

Environmental Science Methods

Edited by

Robin Haynes

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*School of Environmental Sciences
University of East Anglia*

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Preface

This book is an introduction to a range of methods and techniques used in the scientific study of the rocks, soils, atmosphere, waters and living organisms of the Earth, and of the relationships of these environmental factors with human activities. It is intended to provide a selection of methods for students taking university courses in geography, geology, meteorology, hydrology, soil science, ecology and other allied environmental sciences. The contributors are all members of the School of Environmental Sciences at the University of East Anglia, Norwich, UK, and the book has developed from part of our course for first year students. It reflects our belief that students of vast complex environmental systems should begin their work with a panoramic view, whatever their ultimate specialization. The emphasis is therefore on breadth of treatment and on the connections between the various sciences. We have summarized and simplified in order to supply a collection of methods that can be managed by a beginning student.

We start from basic principles and do not assume that the reader already has a strong scientific background. Eleven chapters follow, each dealing with a group of closely related methods and techniques. They may be taken in any order, although there are many cross references which demonstrate that the subjects covered are not eleven isolated techniques but a web of related principles.

The first three topics illustrate the point. In the opening chapter, on measurement, we discuss the principles of defining quantities and units and the problems of making representative observations. These concepts lead into both the mathematics and the statistics chapters, which come next. The mathematics chapter begins with a brief review of algebra and an explanation of functions and graphs but then devotes its main attention to calculus and its applications to the clear-cut principles of mechanics and the motion of particles. It is concerned with situations in which physical processes and relationships can be identified relatively free from extraneous 'noise'. The statistics section, on the other hand, is about interpreting numerical information under conditions of uncertainty: summarizing, comparing and drawing

conclusions from measurements containing large doses of variability and error. These two are the longest chapters in the book. Their subject matter may be difficult but it is nonetheless uniquely valuable for giving insight in any science.

After measurement, mathematics and statistics, a chapter on computing follows naturally. This contains an account of how computers work and the various programming languages available, but it concentrates on the main ways in which a student of the environmental sciences is likely to come into contact with computing. The fifth chapter, on laboratory techniques, describes the apparatus and procedures of an environmental chemistry and sediments laboratory. The analysis of sediments appears again in the microscopy chapter, together with techniques used in biological microscopy and the identification of minerals in thin section. Optics and the properties of light (and other forms of radiation) link this material with remote sensing, the subject of chapter seven, where developments in satellite imagery are illustrated together with the interpretation of air photographs with simple instruments. Chapter eight is about maps of different types and how to use them, while chapter nine demonstrates how maps can be constructed in the field with the techniques of surveying.

One feature that should distinguish the environmental sciences approach from that of the purely physical sciences is a concern with the relationships between nature and man. From physical surveys of terrain we move to social surveys designed to map out the characteristics, behaviour and attitudes of the human population. Chapter ten gives advice on planning and interpreting social surveys. Methods that evaluate the advantages and disadvantages of proposed environmental changes are the subject of the last chapter, which introduces the essentials of cost benefit analysis and outlines other, less refined, methods of environmental impact appraisal that address wider questions. Thus, measurement is appropriately the major theme at the end, as at the beginning, of the book, and the final pages focus on the key role of values and judgements when applying the principles of measurement to controversial environmental issues.

Every chapter finishes with a list of further reading, a reminder that each topic alone is the subject of many weighty volumes. By bringing these techniques together in a concise form we aim to give you, the reader, a launching pad for further explorations. We hope that you will want to discover more, not only about the methods but also, through the methods, more about the environmental sciences.

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February, 1981

R.M.H.

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Contents

Preface viii

Acknowledgements x

1 Measurement *R. M. Haynes, J. G. Harvey and T. D. Davies* 1

1.1 Measurement scales 1

1.2 Physical quantities 3

1.3 Accuracy and errors 10

1.4 Sampling 17

1.5 Further reading 24

2 Mathematics *J. G. Harvey with C. E. Vincent
and I. N. McCave* 26

2.1 Basic algebra 26

2.2 Vectors 30

2.3 Functions and graphs 36

2.4 Rate of change, gradients and limits 48

2.5 Differentiation 51

2.6 Maxima and minima: optimization 58

2.7 Partial differentiation 62

2.8 Anti-derivatives and integration 67

2.9 Equations of motion 74

2.10 Differential equations 85

2.11 Further examples of integration in mechanics 87

2.12 Further reading 93

3 Statistics *R. M. Haynes* 95

3.1 Describing data 95

3.2 Probability and the normal distribution 103

3.3 Hypothesis testing 111

vi Contents

- 3.4 Testing differences between means 115
- 3.5 The analysis of variance 121
- 3.6 The chi-square test 127
- 3.7 Correlation 134
- 3.8 Regression 140
- 3.9 Non-linear relationships 149
- 3.10 Multiple regression and correlation 157
- 3.11 Further statistics 164
- 3.12 Further reading 166
- 4 Computing** *C. G. Bentham and C. E. Vincent* 167
 - 4.1 What are computers? 168
 - 4.2 Using the computer 171
 - 4.3 Data analysis using computers 174
 - 4.4 Non-package computing 180
 - 4.5 Glossary of computer terminology 183
 - 4.6 Further reading 185
- 5 Laboratory techniques** *P. Brimblecombe, D. L. Dent and I. N. McCave* 186
 - 5.1 Tools of the trade 186
 - 5.2 Standard chemical techniques 189
 - 5.3 Laboratory analysis of soils and sediments 199
 - 5.4 Further reading 208
- 6 Microscopy** *I. N. McCave, B. Moss and B. W. D. Yardley* 209
 - 6.1 Components and operation of microscopes 210
 - 6.2 Biological microscopy 212
 - 6.3 Minerals in polarized light: the petrological microscope 215
 - 6.4 The identification of minerals in thin section 224
 - 6.5 Identification of detrital grains 225
 - 6.6 Further reading 238
- 7 Remote sensing** *D. L. Dent, J. R. Tarrant and T. D. Davies* 240
 - 7.1 Why remote sensing? 241
 - 7.2 The electromagnetic spectrum 242
 - 7.3 Remote sensing techniques 245
 - 7.4 Viewing aerial photographs 259
 - 7.5 Interpretation of aerial photographs 266
 - 7.6 Further reading 273

- 8 Maps** *R. M. Haynes, B. W. D. Yardley and T. D. Davies* 276
- 8.1 World maps 276
 - 8.2 Local maps 284
 - 8.3 Map conventions 287
 - 8.4 Measurements from maps 295
 - 8.5 Geological maps 299
 - 8.6 Weather maps 307
 - 8.7 Further reading 316
- 9 Surveying** *N. K. Tovey* 317
- 9.1 Instruments 319
 - 9.2 Location of a point—resection 330
 - 9.3 Location of points—traverses 333
 - 9.4 Height measurement 336
 - 9.5 Construction of a map 337
 - 9.6 Organization and planning of a survey 342
 - 9.7 Further reading 346
- 10 Social surveys** *C. G. Bentham and M. J. Moseley* 348
- 10.1 Types of social survey 349
 - 10.2 Planning the survey 355
 - 10.3 Questionnaire design 355
 - 10.4 The selection of respondents 363
 - 10.5 Carrying out the survey 368
 - 10.6 After the survey 370
 - 10.7 Further reading 371
- 11 Project evaluation** *R. K. Turner and T. O'Riordan* 372
- 11.1 Cost benefit analysis 373
 - 11.2 An example of cost benefit analysis 380
 - 11.3 Environmental impact assessment 387
 - 11.4 Impact assessment techniques 388
 - 11.5 Further reading 397
- Index* 399

1 Measurement

A measurement is a value which reflects the presence or magnitude of some characteristic. Without measurements there could be no science: indeed, there could be no knowledge. The nature of our knowledge is entirely conditioned by the nature of our measurements. The definitions of physical characteristics that we make, the procedures we choose to evaluate quantity, the representativeness of our observations and the methods we adopt to minimize error of one kind or another are all part of the technique of measurement and they all influence our interpretation of nature. In the environmental sciences, as in other branches of knowledge, an appreciation of the measurement process is fundamental to real understanding of any subject matter. That is the theme of this chapter.

1.1 Measurement scales

Any characteristic which varies is known as a *variable*. Soil depth, air pressure, water hardness, population density, particle size, leaf shape, mineral colour, carrying capacity (and almost every other characteristic of the environment) are all variables. To assess the state of a variable requires observation and then measurement. Depending on the type of variable, the measurement scale that is appropriate falls into one of four categories: nominal, ordinal, interval and ratio.

Nominal scales are the crudest form of measurement. A nominal scale consists simply of non-overlapping categories into which observations may be classified. A mineral, for example, might be identified as quartz, feldspar, calcite, mica, or some other category. A front in meteorology could be a warm front, a cold front or an occluded front. Households might be classified according to whether they are car owning or non-car owning, and so on. In all these examples the different categories or classes are distinguished by the presence or absence of certain properties. Magnitude or quantity is not relevant. This is measurement at its simplest.

Ordinal scales are slightly more complicated. Observations are assigned to

2 Environmental Science Methods

categories, as in nominal scales, but this time the categories are arranged in an ordered series. One category is 'first', another is 'second', another 'third', and so on. A good example is Moh's scale of hardness, which makes it possible to measure the hardness of any mineral by comparing it with the following list which progresses from extremely soft to extremely hard: 1 talc, 2 gypsum, 3 calcite, 4 fluorite, 5 apatite, 6 orthoclase, 7 quartz, 8 topaz, 9 corundum, 10 diamond. Only the order is important in this list. Gypsum (number 2) is harder than talc (number 1) but *not* twice as hard. Calcite (number 3) is harder than gypsum, but not necessarily by the same amount that gypsum is harder than talc. In other words, the steps between categories are not necessarily of constant magnitude. Another example of this type of measurement is frequently used to classify responses in social surveys. Residents might be asked which of the following categories best describes their feelings following the closure of a street to traffic: 1 very satisfied, 2 satisfied, 3 neither satisfied nor dissatisfied, 4 dissatisfied, 5 very dissatisfied. Here the variable 'satisfaction with traffic rerouting' is measured according to a numerical scale, but it would be a big mistake to try and do any arithmetic with the numbers. Two people each with a value of 2 certainly do not represent the same amount of satisfaction as one person with a value of 4!

Interval scales have equal steps between successive intervals, so that arithmetic becomes possible. What distinguishes interval scales from the next type (ratio scales) is that an interval scale has no zero point to indicate the absence of a particular characteristic. When temperature is measured in degrees centigrade, for instance, a recording of 0°C does not indicate the absence of temperature, it indicates the temperature exactly half-way between -1°C and $+1^{\circ}\text{C}$. Because zero is arbitrarily fixed (at the freezing point of water) we cannot say that 10°C is ten times warmer than 1°C , but we *can* say that the difference between 10°C and 0°C is ten times that of the difference between 0°C and 1°C .

Ratio scales are the highest form of measurement. They not only have equal increments but also a true zero point, which enable them to be manipulated mathematically. A mass of 2 kg is exactly twice that of 1 kg. When the two are added, the result is 3 kg. A measurement of 0 kg means there is no mass at all. Mass, length, time (except when time refers to years BC or AD) and all the other scales which derive from these are ratio scales. This is by far the most common form of measurement in the environmental sciences.

Four scales of measurement have been identified but these can be grouped further. The main difference is between nominal and ordinal scales on the one hand and interval and ratio scales on the other. While nominal and ordinal scales involve identifying to which of a number of mutually exclusive categories an observation belongs, interval and ratio measurements compare observations with a fixed but arbitrary standard, known as a unit. Furthermore, interval and ratio measurements occupy *continuous* (as opposed to *discrete*) scales. That is to say, a measurement need not be 0, 1, 2, 3, . . . , but

could just as easily be 1.5, 2.3 or 3.7. According to the accuracy of measurement, an interval or ratio measure could be given as 1.5, 1.51, 1.509 or 1.5092. It is to these questions of defining the units and determining the accuracy of interval and ratio measurements that we now turn.

1.2 Physical quantities

1.2.1 Units

Measurements made on interval and ratio scales are known as physical quantities. The value of a physical quantity is equal to the product of a numerical value and a unit, that is:

$$\text{physical quantity} = \text{numerical value} \times \text{unit}$$

Some of the physical quantities which we measure will be numbers of particular organisms or objects, such as oak trees, field mice, people, cars, and so on, and then the unit will be the organism or the object concerned. In many cases, however, we are dealing with quantities which must be compared with an arbitrarily defined standard amount of that quantity which is termed the unit. Particular symbols are widely used to denote such physical quantities (for example, t is used to denote time, l to denote length, s to denote distance, m to denote mass, V to denote volume and ρ to denote density = mass/volume), but neither the physical quantity nor the symbol used to denote it should imply a particular choice of unit. Different systems of units may be used in making measurements and in specifying the values of physical quantities. Thus the mean radius of the Earth may be given either by $R = 6371$ km or $R = 3959$ miles, and the speed of flow in a particular river may be measured as 0.5 m s^{-1} or $1.1 \text{ miles hr}^{-1}$.

However, without specifying units, we can recognize the 'dimensions' of physical quantities. In mechanics, for example, there are three base quantities, each with a fundamental dimension:

- length (l)—dimension denoted by [L]
- mass (m)—dimension denoted by [M]
- time (t)—dimension denoted by [T]

Table 1.1 gives these quantities in different systems of units.

Table 1.1 Units of length, mass and time

| System of units | <i>fps</i> | <i>cgs</i> | <i>mks</i> | Other |
|-----------------|------------|-----------------|---------------|-----------|
| Length [L] | foot (ft) | centimetre (cm) | metre (m) | mile (mi) |
| Mass [M] | pound (lb) | gram (g) | kilogram (kg) | tonne (t) |
| Time [T] | second (s) | second (s) | second (s) | day (d) |

4 Environmental Science Methods

1.2.2 SI units

In order to rationalize units, the *Système International* (SI), in which the metre, kilogram and second are the units of the base quantities of mechanics, has been introduced, and this should be used wherever possible although other units will still be encountered. For other branches of science, further dimensionally independent base quantities must be introduced with their corresponding SI base units, such as:

- electric current (I) with SI unit ampere (A);
- thermodynamic temperature (T) with SI unit kelvin (K);
- amount of substance (n) with SI unit mole (mol); and
- luminous intensity (I_v) with SI unit candela (cd)

All physical quantities can be expressed in terms of the base quantities and thus of the base units. When the base units are combined together, more complex measurements are defined. Table 1.2 gives a few examples. The units for such quantities are known as derived units as they are derived from the base units. Not all derived quantities have units, however. All ratios, proportions and percentages are quantities which have been made dimensionless by dividing a measurement by another measurement in the same units, so cancelling the units.

Table 1.2 Examples of derived units

| Physical quantity | Symbol | Dimensional formula | SI units |
|-------------------|----------------------------|---------------------|--------------|
| Volume | $V = l^3$ | $[L]^3$ | m^3 |
| Density | $\rho = mV^{-1} = ml^{-3}$ | $[ML^{-3}]$ | $kg\ m^{-3}$ |
| Speed | $u = lt^{-1}$ | $[LT^{-1}]$ | ms^{-1} |
| Acceleration | $a = lt^{-2}$ | $[LT^{-2}]$ | ms^{-2} |

Note: l^3 means $l \times l \times l$, mV^{-1} means $m \div V$, etc. (see Chapter 2).

A more complete listing of physical quantities and their SI units is given in Table 1.3. It will be noted that special names and symbols have been introduced for some SI derived units (for example, newton (N) as the unit of force, pascal (Pa) as the unit of pressure, and joule (J) as the unit of work and energy), whilst others are expressed only in terms of the base units. Prefixes may be used to construct decimal multiples of SI units. Those which are most commonly encountered are set out in Table 1.4.

1.2.3 Measurements and equations

In a valid physical equation expressing relations between the magnitudes of

Table 1.3 The International System (SI) of units, and other units in common usage in the environmental sciences

| Quantity | SI Unit | Symbol | Other units |
|----------------------|----------------------------|----------------------------|---|
| <i>Base units</i> | | | |
| Length | metre | m | inch (in), foot (ft) mile (mi) nautical mile (nmi) |
| Mass | kilogram | kg | tonne (t), pound (lb) |
| Time | second | s | minute (min), hour (h), day (d), year (yr) |
| Electric current | ampere | A | |
| Temperature | degree Kelvin | K | degree Celsius ($^{\circ}\text{C}$) degree Fahrenheit ($^{\circ}\text{F}$) |
| Luminous intensity | candela | cd | |
| <i>Supplementary</i> | | | |
| Plane angle | radian | rad | degree ($^{\circ}$) |
| Solid angle | steradian | sr | |
| <i>Derived</i> | | | |
| Area | square metre | m^2 | hectare (ha) |
| Volume | cubic metre | m^3 | litre (l), gallon |
| Speed, velocity | metre per second | m s^{-1} | knot (kn) |
| Angular velocity | radian per second | rad s^{-1} | |
| Acceleration | metre per second squared | m s^{-2} | |
| Angular acceleration | radian per second squared | rad s^{-2} | |
| Frequency | hertz | Hz ($=\text{s}^{-1}$) | |
| Density | kilogram per cubic metre | kg m^{-3} | |
| Unit weight | kilonewton per cubic metre | kN m^{-3} | |
| Force | newton | N ($=\text{kg ms}^{-2}$) | |

6 Environmental Science Methods

Table 1.3 (Contd.)

| Quantity | SI Unit | Symbol | Other units |
|--------------------------------|----------------------------------|---------------------------------|---------------------------|
| Pressure | pascal (newton per square metre) | Pa (= N m^{-2}) | bar (b), atmosphere (atm) |
| Viscosity (dynamic) | newton-second per square metre. | N s m^{-2} (= Pa s) | |
| Viscosity (kinematic) | metre squared per second | $\text{m}^2 \text{s}^{-1}$ | |
| Work, energy, quantity of heat | joule | J (= N m) | calorie (cal), BTU, therm |
| Power | watt | W (= J s^{-1}) | |
| Quantity of electricity | coulomb | C (= A s) | |
| Electric potential | volt | V (= W A^{-1}) | |
| Electromotive force | volt | V (= W A^{-1}) | |
| Resistance (electric) | ohm | (= V A^{-1}) | |
| Capacitance | farad | F (= A s V^{-1}) | |
| Inductance | henry | H (= V s A^{-1}) | |
| Electric field strength | volt per metre | V m^{-1} | |
| Magnetic field strength | ampere per metre | A m^{-1} | |
| Magnetic flux | weber | Wb (= Vs) | |
| Magnetic flux density | tesla | T (= Wb m^{-2}) | |
| Magnetomotive force | ampere | A | |
| Luminous flux | lumen | lm (= cd sr) | |
| Luminance | candela per square metre | cd m^{-2} | |
| Illumination | lux | l (= lm m^{-2}) | |

Table 1.4 The more commonly used prefixes to construct decimal multiples of SI units.

| <i>Multiple</i> | <i>Prefix</i> | <i>Symbol</i> |
|-----------------|---------------|-----------------|
| 10^6 | mega | M |
| 10^3 | kilo | k |
| 10 | deca | da [†] |
| 10^{-1} | deci | d [†] |
| 10^{-2} | centi | c [†] |
| 10^{-3} | milli | m |
| 10^{-6} | micro | μ |
| 10^{-9} | nano | n |

[†] *Note:* these prefixes are not encouraged, but are still widely encountered.

physical quantities both sides of the equation must represent the same kind of physical quantity. It would, for example, be meaningless to state that

$$1 \text{ day} = 431 \text{ metres}$$

but it would be correct to state that

$$1 \text{ day} = 86\,400 \text{ s}$$

If physical quantities are of the same kind, they have the same dimensional formula and can be expressed in the same units. Thus a condition which an equation must fulfil for it to be a valid physical relationship is that it must be possible to express both sides of the equation in the same units. That is to say, the equation must be dimensionally homogeneous. To test whether this is so we consider the dimensional formulae of the various terms which appear in the equation.

Take, for example, the relationship that the square of the speed u at which an object is travelling is equal to some numerical constant c multiplied by the distance s which it has travelled from rest, multiplied by its acceleration a . That is,

$$u^2 = c \times a \times s$$

The constant c is a pure number without units and therefore without dimensions. The dimensional formula of the left-hand side is $[LT^{-1}]^2$, whilst that of the right-hand side, noting that c is dimensionless, is $[LT^{-2}][L] = [LT^{-1}]^2$. Hence we have confirmed that this equation is dimensionally homogeneous.

8 Environmental Science Methods

No other form of equation between speed, distance and acceleration would be physically possible. For example, u could not be equal to $c \times a \times s$ because then the dimensional formula of the left-hand side, $[LT^{-1}]$, would not be the same as that of the right-hand side and the equation would not be dimensionally homogeneous. This requirement provides a useful check that a physical equation has been written down correctly.

As an illustration, suppose that we need to know the rate w at which the water level will rise in a rectangular-shaped reservoir of length l , breadth b , as a result of inflow at a speed u through a pipe of cross-sectional area A , assuming that there is no outflow or other loss of water from the reservoir.

We may check that the relationship

$$w = \frac{u \times A}{l \times b}$$

is dimensionally balanced as follows:

dimensional formula of LHS ($= w$) is $[LT^{-1}]$

dimensional formula of RHS ($= \frac{u \times A}{l \times b}$) is $\frac{[LT^{-1}][L^2]}{[L][L]} = [LT^{-1}]$

When numerical values are inserted into a physical equation such as the one above, they must all be in the same system of units (preferably SI) which will determine the units of a numerical answer, for example, if we measure $u = 30 \text{ cm s}^{-1}$, $A = 1.5 \text{ m}^2$, $l = 1 \text{ km}$, $b = 200 \text{ m}$, we may convert all of these to SI units and w will be given by:

$$\begin{aligned} w &= \frac{0.3 \text{ m s}^{-1} \times 1.5 \text{ m}^2}{1000 \text{ m} \times 200 \text{ m}} \\ &= 0.00000225 \text{ m s}^{-1} \\ &\text{or } 0.1944 \text{ m day}^{-1} \\ &\text{or } 19.44 \text{ cm day}^{-1} \end{aligned}$$

1.2.4 Dimensional analysis

This need to ensure that a physical equation is dimensionally homogeneous can be taken a step further and used to derive physical equations if certain information is available, a method known as dimensional analysis. Consider, for example, an object being projected at a speed u and at a given angle of elevation above the Earth's surface. Its horizontal range R (the distance it will travel before it falls back to the surface) might be expected to be related to the speed u , the acceleration due to gravity g , and the mass of the object m . If there is no air resistance, and we know that no other factors affect R , we can write down the information which we have in the form of an expression:

$$R = k u^a g^b m^c$$