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Sliding Mode Control of Uncertain Parameter-Switching Hybrid Systems

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SLIDING MODE CONTROL OF UNCERTAIN PARAMETER-SWITCHING HYBRID SYSTEMS

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To Jingyan and Zhixin
L. Wu

To my family
P. Shi

To my family
X. Su

Series Preface

Electromechanical systems permeate the engineering and technology fields in aerospace, automotive, mechanical, biomedical, civil/structural, electrical, environmental, and industrial systems. The Wiley Book Series on dynamics and control of electromechanical systems will cover a broad range of engineering and technology within these fields. As demand increases for innovation in these areas, feedback control of these systems is becoming essential for increased productivity, precision operation, load mitigation, and safe operation. Furthermore, new applications in these areas require a reevaluation of existing control methodologies to meet evolving technological requirements, for example the distributed control of energy systems. The basics of distributed control systems are well documented in several textbooks, but the nuances of its use for future applications in the evolving area of energy system applications, such as wind turbines and wind farm operations, solar energy systems, smart grids, and the generation, storage and distribution of energy, require an amelioration of existing distributed control theory to specific energy system needs. The book series serves two main purposes: 1) a delineation and explication of theoretical advancements in electromechanical system dynamics and control, and 2) a presentation of application-driven technologies in evolving electromechanical systems.

This book series will embrace the full spectrum of dynamics and control of electromechanical systems from theoretical foundations to real-world applications. The level of the presentation should be accessible to senior undergraduate and first-year graduate students, and should prove especially well-suited as a self-study guide for practicing professionals in the fields of mechanical, aerospace, automotive, biomedical, and civil/structural engineering. The aim is to provide an interdisciplinary series, ranging from high-level undergraduate/graduate texts, explanation and dissemination of science and technology and good practice, through to important research that is immediately relevant to industrial development and practical applications. It is hoped that this new and unique perspective will be of perennial interest to students, scholars, and employees inthe engineering disciplines mentioned. Suggestions for new topics and authors for the series are always welcome.

This book, *Sliding Mode Control of Uncertain Parameter-Switching Hybrid Systems*, has the objective of providing a theoretical foundation as well as practical insights on the topic at hand. It is broken down into three parts: 1) sliding mode control (SMC) of Markovian jump singular systems, 2) SMC of switched state-delayed hybrid systems, and 3) SMC of switched stochastic hybrid systems. The book provides detailed derivations from first principles to allow the reader to thoroughly understand the particular topic. This is especially useful for Markovian jump singular systems with stochastic perturbations because a comprehensive knowledge of stochastic analysis is not required before understanding the material. Readers can simply dive into the material. It also provides several illustrative examples to bridge the gap between theory and practice. It is a welcome addition to the Wiley Electromechanical Systems Series because no other book is focused on the topic of SMC with a specific emphasis on uncertain parameter-switching hybrid systems.

Mark J. Balas
John L. Crassidis
Florian Holzapfel
Series Editors

Preface

Since the 1950s, sliding mode control (SMC) has been recognized as an effective robust control strategy for nonlinear systems and incompletely modeled systems. In the past two decades, SMC has been successfully applied to a wide variety of real world applications such as robot manipulators, aircraft, underwater vehicles, spacecraft, flexible space structures, electrical motors, power systems, and automotive engines. Basically, the idea of SMC is to utilize a discontinuous control to force the system state trajectories to some predefined sliding surfaces on which the system has desired properties such as stability, disturbance rejection capability, and tracking ability. Many important results have been reported for this kind of control strategy. However, when the controlled plants are uncertain parameter-switching hybrid systems including parameter-switching (Markovian jump or arbitrary switching), state-delay, stochastic perturbation, and singularly perturbed terms, the common SMC methodologies cannot meet the requirements.

It is known that the SMC of uncertain parameter-switching hybrid systems is much more complicated because sliding mode controllers must be designed so that not only is the sliding surface robustly reachable, but also the sliding mode dynamics can converge the system's equilibrium automatically by choosing a suitable switching function. This book aims to present up-to-date research developments and novel methodologies on SMC of uncertain parameter-switching hybrid systems in a unified matrix inequality setting. The considered uncertain parameter-switching hybrid systems include Markovian switching hybrid systems, switched state-delayed hybrid systems, and switched stochastic hybrid systems. These new methodologies provide a framework for stability and performance analysis, SMC design, and state estimation for these classes of systems. Solutions to the design problems are presented in terms of linear matrix inequalities (LMIs). In this book, a large number of references are provided for researchers who wish to explore the area of SMC of uncertain parameter-switching hybrid systems, and the main contents of the book are also suitable for a one-semester graduate course.

In this book, we present new SMC methodologies for uncertain parameter-switching hybrid systems. The systems under consideration include Markovian jump systems, singular systems, switched hybrid systems, stochastic systems, and time-delay systems.

The content of this book are divided into three parts. The first part is focused on SMC of Markovian jump singular systems. Some necessary and sufficient conditions are derived for the stochastic stability, stochastic admissibility, and optimal performances by developing new techniques for the considered Markovian jump singular systems. Then a set of new SMC methodologies are proposed, based on the analysis results. The main contents are as follows:

Chapter 2 is concerned with the state estimation and SMC of singular Markovian switching systems; Chapter 3 studies the optimal SMC problem for singular Markovian switching systems with time delay; and Chapter 4 establishes the integral SMC method for singular Markovian switching stochastic systems.

In the second part, the problem of SMC of switched state-delayed hybrid systems is investigated. A unified approach of the piecewise Lyapunov function combining with the average dwell time technique is developed for analysis and synthesis of the considered systems. By this approach, some sufficient conditions are established for the stability and synthesis of the switched state-delayed hybrid system. More importantly, a set of SMC methodologies under a unique framework are proposed for the considered hybrid systems. The main contents of this part are as follows: Chapter 5 is devoted to the stability analysis and the stabilization problems for switched state-delayed hybrid systems; Chapter 6 investigates the optimal dynamic output feedback (DOF) control of switched state-delayed hybrid systems; and Chapters 7 and 8 study the SMC of continuous- and discrete-time switched state-delayed hybrid systems, respectively.

In the third part, the parallel theories and techniques developed in the second part are extended to deal with switched stochastic hybrid systems. The main contents include the following: Chapters 9 and 10 are concerned with the control of switched stochastic hybrid systems for continuous- and discrete-time cases, respectively; Chapter 11 studies the observer-based SMC of switched stochastic hybrid systems; and Chapter 12 focuses on the dissipativity-based SMC of switched stochastic hybrid systems.

This book is a research monograph whose intended audience is graduate and postgraduate students, academics, scientists and engineers who are working in the field.

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Abbreviations and Notations

Abbreviations

CCL	cone complementary linearization
CQLF	common quadratic Lyapunov function
DOF	dynamic output feedback
LMI	linear matrix inequality
LQR	linear-quadratic regulator
LTI	linear time-invariant
MIMO	multiple-input multiple-output
MJLS	Markovian jump linear system
MLF	multiple Lyapunov function
SISO	single-input single-output
SMC	sliding mode control
SOF	static output feedback
SQLF	switched quadratic Lyapunov functions

Notations

$\blacksquare$	end of proof
$\blacklozenge$	end of remark
$\triangleq$	is defined as
$\in$	belongs to
$\forall$	for all
$\sum$	sum
$\mathbf{C}$	field of complex numbers
$\mathbf{R}$	field of real numbers
$\mathbf{Z}$	field of integral numbers
$\mathbf{R}^n$	space of n-dimensional real vectors
$\mathbf{R}^{n\times m}$	space of $n \times m$ real matrices
$\mathbf{C}_{n,d}$	set of $\mathbf{R}^n$-valued continuous functions on $[-d, 0]$
$\mathbf{E}\{\cdot\}$	mathematical expectation operator
lim	limit
max	maximum
min	minimum

sup	supremum
inf	infimum
rank$(\cdot)$	rank of a matrix
trace$(\cdot)$	trace of a matrix
$\lambda_{\min}(\cdot)$	minimum eigenvalue of a real symmetric matrix
$\lambda_{\max}(\cdot)$	maximum eigenvalue of a real symmetric matrix
diag	block diagonal matrix with blocks $\{X_1, \ldots, X_m\}$
$\sigma_{\min}(\cdot)$	minimum singular value of a real symmetric matrix
$\sigma_{\max}(\cdot)$	maximum singular value of a real symmetric matrix
I	identity matrix with appropriate dimension
I_n	$n \times n$ identity matrix
0	zero matrix with appropriate dimension
$0_{n\times m}$	zero matrix of dimension $n \times m$
X^T	transpose of matrix X
X^{-1}	inverse of matrix X
$X^{\perp}$	full row rank matrix satisfying $X^{\perp}X = 0$ and $X^{\perp}X^{\perp T} > 0$
$X > (<) 0$	X is real symmetric positive (negative) definite
$X \geq (\leq) 0$	X is real symmetric positive (negative) semi-definite
$\mathcal{L}_2[0, \infty)$	space of square integrable functions on $[0, \infty)$ (continuous case)
$\ell_2[0, \infty)$	space of square summable infinite vector sequences over $[0, \infty)$ (discrete case)
$\lvert\cdot\rvert$	Euclidean vector norm
$\lVert\cdot\rVert$	Euclidean matrix norm (spectral norm)
$\lVert\cdot\rVert_2$	$\mathcal{L}_2$-norm: $\sqrt{\int_0^\infty \lvert\cdot\rvert^2\, dt}$ (continuous case) ℓ_2-norm: $\sqrt{\sum_0^\infty \lvert\cdot\rvert^2}$ (discrete case)
$\star$	symmetric terms in a symmetric matrix

Contents

1

Introduction

1.1 Sliding Mode Control

Sliding mode control (SMC) has proven to be an effective robust control strategy for incompletely modeled or nonlinear systems since its first appearance in the 1950s [70, 103, 197]. One of the most distinguished properties of SMC is that it utilizes a discontinuous control action which switches between two distinctively different system structures such that a new type of system motion, called sliding mode, exists in a specified manifold. The peculiar characteristic of the motion in the manifold is its insensitivity to parameter variations, and its complete rejection of external disturbances [260]. SMC has been developed as a new control design method for a wide spectrum of systems including nonlinear, time-varying, discrete, large-scale, infinite-dimensional, stochastic, and distributed systems [101]. Also, in the past two decades, SMC has successfully been applied to a wide variety of practical systems such as robot manipulators, aircraft, underwater vehicles, spacecraft, flexible space structures, electrical motors, power systems, and automotive engines [60, 77, 199, 259].

In this section, we will first present some preliminary background and fundamental theory of SMC, which will be helpful to some readers who have little or no knowledge on SMC, and then we will give an overview of recent development of SMC methodologies.

1.1.1 Fundamental Theory of SMC

We first formulate the SMC problem as follows. For a general nonlinear system of the form

$$\dot{x}(t) = f(x, u, t), \tag{1.1}$$

where $x(t) \in \mathbf{R}^n$ is the system state vector, $u(t) \in \mathbf{R}^m$ is the control input. We need to design a sliding surface

$$s(x) = 0,$$

where $s(x)$ is called the switching function, and the order of $s(x)$ is usually the same as that of the control input, i.e. $s(x) \in \mathbf{R}^m$, and

$$s(x) = \begin{bmatrix} s_1(x) & s_2(x) & \cdots & s_m(x) \end{bmatrix}^T.$$

Sliding Mode Control of Uncertain Parameter-Switching Hybrid Systems, First Edition. Ligang Wu, Peng Shi and Xiaojie Su.

Then a sliding mode controller $u(t) = [\,u_1(t) \quad u_2(t) \quad \cdots \quad u_m(t)\,]^T$ is designed in the form of

$$u_i(t) = \begin{cases} u_i^+(t), & \text{when } s_i(x) > 0, \\ u_i^-(t), & \text{when } s_i(x) < 0, \end{cases} \qquad i = 1, 2, \ldots, m,$$

where $u_i^+(t) \neq u_i^+(t)$, such that the following two conditions hold:

Condition 1. The sliding mode is reached in a finite time and subsequently maintained, that is, the system state trajectories can be driven onto the specified sliding surface $s(x) = 0$ by the sliding mode controller in a finite time and maintained there for all subsequent time;

Condition 2. The dynamics in sliding surface $s(x) = 0$, that is, the sliding mode dynamics, is stable with some specified performances.

Further consider (1.1) with single input, that is, $u(t) \in \mathbf{R}$ and $s(x) \in \mathbf{R}$, and suppose that the sliding mode can be reached in a finite time, then the solutions of the equation

$$\dot{x}(t) = f(x, u^+(t), t), \quad s(x) > 0,$$

will approach $s(x) = 0$ and reach there in a finite time. During the approaching phase, $\dot{s}(x) < 0$. Similarly, the solutions of the equation

$$\dot{x}(t) = f(x, u^-(t), t), \quad s(x) < 0,$$

will also approach $s(x) = 0$ and reach there in a finite time, thus we have $\dot{s}(x) > 0$. To summarize the above analysis, we have

$$\begin{cases} \dot{s}(x) < 0, & \text{when } s(x) > 0, \\ \dot{s}(x) > 0, & \text{when } s(x) < 0, \end{cases}$$

or, equivalently,

$$s(x)\dot{s}(x) < 0.$$

which is the so-called 'reaching condition'. This is the condition under which the state will move toward and reach a sliding surface. The system state trajectories under the reaching condition is called the reaching phase [77, 101].

In summary, Condition 1 requires the reachability of a sliding mode, which is guaranteed through designing a sliding mode controller, while Condition 2 requires the sliding mode dynamics to be stable with some specified performances, which is assured by designing an appropriate sliding mode surface. Therefore, a conventional SMC design consists of two steps:

Step 1. Design a sliding surface $s(x) = 0$ such that the dynamics restricted to the sliding surface has the desired properties such as stability, disturbance rejection capability, and tracking;

Step 2. Design a discontinuous feedback control $u(t)$ such that the system state trajectories can be attracted to the designed sliding surface in a finite time and maintained on the surface for all subsequent time.