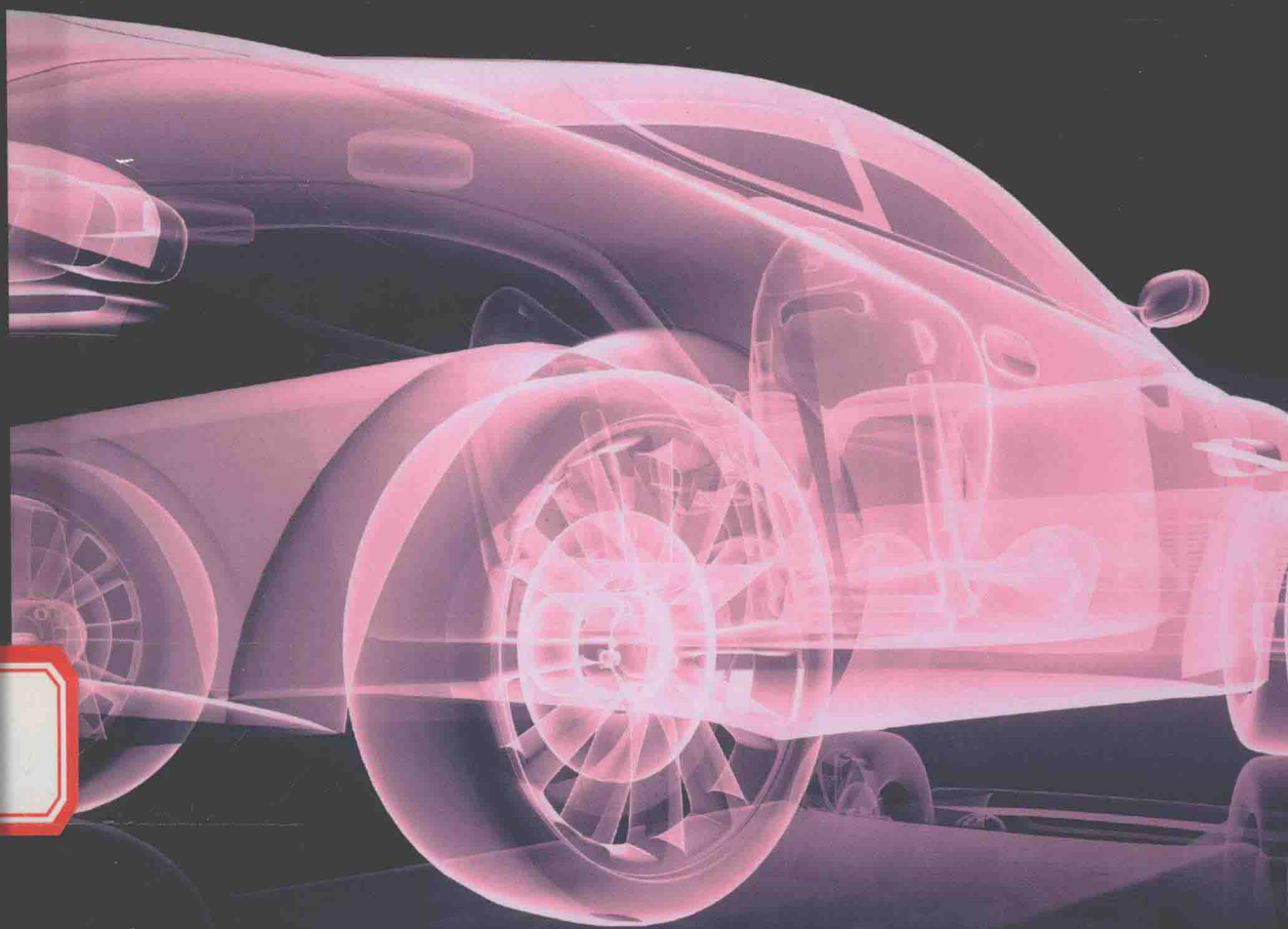


# INTEGRATED VEHICLE DYNAMICS AND CONTROL

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Wuwei Chen | Hansong Xiao | Qidong Wang | Linfeng Zhao | Maofei Zhu



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# INTEGRATED VEHICLE DYNAMICS AND CONTROL

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# Preface

As “the machine that changed the world”, the vehicle has been developed for more than one hundred years. When examining the development history of vehicle technologies, people will find that the development of vehicle technologies had mainly relied on the improvement of mechanical structures and mechanisms during the incipient stage of vehicle. However, by taking advantage of the rapid development of energy, microelectronic, computer, and communication technologies in recent years, it is believed that the vehicle is now experiencing a significant transformation to have attributes of being electric, intelligent, and networked.

Vehicle dynamics studies the basic theory of the motions of various vehicle systems and the performance of the system integration of the vehicle. The design of the dynamic performance of the modern vehicle must meet the multiple requirements of ride comfort, handling stability, safety, and environment-friendliness. These requirements present great challenges to automotive researchers and engineers since the vehicle itself is a complex system consisting of up to several thousands of parts, and often driven under various unpredictable working conditions. Moreover, the traditional vehicle design has focused on improving primarily the mechanical parts and systems through finding a compromised solution amongst the conflicting performance requirements. Thus it is difficult to optimize simultaneously all the performance requirements of the vehicle. To overcome these difficulties, the electronics technology and control theory have been applied to improving the dynamic performance of vehicle systems, especially the vehicle chassis subsystems. As a result, the research topic on vehicle dynamics and control has attracted great attention in recent years. A number of chassis control subsystems have been developed, for example the active suspension, ABS (antilock brake system), and EPS (electrical power steering system), etc.

The dynamic chassis subsystems including tyres, brakes, steering, and suspension, etc., fulfill complex and specific motions, and transmit and exchange energy and information by means of dynamic coupling, and hence realize the basic functions of the vehicle chassis. The fundamental study of the chassis system dynamics focuses on the development of the nonlinear tyre model and nonlinear coupling dynamic model of the full vehicle that describe the combined effects of the longitudinal, lateral, and vertical motions of the tyre and vehicle through analyzing the tyre–road interactions and the coupling mechanisms amongst the brake, steering, and suspension subsystems under different driving conditions.

To date, there are quite a few textbooks and monographs addressing vehicle system dynamics and control. However, most of them explore mainly the stand-alone control of the individual chassis subsystem to improve solely the performance of the subsystem, i.e. brake

control, steering control, suspension control, etc. In addition, there have been numerous achievements in the theoretical research and engineering applications on the multi-objective, multivariate integrated control of multiple vehicle subsystems over the past two decades. It is also demonstrated by a large volume of research articles in this field, covering the modeling of full vehicle dynamics, the architecture of integrated control system, the integrated control strategies, and the decoupling control methods for the integrated system. However, there are few monographs investigating the dynamic model and simulation, design, and experimental verification of the vehicle integrated control systems.

This book provides an extensive discussion on the integrated vehicle dynamics control, exploring the fundamentals and emerging developments in the field of automotive engineering. It was supported by the following research projects: the general programs of the National Natural Science Foundation of China (NSFC), including “Research on integrated control of vehicle electrical power steering (EPS) and active suspension system” (No. 50275045), “Research on methods and key technologies of integrated control of vehicle chassis based on generalized integration” (No. 50575064), “Research on methods and key technologies of integrated control of vehicle chassis integration” (No. 51075112), “A study on control methods of human-machine sharing and key technology for vehicle lateral movement” (No. 51375131), and “Research on integrated control and key technologies of ESP and EPS based on security border control” (No. 51175135). It was also supported by the project of International Cooperation and Exchanges NSFC “Research on methods and key technologies of active integrated control of vehicle chassis system” (No. 50411130486, 50511130365, 50611130371).

The topics presented in this book have been organized in 9 chapters, covering the background of vehicle system dynamics modeling, tyre dynamics, longitudinal vehicle dynamics and control, vertical vehicle dynamics and control, lateral vehicle dynamics and control, analysis of system coupling mechanism, model of full vehicle dynamics, centralized integration control, and multi-layer coordinating control. The major contents of the book are based on the research practice of the authors over the last ten years. Chapters 1, 2 and 3 are written by Professor Qidong Wang, Anhui University of Science and Technology; Chapter 4, Sections 7.1, 7.3, 7.4, 7.5, 7.6, 7.9, Sections 8.1, 8.2, 8.3, 8.7, 8.8, 8.9, and Chapter 9 are written by Professor Wuwei Chen, Hefei University of Technology; Chapter 5 is written by Dr. Linfeng Zhao, associate professor with Hefei University of Technology; Chapter 6 is written by Dr. Maofei Zhu, associate professor with Hefei Institutes of Physical Science, Chinese Academy of Sciences; Sections 7.2, 7.7, 7.8 and Sections 8.4, 8.5, 8.6 are written by Dr. Hansong Xiao, Hanergy Product Development Group. This book is finally compiled and edited by Professor Wuwei Chen.

This book cites a number of research articles published in domestic and international journals, and the scientific publications, all of which enrich the contents of this book. We would like to express gratitude to all the authors of the related references.

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January 2016, Hefei*

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# 1

## Basic Knowledge of Vehicle System Dynamics

### 1.1 Traditional Methods of Formulating Vehicle Dynamics Equations

Traditional methods of formulating vehicle dynamics equations are based on the theories of Newtonian mechanics and analytical mechanics. Some of the definitions used in dynamics are presented first.

#### 1. *Generalized coordinates*

Any set of parameters that uniquely define the configuration (position and orientation) of the system relative to the reference configuration is called a set of generalized coordinates. Generalized coordinates may be dependent or independent. To a system in motion, the generalized coordinates that specify the system may vary with time. In this text, column vector  $\mathbf{q} \equiv [q_1, q_2, \dots, q_n]^T$  is used to designate generalized coordinates, where  $n$  is the total number of generalized coordinates.

In Cartesian coordinates, to describe a planar system which consists of  $b$  bodies,  $n = 3 \times b$  coordinates are needed. For a spatial system with  $b$  bodies,  $n = 6 \times b$  (or  $n = 7 \times b$ ) coordinates are needed.

The overall vector of coordinates of the system is denoted by  $\mathbf{q} \equiv [\mathbf{q}_1^T, \mathbf{q}_2^T, \dots, \mathbf{q}_b^T]^T$ , where vector  $\mathbf{q}_i$  is the vector of coordinates for the  $i$ th body in the system.

#### 2. *Constraints and constraint equations*

Normally, a mechanical system that is in motion can be subjected to some geometry or movement restrictions. These restrictions are called constraints. When these restrictions

are expressed as mathematical equations, they are referred to as constraint equations. Usually these constraint equations are denoted as follows:

$$\Phi \equiv \Phi(\mathbf{q}) = 0 \quad (1.1)$$

If the time variable appears explicitly in the constraint equations, they are expressed as:

$$\Phi \equiv \Phi(\mathbf{q}, t) = 0 \quad (1.2)$$

### 3. *Holonomic constraints and nonholonomic constraints*

Holonomic and nonholonomic constraints are classical mechanics concepts that are used to classify constraints and systems. If constraint equations do not contain derivative terms, or the derivative terms are integrable, these constraints are said to be called holonomic. They are geometric constraints. However, if the constraint equations contain derivative terms that are not integrable in closed form, these constraints are said to be nonholonomic. They are movement constraints, such as the velocity or acceleration conditions imposed on the system.

### 4. *Degrees of freedom*

The generalized coordinates that satisfy the constraint equations in a system may not be independent. Thus, the minimum number of coordinates required to describe the system is called the number of degrees of freedom (DOF).

### 5. *Virtual displacement*

Virtual displacement is an assumed infinitesimal displacement of a system at a certain position with constraints satisfied while time is held constant. Conditions imposed on the virtual displacement by the constraint equations are called virtual displacement equations. A virtual displacement may be a linear or an angular displacement, and it is normally denoted by the variational symbol  $\delta$ . Virtual displacement is a different concept from actual displacement. Actual displacement can only take place with the passage of time; however, virtual displacement has nothing to do with any other conditions but the constraint conditions.

## 1.1.1 *Newtonian Mechanics*

The train of thought used to establish the vehicle dynamics equations using Newton's law can be summarized in a few steps. According to the characteristics of the problem at hand, first, we need to simplify the system and come up with a suitable mathematical model by representing the practical system with rigid bodies and lumped masses which are connected to each other by springs and dampers. Then, we isolate the masses and bodies and draw the free-body diagrams. Finally, we apply the following formulas to the masses and bodies shown by free-body diagrams.

The dynamic equations of a planar rigid body are:

$$m \frac{d^2 \mathbf{r}}{dt^2} = \sum \mathbf{F}_i \quad (1.3)$$

$$J\dot{\omega} = \sum M_i \quad (1.4)$$

where  $m$  is the mass of the body,  $r$  is the displacement of the center of gravity,  $F_i$  is the  $i$ th force acting on the body,  $J$  is the mass moment of inertia of the body about the axis through the center of gravity,  $\omega$  is the angular velocity of the body, and  $M_i$  is the moment of the  $i$ th force acting on the center of gravity of the body.

### 1.1.2 Analytical Mechanics

In solving the dynamics problems of simple rigid body systems, Newtonian mechanics theories have some obvious advantages; however, the efficiency will be low if dealing with constrained systems and deformable bodies. Analytical mechanics theories have been proven to be a useful method in solving these problems. This theory contains mainly the methods of general equations of dynamics, the Lagrange equation of the first kind, and the Lagrange equation of the second kind; the latter being the most widely used.

For a system with  $b$  particles (or bodies), and  $n$  DOF,  $q_1, q_2, \dots, q_n$  is a set of generalized coordinates. Then, the Lagrange equation of the second kind can be expressed as

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_k} \right) - \frac{\partial T}{\partial q_k} + \frac{\partial V}{\partial q_k} = 0 \quad (k = 1, 2, \dots, n) \quad (1.5)$$

where  $T$  is the kinetic energy, and  $V$  the potential energy of the system.

## 1.2 Dynamics of Rigid Multibody Systems

### 1.2.1 Birth and Development

The history of the development of classical mechanics goes back more than 200 years. In the past two centuries, classical mechanics has been successfully used in the theoretical study and engineering practice of relatively simple systems. However, most modern practical engineering problems are quite complicated systems consisting of many parts. Since the middle of the 20th century, the rapid development of aerospace, robotics, automotive and other industries has brought new challenges to classical mechanics. The kinematics and dynamics analysis of complicated systems becomes difficult. Thus, there was an urgent need to develop new theories to accomplish this task.

In the late 1960s and early 1970s, Roberson<sup>[1]</sup>, Kane<sup>[2]</sup>, Haug<sup>[3]</sup>, Wittenburg<sup>[4]</sup>, Popov<sup>[5]</sup> and other scholars put forward methods of their own to solve the dynamic problems of complex systems. Although there were some differences between these methods in describing the position and orientation of the systems, and formulating and solving the equations, one characteristic was common among them: recurring formularization was adopted in all these methods. Computers, which help engineers to model, form, and solve differential equations of motion, were used analyze and synthesize complex systems. Thus, a new branch of mechanics called multibody dynamics was born. This developing

and crossing discipline arises from the combination of rigid mechanics, analytical mechanics, elastic mechanics, matrix theory, graph theory, computational mathematics, and automatic control. It is one of the most active fields in applied mechanics, machinery, and vehicle engineering.

Multibody systems are composed of rigid and/or flexible bodies interconnected by joints and force elements such as springs and dampers. In the last few decades, remarkable advances have been made in the theory of multibody system dynamics with wide applications. An enormous number of results have been reported in the fields of vehicle dynamics, spacecraft control, robotics, and biomechanics. With the development and perfection of the multibody formalisms, multibody dynamics has received growing attention and a considerable amount of commercial software is now available. The first International Symposium on multibody system dynamics was held in Munich in 1977 by IUTAM. The second was held in Udine in 1985 by IUTAM/IFTOMM. After the middle of the 1980s, multibody dynamics entered a period of fast development. A wealth of literature has been published<sup>[6,7]</sup>.

The first book about multibody system dynamics was titled *Dynamics of System of Rigid Bodies*<sup>[4]</sup> written by Wittenburg, was published in 1977. *Dynamics: Theory and applications* by Kane and Levinson came out in 1985. In *Dynamics of Multibody System*<sup>[8]</sup>, printed in 1989, Shabana comprehensively discusses many aspects of multibody system dynamics, with a second edition of this book appearing in 1998. In *Computer-aided Analysis of Mechanical Systems*<sup>[9]</sup>, Nikravesh introduces theories and numerical methods for use in computational mechanics. These theories and methods can be used to develop computer programs for analyzing the response of simple and complex mechanical systems. Using the Cartesian coordinate approach, Haug presented basic methods for the analysis of the kinematics and dynamics of planar and spatial mechanical systems in *Computer Aided Kinematics and Dynamics of Mechanical Systems*<sup>[3]</sup>.

The work of three scholars will also be reviewed in the following section.

1. Schiehlen, from the University of Stuttgart, published his two books in 1977 and 1993 respectively. *Multibody System Handbook*<sup>[10]</sup> was an international collection of programs and software which included theory research results and programs from 17 research groups. *Advanced Multibody Dynamics*<sup>[11]</sup> collected research achievements of the project supported by The German Research Council from 1987 to 1992, and the latest developments in the field of multibody system dynamics worldwide at that time. The content of this book was of an interdisciplinary nature.
2. In *Computational Methods in Multibody Dynamics*<sup>[12]</sup>, Amirouche Farid offered an in-depth analysis of multibody system dynamics with rigid and flexible interconnected bodies, and provided several methods for deriving the equations of motion. Computer methods of tree-like systems and systems with closed loops and prescribed motion were fully discussed.
3. In *Multi-body Dynamics: Vehicles, machines and mechanisms*<sup>[13]</sup>, Rahnejat guided readers through different topics from dynamics principles to the detailed multibody formulation and solution approach. Model analytic solutions were provided for a variety of practical machines and mechanisms such as the suspension of a vehicle and the rotor of helicopter. State-of-the-art modeling and solution methods were presented to investigate complex

dynamics phenomena in the behavior of automobiles and aircraft. Optimal control of multibody systems were also discussed.

Multibody dynamics research in China started late but developed quickly. The inaugural meeting of the Multibody System Dynamics group, part of the General Mechanics Committee of Chinese Society of Mechanics, was held in Beijing in August 1986. Since then, many books on multibody system dynamics have come out. Many researchers have published high-quality papers on modeling theory, computational methods, and other subjects of multibody system dynamics<sup>[14,15]</sup>.

### 1.2.2 Theories and Methods of Multi-Rigid Body System Dynamics

Formulism methods and numerical algorithms are the two most important aspects in multibody system dynamics research. Over the past few decades, many methods have appeared. For example, the New-Euler method by Schiehlen<sup>[10]</sup>, the Kane method by Kane and Huston<sup>[2,16]</sup>, the graph theory method by Roberson and Wittenburg<sup>[1,17]</sup>, and the Lagrangian method by Haug<sup>[3]</sup> are representative. According to the difference in coordinates adopted, formulism methods can be divided into two categories: minimum number of coordinates method and maximum number of coordinates method. The minimum number of coordinate method uses joint coordinates, taking the relative angular or displacement of the adjacent bodies as generalized coordinates. The main advantage of this method is that fewer variables are used, and higher calculation efficiency can be obtained. However, the construction process of coefficient matrix of differential equations is very complex, including a large amount of nonlinear operations. The maximum number of coordinates method uses the Cartesian coordinates of the center of mass of bodies and the Euler angles or Euler parameters as generalized coordinates, combining Lagrangian multipliers to constraint equations to formulate the equations of motion. This method can be easier implemented for coding, but with the features of more variables and lower calculation efficiency.

#### 1. Graph theory (R-W)

Roberson and Wittenburg introduced graph theory into the research of multi-rigid body system dynamics. This method applies some basic concepts and mathematical tools to describe the topological structure of a multibody system. The relative displacements of the adjacent bodies are taken as generalized coordinates, and the unified mathematical formula of the complex tree-like structure is derived and the general forms of the dynamic equations are formulated for multibody systems. Code MESA VERDE based on this method has been developed.

#### 2. Lagrangian method

This method uses the Cartesian coordinates of the center of mass of bodies and the Euler angles or Euler parameters that describe the orientation of the system as generalized coordinates, combining Lagrangian multipliers to constraint equations to formulate the equations of motion. This method has the characteristic of being easier for programming purposes. Orlandea, Chace, Haug, and Nikravesh developed their general purpose codes ADAMS, DADS, DAP. There are still some differences



between them in the detailed formulism and algorithm which is mainly reflected in the different coordinates used. In ADAMS, Cartesian coordinates of the center of mass of bodies and Euler angles that describe the orientation of the system are used as generalized coordinates. In DADS, Cartesian coordinates of the center of mass of bodies and Euler parameters that describe orientation of the system are used as generalized coordinates.

### 3. Multibody dynamics method in ADAMS

For a spatial system with  $b$  bodies, the Cartesian coordinates of the center of mass of body  $i$  are  $x_i, y_i, z_i$ , the Euler angles that describe orientation of the body are  $\psi_i, \theta_i, \phi_i$ , the generalized coordinates of the body can be expressed with a vector  $\mathbf{q}_i$ , such as  $q_i = [x, y, z, \psi, \theta, \phi]_i^T$ .

If vector  $\mathbf{q}$  is used to denote all of the coordinates of the system, then

$$\mathbf{q} = [q_1, q_2, \dots, q_b]^T$$

If the system contains holonomic and non-holonomic constraints, based on the Lagrangian equations with multipliers, the equations of motion, which are a set of differential-algebraic equations (DAE), can be obtained.

$$\frac{d}{dt} \left( \frac{\partial \mathbf{T}}{\partial \dot{\mathbf{q}}} \right)^T - \left( \frac{\partial \mathbf{T}}{\partial \mathbf{q}} \right)^T + \Phi_q^T \boldsymbol{\rho} + \theta_q^T \boldsymbol{\mu} = \mathbf{Q} \quad (1.6)$$

with holonomic constraints equations

$$\varphi(\mathbf{q}, t) = 0$$

and with non-holonomic constraints equations

$$\theta(\mathbf{q}, \dot{\mathbf{q}}, t) = 0$$

where,  $\mathbf{T}$  is the kinetic energy of the system,  $\mathbf{Q}$  is the vector of generalized forces,  $\boldsymbol{\rho}$  is the Lagrange multiplier vector corresponding to holonomic constraints,  $\boldsymbol{\mu}$  is the Lagrangian multiplier vector corresponding to the non-holonomic constraints.

If the kinetic energy is expressed with velocity and mass, the equations can be written in matrix form.

### 4. Multibody dynamics methods in DADS

For a spatial system with  $b$  bodies, the Cartesian coordinates of the center of mass of body  $i$  are  $x_i, y_i, z_i$ , the Euler parameters that describe the orientation of the body are  $p_i = [e_{0i}, e_{1i}, e_{2i}, e_{3i}]^T$ , and the generalized coordinates of the body can be expressed with a vector  $\mathbf{q}_i$ , and  $q_i = [x, y, z, e_0, e_1, e_2, e_3]_i^T = [r, p]_i^T$ .

If a vector  $\mathbf{q}$  is used to denote all of the coordinates of the system, then

$$\mathbf{q} = [q_1, q_2, \dots, q_b]^T$$

For body  $i$ , the mass is  $m_i$ , the inertia matrix in the local coordinate system  $J'_i$  is composed of moments of inertia and products of inertia, the mass characteristics  $N_i = \text{diag}(m, m, m)_i$ ,