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Dedicated to Lord Ganesha

Preface

It seems to be an opportune moment to produce a book on viscoplastic fluid mechanics. There is a vast amount of material covering the theoretical aspects of the subject, as well as numerical modelling. When I began this monograph, I was very surprised to find the treasure that was lying in front of me and this is an account of the voyage of discovery.

The first chapter lays out the essential features of viscoplasticity through a detailed study of the flow of a Bingham fluid in a channel. The influence of the yield stress on the critical pressure drop to sustain the flow, the velocity field, the flow rate and the inherent nonlinearity of the constitutive model are explored in-depth. Non-dimensionalisation, and its use in defining the Bingham number and deriving the Buckingham equation is demonstrated, and the solution to the latter is found. The next section deals with the nature of free boundary problems, such as the Stefan problem. The location of the yield surface in the channel flow of the Bingham fluid is also a free boundary problem, and the corresponding velocity field can be obtained through the minimum of a suitably chosen functional or the solution of its equivalent variational inequality. The chapter closes with a brief review of the experiments which challenge and support the assumption that viscoplastic fluids exist, and a summary of the aim of the rest of the book.

The next two chapters are concerned with the basic kinematics of the flows of fluids and the balance equations of continuum mechanics so that this monograph is self-contained. Chapter 4 examines in-depth the role of pressure in incompressible media and the formulation of constitutive equations to respond to the incompressibility of a material, treating it as a constraint on a given motion. The extension to incompressible viscoplastic fluids is made and the consequence of treating the yield stress effect as a response to a second constraint is explored, leading to the concept of the viscoplasticity constraint tensor. Next, the constitutive equations for compressible viscoplastic fluids are derived. Finally, the correspondence between one-dimensional Bingham, Herschel-Bulkley and Casson models and their three-dimensional versions is exhibited.

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Chapter 5 is concerned mostly with the steady shearing flows of Bingham fluids with a brief mention of modelling the effects of heat transfer. Chapter 6 deals with the unsteady shearing flow in a channel. The lateral movement of the yield surface in the initiation of this flow is described, and the broad question regarding the kinematics and dynamics of this lateral motion is answered through an application of Hadamard's theory of propagating singular surfaces.

Chapter 7 is a sample of analytical approximation techniques to understand the flows of viscoplastic fluids. The lubrication paradox and its resolution through an examination of the flow of a Bingham fluid in a wavy channel are discussed. Next, the equations governing the axisymmetric and asymmetric Hele-Shaw flows of viscous and viscoplastic fluids are derived. Finally, a summary of the results obtained in the study of the linearised stability of the channel and helical flows of a Bingham fluid is given.

In Chap. 8, variational principles and variational inequalities associated to the flows of incompressible viscoplastic fluids are derived through the principle of virtual power. A summary of the results from convex analysis needed to understand this material is included and the equivalences, when they exist, between the minimiser of a functional, the solution of the corresponding variational inequality and that of the equations of motion are explored. Simplifications of the variational inequality occur in several flows and these are listed. Finally, a basic inequality is derived to model the flows of compressible viscoplastic fluids. In Chap. 9, the variational principle is applied to obtain the minimum pressure drop per unit length to sustain the steady flow of a Bingham fluid in a pipe of arbitrary cross-section. Next, the roles of the variational principle and the associated variational inequality are examined to understand when bubbles remain static in viscoplastic fluids, and when rigid bodies move in such materials. Proofs are also provided to show that steady shearing flows in a Bingham fluid come to rest in a finite time when the driving mechanism falls below a critical value, emphasising the role of variational inequalities. Finally, the energy principles are employed in the nonlinear stability analysis of the flow of a Bingham fluid in a channel and a pipe of circular cross-section.

The final chapter is concerned with numerical modelling through the applications of the augmented Lagrangian and the operator-splitting methods. Since the solution of the minimisation problems in finite dimensions through the augmented Lagrangian method leads naturally to its extension to the flow problems in Bingham fluids, this method is described in detail in the first two sections. Next, the operator-splitting method is introduced and employed to study the thermally driven cavity flow of a Bingham fluid. The chapter closes with a section on numerical modelling of flows of compressible viscoplastic fluids with a study of the lid-driven cavity flow of a weakly compressible viscoplastic fluid. Some comments on the use of regularised models in numerical modelling are also offered.

And, the last word. Viscoplastic fluid mechanics means yield stress and the location(s) of yield surface(s). That is, free boundary problems, variational principles, variational inequalities and convex analysis with augmented Lagrangian and operator-splitting methods following from them. In writing this book, apart from including solutions to problems obtained through traditional approaches to fluid mechanics, my aim has been to emphasise the pre-eminence of the modern approach to this subject.

Adelaide, December 2014 Raja R. Huilgol

Acknowledgments

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Turning to the figures in the book, I am deeply indebted to Professor G. Georgiou, University of Cyprus, for producing Figs. 1.1–1.4, 4.1, 4.2, 5.1, 5.2 and 9.1. And, to Mr. G.H.R. Kefayati for Figs. 7.3 and 8.1.

I would also like to thank Professor I.A. Frigaard, University of British Columbia, for sending me Figs. 7.1 and 7.2 used in Sect. 7.2, and to Elsevier for the permission to use these figures. In addition, I wish to thank Elsevier for the permission to reproduce the table on p. 190 and Figs. 9.2–9.6, as well as Figs. 10.1–10.18 from my articles in the Journal of Non-Newtonian Fluid Mechanics.

Finally, I would like to thank Dr. Christoph Baumann, Senior Engineering Editor, Springer, for his enormous help in producing this monograph.

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Chapter 1
The Basic Features of Viscoplasticity

1.1 Bingham Fluid at Rest in a Channel

Consider an incompressible Bingham fluid at rest between two parallel walls. Assume that the domain Ω of the fluid can be described through a region symmetrical about the x-axis as follows:

$$\Omega = \{(x, y) : -\infty < x < \infty, \ -H \leq y \leq H\}. \tag{1.1.1}$$

See Fig. 1.1. Let a constant pressure gradient be applied to the fluid in the x-direction so that we can describe the pressure field in the fluid through $p(x, y) = -Gx + f(y)$, where $G > 0$ is the constant pressure drop per unit length and $f(y)$ is a function of y, which is irrelevant here. Ignoring any body force, the equations of equilibrium lead to

$$-\frac{\partial p}{\partial x} + \frac{\partial \sigma}{\partial y} = 0, \tag{1.1.2}$$

where σ is the shear stress in the fluid. This equation can be integrated for the shear stress and one obtains

$$\sigma = -Gy + b, \tag{1.1.3}$$

where b is the constant of integration. Since the domain is symmetrical about the x-axis, one can assume that $b = 0$. Thus, $\sigma = -Gy$.

The ability of a Bingham fluid to remain at rest under a constant pressure drop per unit length G, albeit for a limited range, requires further investigation, especially since the shear stress distribution in the channel is given by $\sigma = -Gy$ for both purely viscous and viscoplastic fluids. The former class of fluids will flow regardless of how small G is, whereas the latter will not move unless the pressure drop per unit length G exceeds a critical value G_c, which depends on the yield stress of the fluid. To determine G_c, an explanation regarding the change in sign of the shear stress across the channel is given, paving the way for a formula relating G_c to the yield stress τ_y of the fluid.

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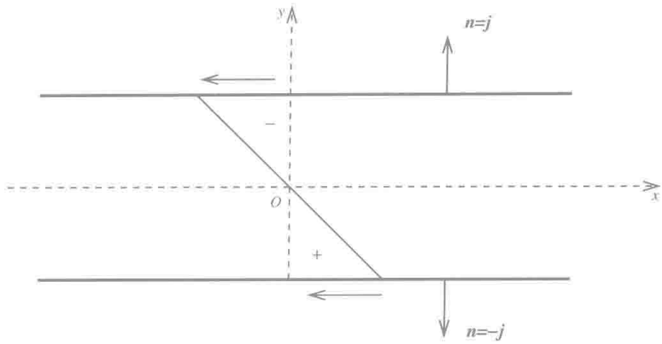

Fig. 1.1 Shear stress distribution across a channel due to a constant applied pressure gradient, with external shear stress vectors and external unit normals

1.2 Sign of the Shear Stress

One can see that the shear stress is *negative* above the x-axis and is *positive* below it. This needs some explanation. First of all, as the pressure drop tries to move the fluid in the positive x-direction, the shear stresses on the two walls oppose it. See Fig. 1.1. While a more detailed description of Cauchy's stress principle is provided in Chap. 3, at present it is sufficient to assume that the stress tensor $\mathbf{T}$ in the fluid is symmetric and two-dimensional, given in matrix form through:

$$\mathbf{T} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}, \quad T_{12} = T_{21}. \tag{1.2.1}$$

On the plane $y = H$, the external unit normal $\mathbf{n} = \mathbf{j}$ is oriented towards the positive y-direction. Cauchy's stress principle says that the external stress vector $\mathbf{t}$ on this plane is given by $\mathbf{t} = \mathbf{Tn}$. So,

$$\mathbf{t} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} T_{12} \\ T_{22} \end{bmatrix} = \begin{bmatrix} -\sigma_w \\ T_{22} \end{bmatrix}, \tag{1.2.2}$$

where σ_w is the magnitude of the shear stress at the wall. Since this external stress points in the negative x-direction, the shear stress $T_{12} < 0$ in the fluid. This negative value persists till it changes from a negative to a positive value, as one moves from the plane $y = H$ to the plane $y = -H$. Now, why is the shear stress on the plane $y = -H$ positive? This is because on this plane, the external unit normal is given by $\mathbf{n} = -\mathbf{j}$. So, the external stress vector is given by

$$\mathbf{t} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -T_{12} \\ T_{22} \end{bmatrix} = \begin{bmatrix} -\sigma_w \\ T_{22} \end{bmatrix}. \tag{1.2.3}$$

Obviously, the shear stress $T_{12} > 0$ here.

Once again, note that the sign of the shear stress is independent of the constitutive equation and applies to all continuous media.

1.3 Critical Pressure Drop and the Constitutive Relation

Now, let the pressure drop G be increased slowly. The shear stress will grow in magnitude till the magnitude of the wall shear stress, σ_w, equals the yield stress, τ_y, of the fluid. That is $\sigma_w = \tau_y$. Consider the axial force acting on the fluid over a cube of height $2H$ in the y-direction, unit width in the z-direction and unit length in the x-direction. This force is given by $2GH$. Opposing it are the forces on the boundaries of the channel at the top and bottom. Per unit length in the x-direction and unit width in the z-direction, these forces are given by $2\tau_y$. Thus, the flow is incipient when the critical pressure drop per unit length is given by

$$G_c = \frac{\tau_y}{H}. \tag{1.3.1}$$

Note that the fluid does not flow till this critical value has been exceeded. If the pressure drop per unit length G is increased beyond G_c, the fluid will flow with the yielding occurring at the wall at first. Assuming that the transient effects have died away and that the flow is steady, there will be a boundary layer of the Bingham fluid moving as a liquid, while away from the wall, the Bingham material will flow as a solid plug; these phenomena require some explanation.

The yield stress and the adherence condition at the wall together prevent the Bingham fluid from undergoing a deformation, i.e., shearing, till the magnitude of the shear stress at the wall, due to the applied pressure gradient, exceeds the yield stress. Elsewhere in the flow domain, the yield stress prevents the fluid from undergoing a deformation, i.e., shearing, where the magnitude of the shear stress is less than or equal to the yield stress. From Figs. 1.1 and 1.2, one sees that this situation arises in a symmetrical region around the centre of the channel. Since there is no fixed boundary at the centre, the only way the fluid can undergo zero deformation is to move as a solid plug.

To understand these matters in detail, let the flow occur in the x-direction with a velocity field given by $u = u(y)$. Since a plug flow exists around the x-axis, we see that

$$u(y) = u(0), \quad 0 \le y \le h, \tag{1.3.2}$$

where h is the semi-width of the plug. Note that in the rigid core, $du/dy = 0$. In $h \le y \le H$, the fluid moves like a viscous liquid. Obviously, one does not know the exact nature of the velocity distribution in this boundary layer. Clearly, one needs a constitutive equation to proceed.

The commonly used constitutive assumption is that the magnitude of the shear stress in the plug is less than or equal to the yield stress τ_y, while in the yielded

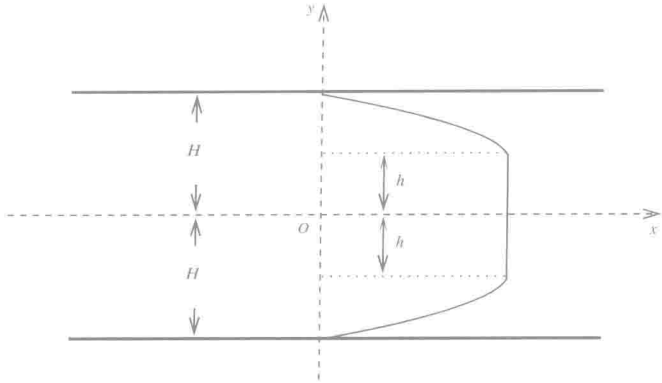

Fig. 1.2 Steady flow in a channel due to a constant applied pressure drop per unit length, with a moving rigid core and yielded zones next to the walls

domain, the magnitude of the shear stress exceeds the yield stress, augmented by a shear rate dependent stress. So, we have

$$du/dy = 0, \quad |\sigma| \leq \tau_y, \tag{1.3.3}$$

And,

$$\sigma = \eta \frac{du}{dy} + \frac{\tau_y}{|du/dy|} \frac{du}{dy}, \tag{1.3.4}$$

where η is the viscosity of the fluid. Since the velocity in the fluid increases from zero at the boundary to the plug velocity at $y = h$, it is clear that $du/dy \leq 0$ in the yielded region, $h \leq y \leq H$. So, we can write the constitutive equation as

$$-\tau_y \leq \sigma \leq 0, \quad 0 \leq y \leq h, \tag{1.3.5}$$

$$-\tau_y + \eta \frac{du}{dy} = \sigma, \quad h \leq y \leq H. \tag{1.3.6}$$

Keeping in mind that the pressure drop per unit length $G > G_c > 0$ is a constant, one is faced with the following questions:

1. How wide is the plug, or how can one find h?
2. What is the constant speed $u(0)$ of the plug?
3. What is the velocity distribution $u = u(y)$ in $h \leq y \leq H$?
4. What are the boundary conditions on $u = u(y)$ at the interface between the plug flow and the boundary layer?

The answers to these questions can be found easily in the problem at hand as can be seen next.

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