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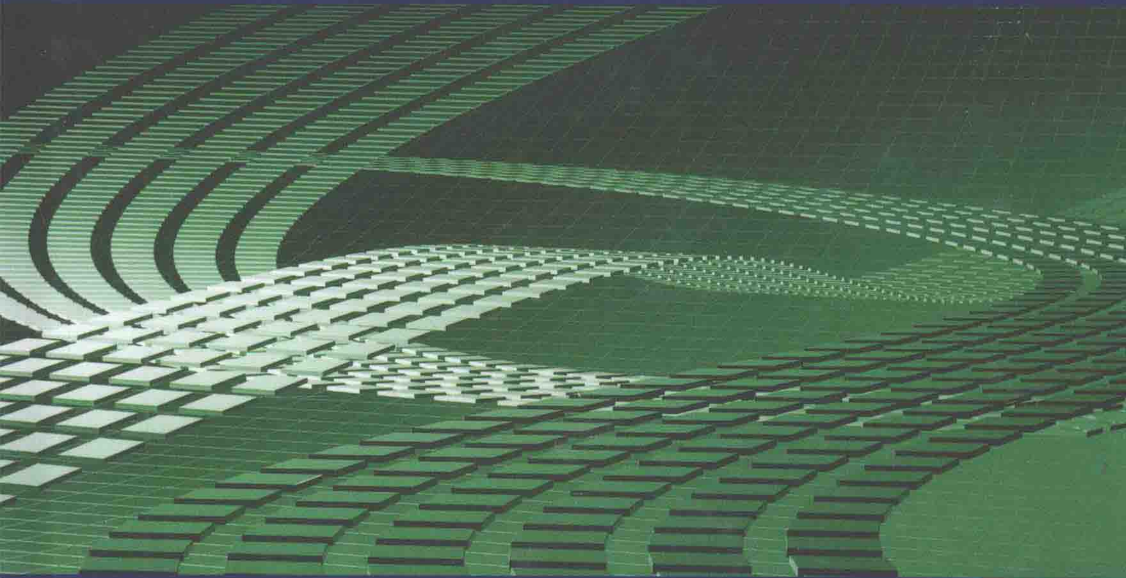

Random Vibration

Mechanical Vibration and Shock Analysis

Revised and Updated 3rd Edition

Volume 3

Christian Lalanne

iSTE

WILEY

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Third edition – Volume 3

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Foreword to Series

In the course of their lifetime simple items in everyday use such as mobile telephones, wristwatches, electronic components in cars or more specific items such as satellite equipment or flight systems in aircraft, can be subjected to various conditions of temperature and humidity, and more particularly to mechanical shock and vibrations, which form the subject of this work. They must therefore be designed in such a way that they can withstand the effects of the environmental conditions to which they are exposed without being damaged. Their design must be verified using a prototype or by calculations and/or significant laboratory testing.

Sizing, and later, testing are performed on the basis of specifications taken from national or international standards. The initial standards, drawn up in the 1940s, were blanket specifications, often extremely stringent, consisting of a sinusoidal vibration, the frequency of which was set to the resonance of the equipment. They were essentially designed to demonstrate a certain standard resistance of the equipment, with the implicit hypothesis that if the equipment survived the particular environment it would withstand, undamaged, the vibrations to which it would be subjected in service. Sometimes with a delay due to a certain conservatism, the evolution of these standards followed that of the testing facilities: the possibility of producing swept sine tests, the production of narrowband random vibrations swept over a wide range and finally the generation of wideband random vibrations. At the end of the 1970s, it was felt that there was a basic need to reduce the weight and cost of on-board equipment and to produce specifications closer to the real conditions of use. This evolution was taken into account between 1980 and 1985 concerning American standards (MIL-STD 810), French standards (GAM EG 13) or international standards (NATO), which all recommended the *tailoring of tests*. Current preference is to talk of the *tailoring of the product to its environment* in order to assert more clearly that the environment must be taken into account from the very start of the project, rather than to check the behavior of the material *a* *posteriori*. These concepts, originating with the military, are currently being increasingly echoed in the civil field.

Tailoring is based on an analysis of the life profile of the equipment, on the measurement of the environmental conditions associated with each condition of use and on the synthesis of all the data into a simple specification, which should be of the same severity as the actual environment.

This approach presupposes a proper understanding of the mechanical systems subjected to dynamic loads and knowledge of the most frequent failure modes.

Generally speaking, a good assessment of the stresses in a system subjected to vibration is possible only on the basis of a finite element model and relatively complex calculations. Such calculations can only be undertaken at a relatively advanced stage of the project once the structure has been sufficiently defined for such a model to be established.

Considerable work on the environment must be performed independently of the equipment concerned either at the very beginning of the project, at a time where there are no drawings available, or at the qualification stage, in order to define the test conditions.

In the absence of a precise and validated model of the structure, the simplest possible mechanical system is frequently used consisting of mass, stiffness and damping (a linear system with one degree of freedom), especially for:

– the comparison of the severity of several shocks (shock response spectrum) or of several vibrations (extreme response and fatigue damage spectra);

– the drafting of specifications: determining a vibration which produces the same effects on the model as the real environment, with the underlying hypothesis that the equivalent value will remain valid on the real, more complex structure;

– the calculations for pre-sizing at the start of the project;

– the establishment of rules for analysis of the vibrations (choice of the number of calculation points of a power spectral density) or for the definition of the tests (choice of the sweep rate of a swept sine test).

This explains the importance given to this simple model in this work of five volumes on “Mechanical Vibration and Shock Analysis”.

Volume 1 of this series is devoted to *sinusoidal vibration*. After several reminders about the main vibratory environments which can affect materials during their working life and also about the methods used to take them into account, following several fundamental mechanical concepts, the responses (relative and absolute) of a mechanical one-degree-of-freedom system to an arbitrary excitation are considered, and its transfer function in various forms are defined. By placing the properties of sinusoidal vibrations in the contexts of the real environment and of laboratory tests, the transitory and steady state response of a single-degree-of-freedom system with viscous and then with non-linear damping is evolved. The various sinusoidal modes of sweeping with their properties are described, and then, starting from the response of a one-degree-of-freedom system, the consequences of an unsuitable choice of sweep rate are shown and a rule for choice of this rate is deduced from it.

Volume 2 deals with *mechanical shock*. This volume presents the shock response spectrum (SRS) with its different definitions, its properties and the precautions to be taken in calculating it. The shock shapes most widely used with the usual test facilities are presented with their characteristics, with indications how to establish test specifications of the same severity as the real, measured environment. A demonstration is then given on how these specifications can be made with classic laboratory equipment: shock machines, electrodynamic exciters driven by a time signal or by a response spectrum, indicating the limits, advantages and disadvantages of each solution.

Volume 3 examines the analysis of *random vibration* which encompasses the vast majority of the vibrations encountered in the real environment. This volume describes the properties of the process, enabling simplification of the analysis, before presenting the analysis of the signal in the frequency domain. The definition of the power spectral density is reviewed, as well as the precautions to be taken in calculating it, together with the processes used to improve results (windowing, overlapping). A complementary third approach consists of analyzing the statistical properties of the time signal. In particular, this study makes it possible to determine the distribution law of the maxima of a random Gaussian signal and to simplify the calculations of fatigue damage by avoiding direct counting of the peaks (Volumes 4 and 5). The relationships that provide the response of a one-degree-of-freedom linear system to a random vibration are established.

Volume 4 is devoted to the calculation of *damage fatigue*. It presents the hypotheses adopted to describe the behavior of a material subjected to fatigue, the laws of damage accumulation and the methods for counting the peaks of the response (used to establish a histogram when it is impossible to use the probability density of the peaks obtained with a Gaussian signal). The expressions of mean damage and its standard deviation are established. A few cases are then examined using other hypotheses (mean not equal to zero, taking account of the fatigue limit, non-linear accumulation law, etc.). The main laws governing low cycle fatigue and fracture mechanics are also presented.

Volume 5 is dedicated to presenting the method of *specification development* according to the principle of tailoring. The extreme response and fatigue damage spectra are defined for each type of stress (sinusoidal vibrations, swept sine, shocks, random vibrations, etc.). The process for establishing a specification as from the lifecycle profile of the equipment is then detailed taking into account the uncertainty factor (uncertainties related to the dispersion of the real environment and of the mechanical strength) and the test factor (function of the number of tests performed to demonstrate the resistance of the equipment).

First and foremost, this work is intended for engineers and technicians working in design teams responsible for sizing equipment, for project teams given the task of writing the various sizing and testing specifications (validation, qualification, certification, etc.) and for laboratories in charge of defining the tests and their performance following the choice of the most suitable simulation means.

Introduction

The vibratory environment found in the majority of vehicles essentially consists of random vibrations. Each recording of the same phenomenon results in a signal different from the previous ones. The characterization of a random environment therefore requires an infinite number of measurements to cover all the possibilities. Such vibrations can only be analyzed statistically.

The first stage consists of defining the properties of the processes comprising all the measurements, making it possible to reduce the study to the more realistic measurement of single or several short samples. This means evidencing the stationary character of the process, making it possible to demonstrate that its statistical properties are conserved in time, thus its ergodicity, with each recording representative of the entire process. As a result, only a small sample consisting of one recording has to be analyzed (Chapter 1).

The value of this sample gives an overall idea of the severity of the vibration, but the vibration has a continuous frequency spectrum that must be determined in order to understand its effects on a structure. This frequency analysis is performed using the power spectral density (PSD) (Chapter 2) which is the ideal tool for describing random vibrations. This spectrum, a basic element for many other treatments, has numerous applications, the first being the calculation of the rms (root mean square) value of the vibration in a given frequency band (Chapter 3).

The practical calculation of the PSD, completed on a small signal sample, provides only an estimate of its mean value, with a statistical error that must be evaluated. Chapter 4 shows how this error can be evaluated according to the analysis conditions and how it can be reduced, before providing rules for the determination of the PSD.

The majority of signals measured in the real environment have a Gaussian distribution of instantaneous values. The study of the properties of such a signal is extremely rich in content (Chapter 5). For example, knowledge of the PSD alone gives access, without having to count the peaks, to the distribution of the maxima of a random signal (Chapter 6), and to the law of distribution of the largest peaks, in itself useful information for the pre-sizing of a structure (Chapter 7).

It is also used to determine the response of a system with one degree-of-freedom (Chapters 8 and 9), which is necessary to calculate the fatigue damage caused by the vibration in question (Volume 4).

The study of the first crossing of a given response threshold for a one-degree-of-freedom system can also be useful in estimating the greatest stress value over a given duration. Different methods are presented (Chapter 10).

List of Symbols

The list below gives the most frequent definition of the main symbols used in this book. Some of the symbols can have another meaning which will be defined in the text to avoid any confusion.

a	Threshold value of $\ell(t)$ or maximum of $\ell(t)$
A	Maximum of $A(t)$
$A(t)$	Envelope of a signal
b	Exponent
c	Viscous damping constant
$e(t)$	Narrow band white noise
$E(\;)$	Expectation of...
$E_1(\;)$	First definition of error function
$E_2(\;)$	Second definition of error function
Erf	Error function
$E(\;)$	Expected function of ...
f	Frequency of excitation
$f_{samp.}$	Sampling frequency
f_{max}	Maximum frequency
f_0	Natural frequency
g	Acceleration due to gravity
G	Particular value of power spectral density
$G(\;)$	Power spectral density for $0 \le f \le \infty$
$\hat{G}(\;)$	Measured value of $G(\;)$
$G_{\ell u}(\;)$	Cross-power spectral density
h	Interval (f/f_0) or f_2/f_1
$h(t)$	Impulse response
$H(\;)$	Transfer function
i	$\sqrt{-1}$
k	Stiffness
K	Number of subsamples
ℓ	Value of $\ell(t)$
$\bar{\ell}$	Mean value of $\ell(t)$
$\bar{\ell}_N$	Average maximum of N_p peaks
ℓ_{rms}	Rms value of $\ell(t)$
$\ddot{\ell}_{rms}$	Rms value of $\ddot{\ell}(t)$
$\ell(t)$	Generalized excitation (displacement)
$\dot{\ell}(\;)$	First derivative of $\ell(t)$
$\ddot{\ell}(t)$	Second derivative of $\ell(t)$
L	Given value of $\ell(t)$
L_{rms}	Rms value of filtered signal

$L(\Omega)$ Fourier transform of $\ell(t)$
$\dot{L}(\Omega)$ Fourier transform of $\dot{\ell}(t)$
m Mean
M Number of points of PSD
M_a Average number of maxima which exceeds threshold per unit time
M_n Moment of order n
n Order of moment or number of degrees of freedom
n_a Average number of crossings of threshold a per unit time
n_a^+ Average number of crossings of threshold **a** with positive slope per unit time
n_0 Average number of zero-crossings per unit time
n_0^+ Average number of zero-crossings with positive slope per second (average frequency)
n_p^+ Average number of maxima per unit time
N Number of curves or Number of points of signal or Numbers of dB
N_p Number of peaks
N_a^+ Average number of crossings of threshold **a** with positive slope for given length of time
N_0^+ Average number of zero-crossings with positive slope for given length of time
N_p^+ Average number of positive maxima for given length of time
p( ) Probability density
$p_N(\ )$ Probability density of largest maximum over given duration
P Probability
PSD Power spectral density
q $\sqrt{1-r^2}$
q_E $\dot{R}_{rms}/\dot{u}_{rms}$
q_{max}^+ Probability that a maximum is positive
q_{max}^- Probability that a maximum is negative
q( ) Probability density of maxima of $\ell(t)$
$q(\theta)$ Reduced response
$\dot{q}(\theta)$ First derivative of $q(\theta)$
$\ddot{q}(\theta)$ Second derivative of $q(\theta)$
Q Q factor (quality factor)
Q( ) Distribution function of maxima of $\ell(t)$
Q(u) Probability that a maximum is higher than a given threshold
r Irregularity factor
rms Root mean square (value)
r(t) Temporal window
R Slope in dB/octave or Ratio of the number of minima to the number of maxima
$R_E(\ )$ Auto-correlation function of envelope
$R_{\ell u}$ Cross-correlation function between $\ell(t)$ and $u(t)$
R(f) Fourier transform of r(t)
R(t) Envelope of maxima of u(t)

$\dot{R}(t)$	First derivative of R(t)
$R(\tau)$	Auto-correlation function
s	Standard deviation
S_0	Value of constant PSD
$S(\ )$	Power spectral density for $-\infty \le f \le +\infty$
t	Time
T	Duration of sample of signal or duration of vibration
T_a	Average time between two successive maxima
u	Ratio of threshold a to rms value ℓ_{rms} of $\ell(t)$
u_0	Initial value of $u(t)$
$\dot{u}_0$	Initial value of $\dot{u}(t)$
$\overline{u}_0$	Average of highest peaks
u_{rms}	Rms value of $u(t)$
$\dot{u}_{rms}$	Rms value of $\dot{u}(t)$
$\ddot{u}_{rms}$	Rms value of $\ddot{u}(t)$
$u(t)$	Generalized response
$\dot{u}(t)$	First derivative of $u(t)$
$\ddot{u}(t)$	Second derivative of $u(t)$
v	Ratio a/u_{rms}
v_{rms}	Rms value of $\dot{x}(t)$
x_{rms}	Rms value of $x(t)$
$\ddot{x}(t)$	Absolute acceleration of base of one-degree-of-freedom system
$\ddot{x}_{rms}$	Rms value of $\ddot{x}(t)$
$\ddot{x}_m$	Maximum value of $\ddot{x}(t)$
y_{rms}	Rms value of $y(t)$
$\dot{y}_{rms}$	Rms value of $\dot{y}(t)$
$\ddot{y}_{rms}$	Rms value of $\ddot{y}(t)$
z_{rms}	Rms value of $z(t)$
$\dot{z}_{rms}$	Rms value of $\dot{z}(t)$
$\ddot{z}_{rms}$	Rms value of $\ddot{z}(t)$
α	Time-constant of the probability density of the first passage of a threshold or Risk of up-crossing or $2\sqrt{1-\xi^2}$
β	$2\left(1-2\xi^2\right)$
χ_n^2	Variable of chi-square with n degrees of freedom
δt	Time step
$\delta(\ )$	Dirac delta function
$\Delta\tau$	Effective time interval
Δf	Frequency interval between half-power points or frequency step of the PSD
ΔF	Bandwidth of analysis filter
$\Delta\ell$	Interval of amplitude of $\ell(t)$
Δt	Time interval
ε	Statistical error or Euler's constant (0.57721566490…)
$\gamma_{\ell u}$	Coherence function between $\ell(t)$ and $u(t)$
φ	Phase
$\Phi(t)$	Gauss complementary distribution function
$\lambda(\ )$	Reduced excitation
π	3.14159265...
θ	Reduced time ($\omega_0\, t$)
μ_n	Central moment of order n
μ'_n	Reduced central moment of order n
π	3.14159265 ...
ρ	Correlation coefficient
τ	Delay

τ_m	Average time between two successive maxima
ω_0	Natural pulsation ($2\pi f_0$)
Ω	Pulsation of excitation ($2\pi f$)
ξ	Damping factor
ψ	Phase

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