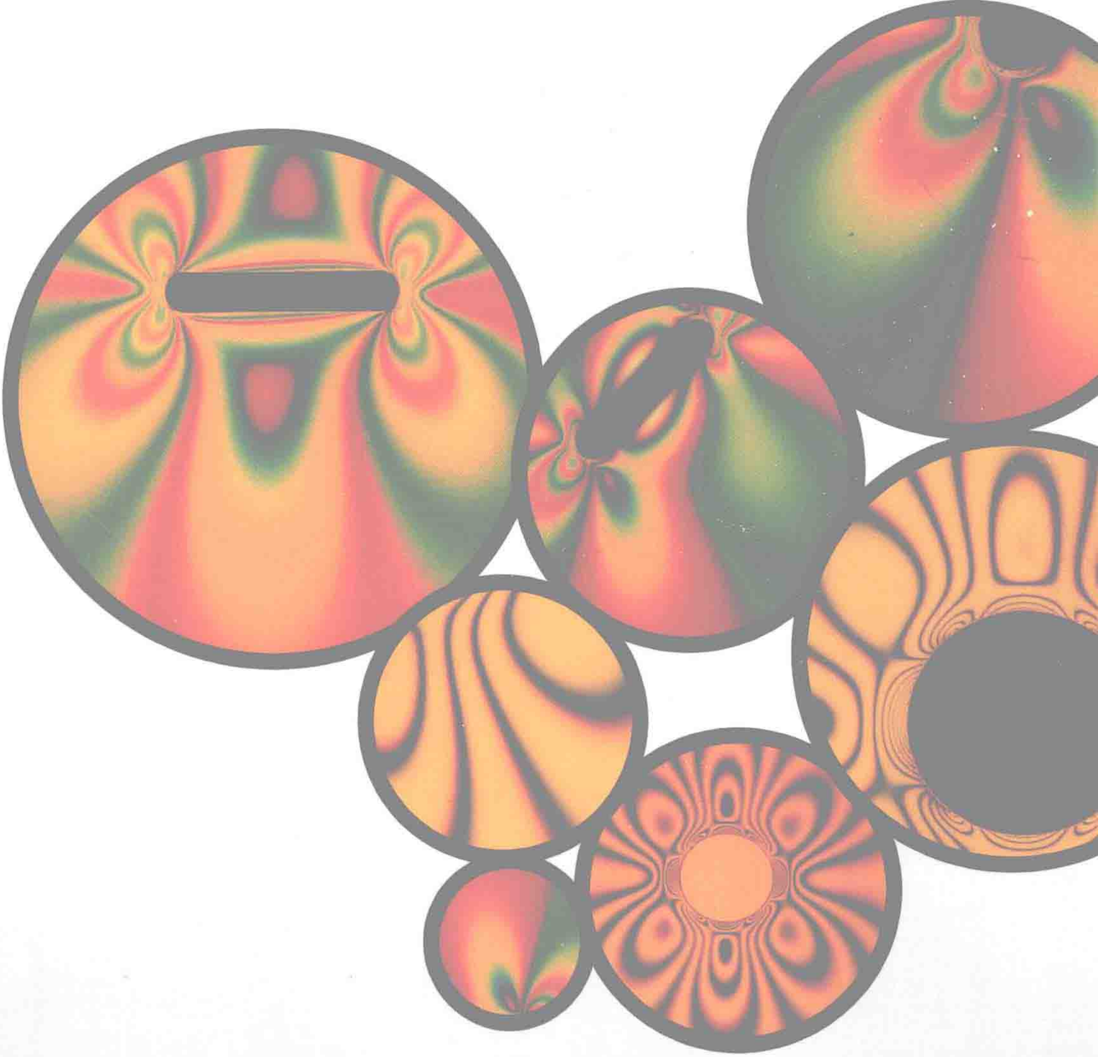

The Mechanics of Engineering Structures

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PREFACE

This book has been compiled from lecture notes and examples that I have used in my teaching of solid mechanics in various forms (including strength of materials, stress and structural analysis), over many years. It is intended for undergraduate and postgraduate engineering courses in which statics, solid mechanics and structures are taught from an intermediary to advanced level. The contents should serve most courses in mechanical, civil, aeronautical and materials engineering. The approach employed is to intersperse theory with many illustrative examples and exercises. As readers work through these it will become apparent what the engineer's practical interests in structural mechanics are. They will see that all calculations made are related to a safe load-carrying capacity and the deformation that materials used in structural design undergo. Amongst the specific design considerations are: the choice of material, its physical shape, the nature of imposed loading and its effect on the internal stress and strain. The loadings refer to: tension, compression, bending, torsion and shear. Typical structures upon which these loadings are applied in a multitude of applications include: bars, columns, struts, tubes, vessels, beams, springs and frames.

The chapters follow an orderly sequence, loosely connected to their degree of difficulty, in which the more fundamental material appears first. Thus, the properties of areas, the conditions for static equilibrium, definitions of stress and strain and linear elasticity theory underpin the structural analyses that follow. Therein lie those structures commonplace in many applications: beam bending, torsion of bars and tubes, buckling of struts and plates and tubes under pressure. The final four chapters examine more advanced analytical techniques, including the use of energy methods, plane stress and strain analyses, yield and failure criteria and finite elements. The analyses given of stress, strain, load and deflection employ various techniques with which the reader should soon become familiar. For example, amongst these are: Mohr's circle, the free-body diagram, Hooke's law, Macaulay's step-function method and Castigliano's theorems. The text illustrates where and how to employ each technique effectively within a logical presentation of the subject matter.

In general, a unique solution to the stress and strain borne by a loaded structure will satisfy three requirements: equilibrium, compatibility and the boundary conditions. Throughout this book these three conditions have been imposed upon many structures to provide closed solutions. However, it may not always be possible to achieve a closed-form solution as the loading and geometry become more complex. The final chapter shows how the known stiffness matrix for simpler types of finite elements can be embodied within a numerical solution to displacement, stress and strain. The three aforementioned conditions are satisfied but, because it is necessary to assume a displacement function, the solutions found remain approximations. Because finite elements cannot improve the accuracy of structural analyses that appear in closed-form the latter are often used to validate the numerical solutions as confidence measure. Finally, it must be mentioned that all that appears in a book of this kind will serve the basic need to design safe structures. The text revisits this basic objective throughout, particularly in its examination of safe stress levels through the use of safety factors. The point is often made that it is only through having a complete grasp of the subject can one exercise a proper control upon the degree of safety required from a structure, especially where the design imposes an economical use of material.

Worked examples and exercise sections have been devised and compiled by the author to support the topics within each chapter. Some have been derived, often with a conversion to SI units, from past examination papers set by institutions with which the author has been associated, namely: Brunel, Dublin, Kingston and Surrey Universities, and the Council of Engineering Institutions (CEI).

D. W. A. REES

CONTENTS

CHAPTER 3 BASIC STRESS AND STRAIN

CHAPTER 4 LINEAR ELASTICITY

CHAPTER 5 BENDING AND SHEAR IN BEAMS

CHAPTER 6 SLOPE AND DEFLECTION OF BEAMS

CHAPTER 7 THEORIES OF TORSION

CHAPTER 8 BUCKLING OF STRUTS

CHAPTER 9 BUCKLING OF PLATES AND TUBES

CHAPTER 10 ENERGY METHODS

CHAPTER 11 PLANE STRESS AND STRAIN

CHAPTER 12 YIELD, STRENGTH AND FAILURE CRITERIA

CHAPTER 13 FINITE ELEMENTS

HISTORICAL OVERVIEW

I Introduction

The subject of structural mechanics has had a long history within the role it plays in engineering design. Thus it has long been recognised that the engineer needs to design a safe structure, be it a bridge, a pressure vessel, a ship or an aeroplane. Not only is the choice of material important to this goal but also is the correct analysis of the manner in which the external loads are to be supported. It is via this route that a safety factor is decided upon, given other constraints such as minimising weight while retaining stiffness and resistance to corrosion. Solid mechanics is concerned with an understanding of what happens within a body when it is expected to carry loads. We identify the external loads within applied forces, moments and torques and their transmission into an internal stress and an accompanying strain. Often, the subject is employed with a re-design, say in a beam (see Chapters 5 and 6), where the area is to be increased and the length shortened to reduce the maximum stress and deflection to an acceptably safe level for a chosen material. Alternatively, we may select a stronger, stiffer material where an alteration to shape is not permitted if we are to increase the margin of safety. This book shows how mechanics plays an important role within the analysis stage of structures required to bear load often requiring many iterations before the synthesis stage can begin.

A wide variety of structures under different loading modes are presented within the twelve chapters of this book. Throughout the reader will see the names of those men whose contributions to engineering mechanics have shaped the subject into its present form. What follows here is not intended as a detailed biography of each of them, only to recognise that their associations with the following elements of this text has ensured their immortality.

II Units and Conventions

Firstly, within the SI system of units we acknowledge Sir Isaac Newton (1642–1727) [1] for our unit of force. A Newton (N) is the force required to give a mass of 1 kg an acceleration of 1 m/s^2. Our measures of energy and work recognise the pioneering work of James Prescott Joule (1818–1889). The Joule (J) refers to the work done when a force of 1 N moves its point of application through a distance of 1 m. Thus 1 J = 1 N × 1 m. The unit of energy is identical to that of work. This we should expect from the conservation law: that energy can neither be created nor destroyed but only converted from one form to another. For example, in loading a beam, work is done in deflecting the beam and this is stored internally as strain energy. James Watt (1736–1819) [2] is remembered for the unit Watt (W) now assigned to power. Power is the rate of doing work: 1 W = 1 J/s. Two further fundamental units that appear occasionally in solid mechanics (statics) are: the measurement of temperature on the Kelvin scale (K), after Lord Kelvin (1824–1907), and frequency in Hertz (Hz), where 1 Hz = 1 cycle/s, after Rudolph Heinrich Hertz (1857–1894).

Other important derived units, which the reader will meet, refer to our measures of stress and pressure. Both of these are measures of force intensity, referring to a unit of area lying either normal or parallel to that force. The chosen unit of stress and pressure in the SI system of units is the Pascal (Pa) where 1 Pa = 1 N/m^2, after Blaise Pascal (1623–1662). In fact we have become more accustomed to working in multiples in this unit to make the numbers more manageable and meaningful. Thus the pressure unit 1 bar = 10^5 Pa, is just a little less than atmospheric pressure (1.01325 bar). The preferred stress unit is the Mega Pascal (MPa) where 1 MPa = 10^6 Pa. The use of the MPa has the advantage of providing the same numerical stress value when a force in Newton is referred to a unit area of 1 mm^2. That is, 1 MPa = 1 N/mm^2. Strength, in the context of resisting failure from tension, tearing, crushing and shear, refers to the limiting stress values of a material. Strength therefore carries the same unit (MPa) as stress. Usually, a three-figure number applies to the strengths of metals. Typically, the tensile yield strength of a low-carbon steel is 300 MPa and its ultimate strength is 450 MPa.

In our definition of stress we refer the force to the original area despite the small changes in dimensions brought about by strain. There are two types of stress: direct and shear. The former arises when the force lies normal to the area and the latter when it is tangential (see Chapter 3). The sense of each is given a sign so that direct tensile stress is positive and direct compressive stress is negative. Clockwise shear stress is positive and anticlockwise shear stress is negative. Strain is the measure of the change to the original dimensions which occurs when a material is stressed. It is defined as the non-dimensional ratio between the change in length and the original length, often expressed as a percentage. Within the elastic limit metallic materials remain stiff and dimensions will not have changed (i.e. strained) by much more than 0.2% at the yield point. However, when the stresses exceed the yield point a metal loses its spring-like behaviour and becomes plastic. The metal lattice distorts permanently through shear slippage along planes most closely packed with atoms. What we can see in the plastic range of a very ductile material like aluminium are dimensional changes (strains) of the order of 50%. Similar strains are reached when forming a steel sheet at a high speed in a press. The need to refer the applied loading to the current dimensions as a material suffers larger strain was recognised by Augustin-Louis Cauchy (1789–1857). It was he who first analysed stress where a force is applied obliquely to a plane. Resolving this force normal and parallel to the plane he identified the stress state for a unit area of that plane. In general, the stress state consists of three components: one normal and two shear stresses for any plane with similar loading. In a three-dimensional analysis of stress at a point within a body loaded in multiple directions we may take three such reference planes to form a cube and then let their areas tend to zero. This reveals that six stress components, from the total of nine acting over the three faces, are independent and sufficient to define the stress at a point uniquely. A particularly convenient representation of these components has been borrowed from relativity theory (Albert Einstein, 1879–1955 [3]). Thus stress is denoted simply as σ_{ij} where i and j may take any value between 1 and 3. Stress at a point in the body is said to have a tensorial character because it can only be defined completely when the magnitude of the components σ_{ij} are connected to its reference planes set in the orthogonal directions 1, 2 and 3. Einstein's tensor subscript notation appears in Chapters 10–13 alongside an equivalent matrix notation, the latter being more popular nowadays for finite element analyses.

III Elasticity

The spring-like behaviour of metals, in which stress and strain are proportional, is expressed within the law attributed to Robert Hooke (1635–1693) [4]. Hooke's law was initially concealed within a Latin anagram *ceiiinosssttuu*. The letters were arranged later in 1678 into the phrase *ut tensio sic vis*, meaning: 'as the extension, so the force.' Written in this form Hooke's law embraces all elastic structures given in this book including: a bar in tension, a spring under load and a beam under bending. All display the proportionality implied within Hooke's secretive discovery. Despite there being no surviving portraits of Robert Hooke his law has ensured his immortality though, evidently, this would not have been of his own choosing! Once its importance was recognised, there followed applications of Hooke's law to describe the elastic behaviour of many metallic materials under axial tension. So it was that Thomas Young (1773–1829) [5] identified a modulus of elasticity (symbol E) as the constant ratio of proportionality between axial stress and strain. That each material has a different value of Young's modulus provides a measure of its spring-like stiffness. For example, the ratio shows that steel is three times stiffer than aluminium with their respective moduli being 210 GPa and 70 GPa. Further elastic constants were identified in the corresponding ratio between stress and strain for a shear force and for a uniform hydrostatic pressure. These constants are called the shear and bulk moduli, respectively (symbols G and K) . If we wish to extend Hooke's law to two- and three-dimensional stress states an account of the lateral strain induced by an axial stress is also required. Simon-Denis Poisson (1781–1840) recognised that a bar in tension will contract in its lateral dimensions as its length increases. Poisson's ratio (symbol ν) is the constant ratio between the corresponding lateral and axial elastic strains. The three moduli mentioned above, together with

Poisson's ratio, account for the elastic response of a material under any loading combination. With restricted loading fewer constants are needed. Originally, Gabriel Lamé (1795–1870) identified two independent (mathematical) elastic constants even for the general loading condition. This is because of the relationships that exist between the four engineering constants. The derivation of Lamé's constants is given in the book by August Edward Hough Love (1863–1940) [6]. Love also persues the history of the subject, as has Timoshenko [7] and others [8–10], in greater detail than is given here. Among today's engineers it is more common to adopt the four engineering constants E, G, K and v that appear in this book. Typical values for ten materials are given in Table 4.4, (see p. 113).

The theory of elasticity is concerned with these relations between stress and strain and the two accompanying conditions: (i) that variations in stress remain in equilibrium and (ii) strains are compatible with displacements. In addition (i) and (ii) must be matched to the respective forces and displacements that are known to exist around the boundary. This is the approach adopted throughout this book to give the analytical solutions to stress, strain and displacements within loaded bodies. However, it is recognised that not all problems lend themselves to a closed solution. In Chapter 13 it is shown how the stated conditions can then be implemented within a numerical procedure centred upon the structural stiffness matrix [11], more commonly known as the finite element method [12].

IV Structures

Many have examined the manner in which particular structures deform elastically under point and distributed loadings. The distinction can be made between large displacements, in the case of an unstable structure and small displacements, that conform to Hooke's law, in a stable structure.

In the former category, Leonhard Euler (1707–1783) [13] considered how a long thin strut behaved when subjected to an axial compressive load. The solution to this problem revealed that here Hooke's law does not apply, in that the lateral deflection within the length is not directly proportional to the load. Euler was able to show mathematically that beyond a critical load a strut would become unstable and buckle. Clearly, this is an important design consideration in engineering construction whenever pillars and columns are used as supports. For shorter strut designs, William John Macquorn Rankine (1820–1872) predicted the critical buckling load empirically. He and Lewis Gordon showed that Euler's mathematics of buckling does not account for a limiting strength of the strut material. In fact, Chapter 8 shows an empirical basis of strut design, is often the preferred approach.

In the latter category it is often necessary, when designing structures, to allow for the smaller elastic deflections that occur beneath the loads that are applied to it. Otto Mohr (1835–1918) proposed two theorems that enable both the slope and deflection of a beam to be found when carrying lateral loading (see Chapter 6). Mohr's method employs the bending moment diagram for the applied loading and is particularly useful for cantilever beams. Later, Rudolf Freidrich Alfred Clebsch (1833–1872) and William Herrick Macaulay (1853–1936) overcame the problem of the discontinuities that arise in this diagram at concentrated load points. Their elegant step-function approach can be applied to find deflections for a beam whose loading is supported in any manner. One of the major advances to our understanding of the deformation behaviour of any structure was to link the displacement and the applied loading to its store of energy. Albert Castigliano (1847–1884) [14] set out to discover the link within an equilibrium structure for his degree thesis in 1873. He showed that both the load and the displacement beneath it were separate partial derivatives of the structure's complementary energy (see Chapter 10). This was a remarkable achievement for a young Italian railway engineer of 28 years whose interest in mechanics was largely self-taught. We shall see how his two theorems are applied to a Hookean structure where strain energy and complementary energy take on the same meaning. At that time it was realised that a *statically determinate* structure need only its equilibrium equations when finding the displacements, stresses and strains arising from the applied forces. On the other hand, a *statically indeterminate* structure is insoluble from applying equilibrium principles alone, where an additional compatibility condition is required. Examples of both these structures appear throughout this text, where a similar division in determinancy applies to any load-bearing device including frames, pressure vessels, beams, torsion bars and tubes.

V Yielding

Long has it been known that a metallic material will continue to support load levels beyond its elastic limit, a property that has ensured their survival in the face of many man-made materials. What the early engineers were less clear about was how a combination of loads would affect the yield point (see Chapter 12). We could present the problem in general terms by asking what magnitudes of the six independent components of the stress tensor are required to produce yielding? Richard von Mises (1883–1953) [15] proposed a criterion of yielding admitting all stress states. His criterion recognises that the stress tensor has invariants which do not depend upon the co-ordinate directions. Because yielding should not depend upon co-ordinates it becomes linked to critical values of the stress invariants. This general condition for yielding envelopes many of the earlier proposals for yielding under two- and three-dimensional stress states. For example, by omitting shear stresses, James Clerk Maxwell (1831–1879) [16] believed intuitively that yielding would commence when the root mean square of the principal stresses attains a critical value. This mean value, natural to an electrical engineer, has provided us with a dependable, simpler form of the von Mises yield criterion. There are alternative yield criteria however. One, in particular, is based upon the maximum shear stress in a simple tension test attaining a critical value at yield. Engineers have always been confident in extending this approach to multi-axial yielding because its predictions are known to be conservative. Thus, the original hypothesis of

Charles Augustin Coulomb (1736–1806), re-discovered in 1865 by H. Tresca, continues to this day to serve engineers with safe designs. Less use is made now of the Barre de Saint-Venant (1797–1886) criterion, based upon a critical strain at yield but, as is the case with many of these early engineers, his name will appear elsewhere. In particular, Chapter 7 refers to the St.Venant torsion constant for providing the angular twist of thin tubes and rectangular strips under torsion.

VI Concluding Remarks

The pioneering work of these early engineers, physicists and mathematicians sets the scene for the research conducted today in many areas of solid mechanics. Amongst these are: numerical techniques of stress analysis including the finite element and boundary element methods, and the development of the mechanics appropriate to plasticity, creep, fatigue and fracture. The continued interest in the subject arises from the development of new materials to support any manner of applied loading. We might require, for example, an enhanced strength from a structure with reduced weight [17]. Alternatively, a life prediction is imposed upon the design of a component that is expected to become damaged operating at a high temperature under fluctuating loading [18].

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CHAPTER 1

PROPERTIES OF AREAS

Summary: In this chapter the basic properties of areas that underpin many of the topics that appear throughout this book are introduced. It will be seen later how the present topic is applied to the cross-sections of beams in bending, shafts in torsion and long struts under compressive loading. Examples are selected to show what is meant by the terms 'centroid', the 'first and second moments' of their plane section areas. Two theorems are derived that enable the transfer of second moments of area between parallel and perpendicular axes within a given cross-section. Both analytical and graphical techniques are available for dealing with co-ordinate rotations. Exercises are given to enable the reader to gain familiarity with these terms and techniques.

1.1 Centroid and Moments of Area

The properties of a cross-section that resist loading applied externally loading are the area A of that section and its first and second moments of area i and I respectively, about the centroidal axes x and y shown in Figure 1.1.

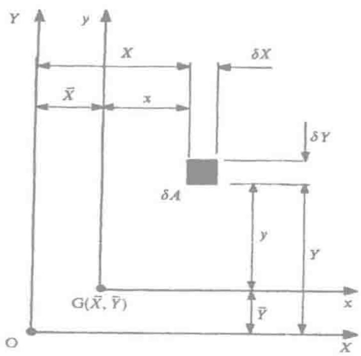

Figure 1.1 Elemental area δA set in Cartesian axes x, y