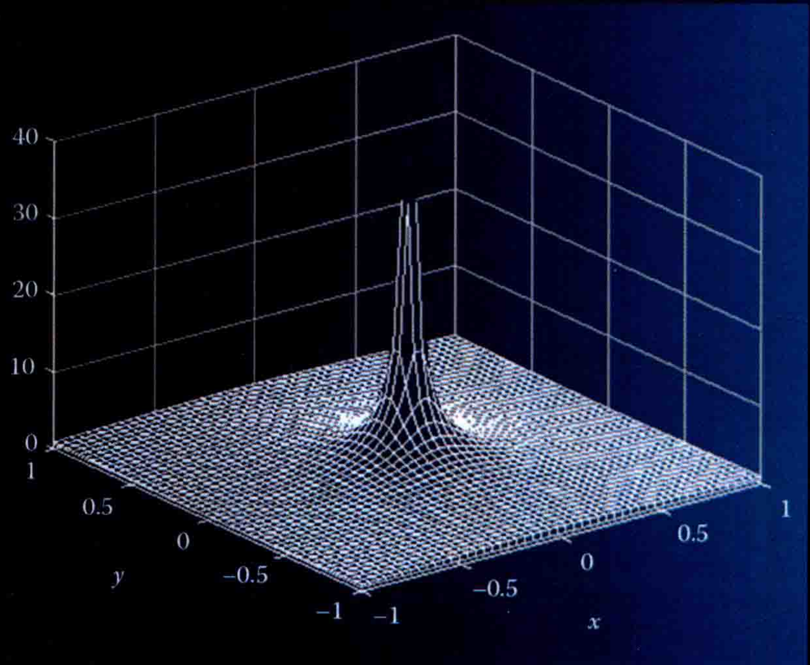
TEXTBOOKS IN MATHEMATICS

A MATLAB® COMPANION TO COMPLEX VARIABLES



A. David Wunsch

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A. David Wunsch
University of Massachusetts Lowell

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Michael F. Brown ably assisted me on the last two editions of my book on complex variables and kindly found time to help me with this text. He is a good mathematician and a careful reader.

Finally, MathWorks, publisher of MATLAB®, provides a team of consultants to help authors who are writing MATLAB-based books. These people have responded quickly to my e-mails and have almost always managed to correct my work and supply helpful advice.

Introduction

I hope the reader will enjoy using this book. It is for someone with at least a beginner's knowledge of MATLAB® who is learning the branch of advanced calculus called "functions of a complex variable." Complex variable theory is the calculus of functions dependent on variables that can assume complex numerical values. I want you to discover that MATLAB is your friend when you are learning this type of mathematics.

One of the daunting aspects of complex numbers (and variables) is that simple arithmetic operations that are easily done in one's head, if they involve real quantities, become tedious when the numbers are complex. You can ask a class of third graders for the product of the numbers 3, 4, and 5, and someone with the answer will raise his or her hand in less than 5 seconds. Ask a college student for the product of the numbers $(1 + 3i)$, $(1 + 4i)$, $(1 + 5i)$ and he or she will, after a little squirming, probably resort to pencil and paper, although the problem is solvable in one's head with some serious concentration. If you were to ask me this question as I'm typing this sentence, I would open the MATLAB window on my computer and type in the product. Here we see a glimpse of the utility of MATLAB in complex arithmetic.

If you glance at the table of contents, you will see how MATLAB can be your companion in such staples of complex variable theory as conformal mapping, infinite series, contour integration, and Laplace and Fourier transforms. Fractals, the most recent interesting topic involving complex variables, cries out to be treated with a language such as MATLAB, and you might want to begin this book at its end, The Coda, which is devoted entirely to this visually intriguing subject. However, I must add that as you progress through the book, the MATLAB skill required increases gradually, and leaping to the end is not for everyone.

Sometimes while working with MATLAB and complex algebra, you may be puzzled at what you find. For example, if you ask a class of high school students to compute $\left(5^3\right)^{1/3}$, someone will quickly call out "five." He or she will know to multiply the exponents together. Now asking the same class for $\left((-1+i)^2\right)^{1/2}$, a student will follow the same logic and produce a correct answer: $-1 + i$. But another student, using MATLAB, will say $1 - i$. Both answers are correct, but why did MATLAB choose this one? This book answers such questions and many similar ones.

This textbook does not purport to present MATLAB as a substitute for a knowledge of the functions of a complex variable any more than MATLAB can be used as a replacement for an actual understanding of elementary calculus or linear algebra. This is also not a text from which one learns the elements of MATLAB, although if you already know a little of the language, it will expand your knowledge. Some books that will get the reader started in the elements of MATLAB programming are listed as references [1,2]. MATLAB is not without constraints, assumptions, limitations, irritations, and quirks, and there are subtleties involved in performing the calculus of complex variable theory with this language that will be made evident here. Without knowledge of these subtleties, the engineer or scientist who is attempting to use MATLAB for solutions of practical problems in complex variable theory suffers the real risk of making major mistakes. This book should serve as an early warning system about these pitfalls.

This book should be read as a companion to standard texts on functions of a complex variable. Throughout what follows, we refer to two of the author's favorites. Not surprisingly, one volume is his own *Complex Variables with Applications* (3rd edition) published by Addison-Wesley in 2005. We refer to this book as "W" in the text. In most cases, the section numbers referred to apply to the second edition as well. There is a Spanish translation available for those who prefer that language: *Variable Compleja con Aplicaciones*. The reference book in the Schaum's outline series, *Complex Variables*, 2nd edition by M. Spiegel et al. is an old favorite of mine, and although it is more handbook than textbook, it is remarkably well done, and I will refer to it with the letter "S." Notice that the section numbers that I refer to apply to the second edition only. *Complex Variables and Applications* by R. V. Churchill and colleagues is very well written, but the book has had so many editions that it is difficult to refer the student to any particular section.

References

1. Hunt, B., Lipsman, R., and Rosenberg, J. *A Guide to MATLAB for Beginners and Experienced Users*, 3rd edition. New York: Cambridge University Press, 2014.
2. Hahn, B. and Valentine, D. *Essential MATLAB for Engineers and Scientists*, 5th edition. New York: Academic Press, 2013.

A. David Wunsch
University of Massachusetts Lowell
Lowell, Massachusetts

A Note to the Reader

To keep the cost of this book reasonable, all figures are rendered in black and white and shades of gray. Some of the codes provided here will produce only these colors, while other programs will produce color plots on your screen even though the plots are in grayscale in your book. In the solutions manual I have not avoided color.

MATLAB® is upgraded at least once a year with new releases. Its capabilities change and for the most part are improved. This book is based on R2015a, the release available through most of the year 2015.

Corrections to this book as well as to the solutions manual will be posted at the author's web page hosted by the University of Massachusetts Lowell. Here is the URL: http://faculty.uml.edu/awunsch/wunsch_complex_variables/faculty.htm.

The author invites corrections and comments for his work. An email address can be found at the above website where you will also find solutions to the odd numbered problems. To assist readers with data entry in MATLAB, code text can be found for download at https://www.crcpress.com/A-MatLab-Companion-to-Complex-Variables/Wunsch/9781498755672.

Instructors using my book in the classroom can also receive a copy of a complete solutions manual, for all problems, if they write to me on their college stationary at the ECE Department, University of Massachusetts Lowell, Lowell, MA 01854, USA.

Contents

1

Complex Arithmetic

1.1 The Rectangular Form

A complex number z can be stated in the form

$$z = x + iy \tag{1.1}$$

where x and y are real numbers, and there is multiplication between the i and the y. An equally valid representation is

$$z = x + yi \tag{1.2}$$

since complex numbers obey the commutative law of multiplication so that the order of multiplying y with i is immaterial. Right away we must deal with an idiosyncrasy of MATLAB®. The statement $z = 3 + 4i$ can be entered in the MATLAB command window with the result given in the following example. Note that >> preceding a line of code indicates that the expression was *entered from the keyboard* into the command window of MATLAB.

Example 1.1

```
>> z=3 + 4i
z = 3.0000 + 4.0000i
```

which is exactly what we hoped for: MATLAB has returned 3.0000 + 4.0000i. However, entering $z = 3 + i4$ in the command window will result in an error message. The value i must appear as the second factor in the multiplication if i is to be interpreted by MATLAB as a multiplicative factor in $z = x + y$i. The practice is only valid if i is *preceded* by an explicitly stated *real* (not complex) number. A symbol cannot be used for that number. This entire convention can be overlooked provided we employ the MATLAB * for multiplication. We will adopt that practice throughout this book even though the authors of MATLAB claim that eliminating * where allowed will speed up calculations; the advantage is often slight.

Note that in general, all the arithmetical operations that one does with real numbers in MATLAB can be carried out for complex numbers, using the same operators (i.e., the + and − signs for addition and subtraction, the / for division, the ^ for raising a number to a power, and as noted, the * for multiplication). The precise meaning of what the ^ will yield when followed by a fraction will be treated in section 1.3.

To get the magnitude (or absolute value or modulus) of a complex number, we apply the operation **abs** to that number. Thus,

```
>> abs(3 + 4*i)
ans = 5
```

Example 1.2

Multiply $3 + i4$ by $(1 + 2i)$ and add $i2$ to that result. Then find the absolute value of that quantity.

Solution:

```
>> (3 + i*4)*(1 + 2*i) + 2*i
ans = −5.0000 + 12.0000i
>> abs(ans)
ans = 13
```

Electrical engineers often prefer to use j instead of i, and this notation can be used automatically in MATLAB. We just use j instead of i and follow the same conventions as above.

You should not use both i and j in the same code, or you will confuse yourself and whoever reads what you wrote.

Example 1.3

Use j instead of i, and raise $(1 + j3)$ to the −2 power, divide that result by $2 - j3$, and subtract $3 + 4j$ from that result.

Solution:

```
>> (1 + 3*j)^(−2)/(2−j*3)−(3 + 4*j)
ans = −2.9985 − 4.0277i
```

Note that although we used j instead of i, MATLAB returned an answer employing i.

For some purposes, it is more convenient to put $j = -i$, and you should make this statement at the start of your work if that is your preference. (Note that $j^2 = -1$ as before.) Recall that physicists prefer $e^{-i\omega t}$ in lieu of the electrical engineer's $e^{j\omega t}$, where ω is a radian frequency and t is the time.

The real and imaginary parts of a complex number, for example z, are found from the functions **real**(z), **imag**(z), and the conjugate of z is given by

conj(z) as shown by the following sequence of calculations. Note that in the text material of this book, we will designate the conjugate of a quantity by an overbar, as in $\bar{z}$, which in MATLAB would of course be **conj**(z).

Example 1.4

```
>> z=3 + 4*i
z = 3.0000 + 4.0000i
>> x=real(z)
x = 3
>> y=imag(z)
y = 4
>> w=conj(z)
w = 3.0000 – 4.0000i
```

1.1.1 A Caveat on Complex Numbers as Matrix Elements

MATLAB, as its name might suggest, is a computer language for technical computation that is based on matrices (i.e., rectangular arrays of numbers). Sometimes the matrix consists of just a horizontal row of numbers, in a specific order, or a column of numbers in a certain order. Such matrices are known as vectors and more specifically as row or column vectors. The numbers that make up matrices, the elements, can be complex or real numbers, or symbols. When entering complex numbers as the elements, it is useful to state them with parentheses surrounding each complex number. In other words, instead of entering 3 + 4*i, you should enter (3 + 4*i). Otherwise, an inadvertent use of the space bar can result in an error, as shown by the following where we want to enter a row vector *a* having two elements, one being 3 + *i* and the other 2–*i*. In the first instance, we will do this correctly without parentheses, then incorrectly without parentheses, because we have placed a space where it does not belong, and finally correctly with parentheses even though there is an unnecessary space:

```
>> a=[3 + i  2–i]
a = 3.0000 + 1.0000i  2.0000 – 1.0000i
>> a=[3  + i  2–i]
a = 3.0000  0 + 1.0000i  2.0000 – 1.0000i
>> a=[(3  + i)  (2–i)]
a = 3.0000 + 1.0000i  2.0000 – 1.0000i
```

Comment: Note that in the second instance, the value obtained for *a* is

3.0000 0 + 1.0000i 2.0000 – 1.0000i

which is a row vector with *three* elements, not the desired two elements. The first element is 3, and the second is 0 + 1.0000i=*i*. The error was caused by our having a space after the 3 and before the adjacent + sign.

In the third output for a, we got the correct value even though we typed a space between the 3 and the plus sign following it.

Note that you can always tell the number of elements in a row or column vector, let us call it a, with the command **length**(a). If we apply this operation to the three values of a described above, we get 2, 3, and 2, respectively.

Exercises

1. Using MATLAB, determine

   a. $\dfrac{3-4i}{1+i2}+i2$

   b. $\dfrac{2+i}{3+5i}-1-i/2$

   c. $(1+i2/3)^3-\dfrac{i}{3-4i}$

   d. $\left(1+\dfrac{2}{i3}\right)^{11}$

   e. $\left(\dfrac{2i}{(3+4i)}+\overline{\left(\dfrac{1}{5-4i}\right)}\right)^5$

2. The numerical value of the expression $(1+i)^{37}$ can be determined without recourse to MATLAB by the following technique: $(1+i)^{37}=(1+i)^{36}(1+i)=\left((1+i)^2\right)^{18}(1+i)$. Now complete the calculation by evaluating $(1+i)^2$ and noting that i raised to any even power is easily computed. Check your answer by using MATLAB to find $(1+i)^{37}$ directly.

3. Consider the finite series ($N \geq 0$ is an integer) and its sum.
   $\sum_{n=0}^{N} z^n = \dfrac{1-z^{N+1}}{1-z}$ $z \neq 1$, whose derivation is identical to that given in real calculus for the sum of a finite geometric series.

   a. Using MATLAB, sum the series $1+\dfrac{i}{2}+\left(\dfrac{i}{2}\right)^2+\left(\dfrac{i}{2}\right)^3+\ldots+\left(\dfrac{i}{2}\right)^{10}$ directly by using a loop created from the **while** or **for** commands. Use the "long format" of MATLAB as you will need it in part (b).

   b. Verify the answer obtained in (a) by using the closed-form expression for the sum given above, again using the long format so that you can notice any small discrepancy.