniques, or is led to the development of a known procedure. As this is not a book on case studies, the full import and concerns of the decision framework cannot be demonstrated. For example, the problems of data collection, model validity, and model implementation can only be hinted at. The student is cautioned against indiscriminate use of a mathematical model.

Each of the five parts of the text ends with a chapter that combines further discussions, extensions, and exercises. These concluding chapters are designed to clarify and extend the previously discussed material, and to motivate and challenge the student. The teacher should use these chapters in planning the course content and include those sections that best meet the objectives of the course and the needs of the students. The student and teacher must treat the discussions, extensions, and exercises chapters as integral parts of the course and book.

A few teachers and researchers familiar with the totality of decision models might question my apparent emphasis on the linear-programming model. This emphasis is due to a number of reasons: (1) linear programming is the preeminent model of the decision sciences; (2) it is the most widely used modern mathematical optimization technique; and (3) its range of application and use in the practical worlds of business, industry, and government appears to have no bound. Further, as an instructional tool for linear algebra, optimization, and decision making, there is no competitor.

Algorithms imply computation. To learn just how an algorithm works you need to carry out the prescribed steps on a typical set of numbers. The problems you will encounter in this text are readily solved using the old reliable paper and pencil (with eraser) system, possibly augmented with a hand calculator. However, we need to recognize the pervasiveness of electronic computers and their use in solving decision problems. Thus, based on the backgrounds and needs of the students, we encourage the instructor to integrate, as much as possible, the course material with the local computer/software systems. This can be done in a number of ways. If the course involves computer programming (or requires it as a prerequisite), the students can test out their understanding of an algorithm and their programming abilities by writing codes for solving the related decision problems. The numerical exercises could then be solved using these codes. Software packages for some models may already be available, for example, in the case of linear programming. I have found the use of an interactive linear-programming system, such as LINDO (Linear, Interactive, and Discrete Optimizer), to be a valuable instructional aid. I would like to stress, however, that the complete reliance on computer software to solve problems without attempting to understand the associated algorithmic process limits one's future decision-making skills. Such reliance inhibits the ability to raise the right questions as to the structure and assumptions of the algorithm and related model, and leads to blind acceptance of computer printouts.

The level of mathematics required of the student is introductory mathematics, including linear equations and graphs, that is, deterministic mathematics at an elementary level. Introductory probabilistic notions are assumed, but they are not used extensively and can be introduced by the instructor. My target audience is undergraduates, certainly at the junior and senior levels, and advanced lower-division students. The text is for a one-semester course.

In sum, this book presents my approach to how undergraduate students in mathematics, business, computer science, and engineering should be introduced to the science of decision making. It is not a complete review of the subject matter. I have discussed those issues and problems that have been important to me as an analyst and appeal to me as a teacher. The material is designed to prepare the student for more advanced topics. No matter what career path is followed, the concepts of scientific decision making will prove to be of great value.

A text such as this is based on many sources. I would like to acknowledge my indebtedness to the many scientists whose works have formed the basis of my experiences and knowledge. I have tried to give complete references throughout. The explicit and subliminal use of their work is greatly appreciated. The student is encouraged to investigate the original sources to obtain a first-hand view of the ideas and models discussed in the text.

The ever-present encouragement and patience of my wife, Trudy, has always been the fuel that has enabled me to continue—there has never been any shortage here.

SAUL I. GASS

Potomac, Maryland
March 1985

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PART

I A Framework for Decision Making

1

A Framework for Decision Making

1 Decision Making

1.1 THE SCIENCE OF DECISION MAKING

All of us have encountered situations and problems that have required the making of a decision—the choosing between competing opportunities or alternatives. Most everyday problems are resolved without too much difficulty or without serious consequence if, from among the many available alternatives, the "correct" one is not selected. There are problems, however, for which we, as decision makers, want to do our utmost to ensure that the best possible solution is chosen. Such problems occur in most professions, in business, industry, and government, as well as in some personal situations. The medical doctor wants to prescribe the best treatment, the architect wants to produce the best design, the student wants to select the correct career, a company needs to decide where to build the new warehouse, a pension fund manager has to determine what stocks to have in the fund's portfolio and when to buy and sell, a city manager has to select the location of the new incinerator and sanitary landfill. These are hard choices and decisions.

Since the 1940s, our ability to understand, structure, and resolve decision problems has improved greatly. This is due to the increased study of applied problems by mathematicians and other scientists, the development of new mathematical techniques, and the power of the electronic computer. A new science of decision making has been evolving. This has produced a set of ideas, approaches, and procedures that can be considered to form a modern framework for decision making. In this book, we shall describe our view of this framework and focus attention on its centerpiece—the mathematical model. But first, some words of caution.

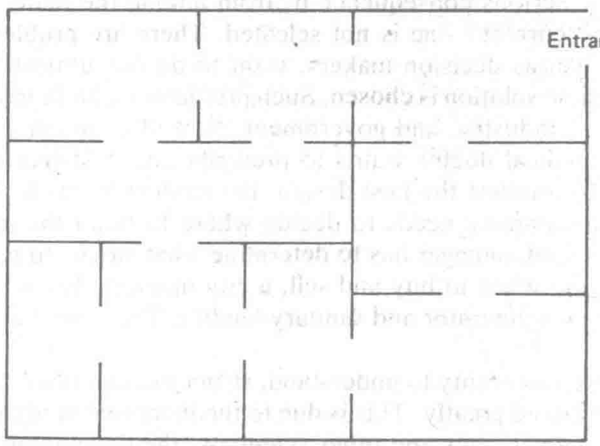
It is claimed that decision making is more art than science and that intuition and experience are the main resources of a decision maker. While we are in no position to refute this view, we do believe that many decision-making situations can be understood better by the application of the more disciplined approach to problem analysis that is imposed by the framework. With better understanding come better decisions.

Although the decision framework can be used in just about any decision situation, its abilities as an aid in solving certain types of problems are limited. This limitation is due to our inadequate knowledge of the underlying physical or behavioral phenomena.

The science of decision making is still developing. Opportunities exist for advancing and improving decision making in all fields of endeavor. However, we feel that students can benefit and learn from the present, incomplete state of decision analysis methodologies. We next discuss the general areas of problems and decisions, mathematical models and modeling, and a framework for decision making.

1.2 PROBLEMS AND DECISIONS

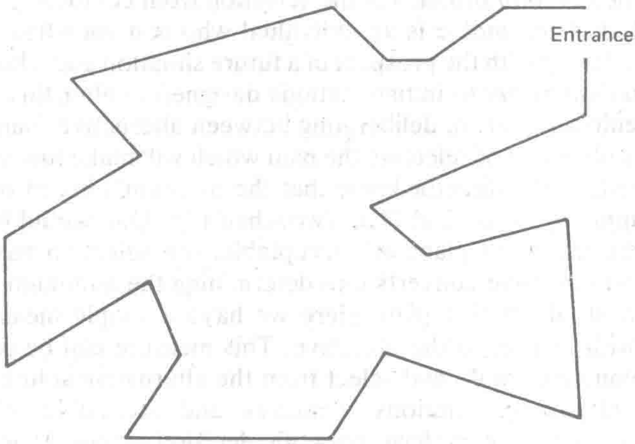
Let us consider the following situation. A museum director has been presented with two floor plans for a proposed art gallery. The first plan uses the available one-story rectangular site in the standard gallery form that has rooms interconnecting with one another (Plan One). The second plan makes



Plan One

a more daring use of the space and consists of one big room whose walls zig and zag to form an irregular polygon (Plan Two).

The competing architects have shown that each plan meets the exhibit wall space requirements and the budgets for construction and future maintenance. One other recurring cost must be considered: the payroll costs for guarding the exhibit space. The director would like to know the minimum number of guards required to safeguard the rooms of Plan One or the walls of Plan Two. All other things being equal, the plan with the fewest guards would then be chosen.



Plan Two

The museum director is faced with a basic situation which defines management: *decision making*. Decision making consists of recognizing that a problem exists, identifying possible causes, developing alternative solutions, choosing among alternative courses of action, and carrying out the action chosen.

The director has a problem to solve—a decision to make and to implement. But to do so, requires more information, that is, the cost of each plan's guards. By what means can the problem be analyzed and the information developed?

Any procedure should determine the minimum number of guards for each plan. Hopefully, the analysis can go further than just calculating these numbers. What if the director wants to add a room or a doorway in Plan One? What if a zig or zag is added or removed from Plan Two? How do such changes influence the number of guards required and the selection decision? If the framework of analysis can handle these variations, it would be of greater value to the questioning mind of the director.

The museum director's problem (which we will return to from time to time) illustrates the more general problem and decision-making concerns that are the subject of this book. *Problem solving* is that form of activity in which an individual or organization, wanting to achieve a desired objective, must make a selection from the set of alternative ways for accomplishing that objective. A decision problem is nonexistent if there is no way to accomplish the objective or if there is only one way. In either of these situations, there is no decision to be made—for, as we note from *Webster's Dictionary*, a decision is a conclusion arrived at after consideration. In the