Springer Tracts in Mechanical Engineering

Jingshan Zhao · Zhijing Feng
Ning Ma · Fulei Chu

Design of Special Planar Linkages

Springer

Jingshan Zhao • Zhijing Feng • Ning Ma • Fulei Chu

Design of Special Planar Linkages

Springer

Jingshan Zhao
Zhijing Feng
Ning Ma
Fulei Chu
Tsinghua University
Beijing
China, People's Republic

ISSN 2195-9862 ISSN 2195-9870 (electronic)
ISBN 978-3-642-38447-9 ISBN 978-3-642-38448-6 (eBook)
DOI 10.1007/978-3-642-38448-6
Springer Heidelberg New York Dordrecht London

Library of Congress Control Number: 2013946254

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

Linkages are widely used in our life and work. Almost all planar linkages can be decomposed with a number of equivalent four-bar linkages which are considered to be one of the fundamental mechanism forms. As the simplest closed chain mechanism, planar four-bar linkages are extensively used from mechanical engineering to aerospace engineering and even civil engineering.

This book focuses on the design of a kind of special planar linkages which consist of a number of pivot jointed scissorlike elements (SLEs) in one or more different planes. These planar linkages are overconstrained in structure but allow to stretch or compact in some directions; therefore, they have the advantages of both truss and mechanism. However, the kinetostatics and dynamics of these linkages are different from their existing counterparts of trusses or mechanisms. The statics, kinematics, and dynamics of such planar linkages are important topics in applications. We sincerely hope the book will be beneficial for mechanical engineers, robot design engineers, and researchers in other related fields.

Chapter 1 presents an overview of the pivot jointed planar linkages that will be investigated in this book. Some definitions, such as planar kinematic chain, mobility, planar linkages, workspace and singularity, design problems including statics design, kinematics design, and structural dynamics of planar linkages, are discussed in this chapter.

Chapter 2 develops a uniform position design equation for planar four-bar linkages. According to Burmester theory, a general planar four-bar linkage could trace five arbitrary accurate positions at most. However, modern design often requires a mechanism to precisely or approximately trace more positions. The text starts from proposing a uniform equation for position design and then provides a proof for the sufficiency and necessity. The algorithm can search the accurate results when there are such solutions for the specified n precise positions; otherwise, the best approximate solutions will be found. This permits the engineers to directly obtain the planar four-bar linkages when the specified positions are prescribed.

Chapter 3 proposes a foldable stair which remains the walking conversions of human being and all advantages of concrete stair in civil engineering. The stair is discussed as a mechanism synthesis example with screw theory in accordance to the common concrete stairs between two floors. It consists of a number of identical deployable SLEs that form the staircases when expanded. The instantaneous screw of every stair represents the relative rotation and the rotation center of the stair. The actuator is a planar four-bar linkage which is synthesized in line with the two extreme positions of the stair, folded and unfolded.

Chapter 4 starts from the discussion of the number of points that a planar four-bar linkage could precisely trace and then proposes a noncircular gear-coupled five-bar linkage as a steering mechanism. It strictly follows the Ackermann steering rule compared with the existing four-bar steering linkages. Therefore, the method can be used to design any kind of planar five-bar linkage to satisfy a specified function of curves.

Chapter 5 introduces the theoretical foundation for the workspace of planar parallel mechanism under rotational actuations and the algorithms to search the reachable and the dexterous workspace. Then singularity workspaces of such mechanisms are presented with an application example of a planar 3-RRR linkage. The method and algorithm discussed in this chapter can be used to analyze other planar linkages.

Chapter 6 addresses the statics of some special planar linkages with the example of foldable stair proposed in Chap. 3. The statics of rigid-body system of the stair mechanism is first discussed. Because of the redundant constraints, the internal forces are examined by using the second Castigliano's theorem. And then the inner forces of every link are analyzed so that the engineers can execute an identical strength design.

Chapter 7 investigates the kinetostatics of overconstraint mechanisms which are made of double planar linkages. The mechanism is redundantly constrained in structure and therefore has both merits of high structural stiffness and strength of a truss structure and motion flexibility of a mechanism. The kinetostatics is discussed with the synthesis of a deployable wing frame and the lift mechanism.

Chapter 8 discusses a structural dynamics algorithm for foldable linkages based on transfer matrix. The foldable stair and deployable wing are all typical planar linkages which are made up of a number of identical units. Therefore, the dynamics of each link between every two adjacent revolute joints is precisely expressed by the transfer matrix of Euler-Bernoulli beam with the variables of boundary conditions of the joints. In this way, the structural dynamics of the whole structure can be developed by using the least variables compared with the traditional methods. In addition, this algorithm avoids the problem of the traditional transfer-matrix method that the number of variables greatly increases when there are a huge number of cross joints within a structure.

Chapter 9 proposes a foldable tower for supporting wind turbine whose height is changeable. The advantage of the flexibility to fold and unfold can be utilized for protecting the wind turbine when destructive weather comes. The dynamic equivalent stiffness of the foldable tower is investigated as the function of deployment angle of the actuator. To expand the applications of the foldable structures, multi-planar linkages and the topology changeable linkages are discussed at the end of this chapter.

Chapter 10 introduces the design of foldable mounting equipment for solar cell panels. The equipment consists of a number of identical brackets to fix the solar cell panels. Therefore, this chapter first investigates the geometry of each foldable unit and then focuses on statics of the structure and simulation experiments. When the bracket is deployed, the frame of each unit forms the space for fastening the solar cell panels through fixtures. The merit of the structure is that the whole structure occupies less space when completely folded. This is most convenient for transporting and manufacturing because of its less occupied space when folded. Meanwhile, it is also available to form arc-shaped or any spatial curved space panels by changing the positions of the middle joints of scissor units in assembly.

I am pleased to express my gratitude for the contribution of many teachers and colleagues whose work over the years has developed and clarified the design and analysis theory for overconstraint planar linkages. This book consists of results of the insight, commitment, and hard work by Jianyi Wang, Houlin Fang, Xiang Liu, Li Ye, Zhengfang Yan, Zheng Cai, and Huichan Zhao. I have also benefitted from the insight of my coauthors Zhijing Feng, Ning Ma, and Fulei Chu. In addition, I am grateful for the inspiration of Wenxiu Lu, Ketao Zhang, Guowu Wei, Ligang Yao, Daniel Martins, Jian S. Dai, Yuefa Fang, and J. Michael McCarthy.

Finally, I gratefully acknowledge the support of the Natural Science Foundation of China, the Natural Science Foundation of Beijing, the Foundation for the Author of National Excellent Doctoral Dissertation of China, and the Program for New Century Excellent Talents in University of Education Ministry of China and the support of State Key Laboratory of Tribology in Tsinghua University.

Tsinghua University, Beijing, China Jingshan Zhao

Contents

1 Design Requirements of Planar Four-Bar Linkages 1
- 1.1 Analysis of Planar Linkages 1
  - 1.1.1 Planar Kinematic Chain 2
  - 1.1.2 Planar Linkages 2
  - 1.1.3 Workspace and Singularity 5
- 1.2 Design of Planar Linkages 5
  - 1.2.1 Position Design 6
  - 1.2.2 Kinetostatic Design 6
  - 1.2.3 Dynamics Design 7
- 1.3 Contents of the Book 10
- 1.4 Conclusions 10
- References 11

2 Design of Planar Linkages with Specified Positions 13
- 2.1 Analysis of Planar Four-Bar Linkages with Multispecified Positions 13
- 2.2 Uniform Equation of Planar Four-Bar Linkages with n Specified Positions 15
  - 2.2.1 General Requirements for Multiposition Design of a Planar Four-Bar Linkage 15
  - 2.2.2 Uniform Equation for the Design of a Planar Four-Bar Linkage with n Specified Positions 20
- 2.3 Application Examples and Discussions 22
  - 2.3.1 Design of Planar Four-Bar Linkages with Four Accurate Positions 23
  - 2.3.2 Design of Planar Four-Bar Linkages with Five Positions 24
  - 2.3.3 Design of Planar Four-Bar Linkages with More Approximate Positions 26
- 2.4 Conclusions 32
- References 33

3 **Design of Two Particular Positions for a Foldable Stair** 35
3.1 Introduction to the Foldable Structure 35
3.2 Mobility of the Foldable Stair 37
3.3 Synthesis of the Foldable Stair 44
3.4 Conclusions 48
References 48

4 **Kinematic Design of Geared Five-Bar Linkage** 51
4.1 Introduction to Vehicle Steering Mechanisms 51
4.2 Restrictions of a Four-Bar Steering Linkage 53
4.2.1 Ackermann Criteria for Turning Wheels 53
4.2.2 The Maximum Number of Points a Planar Four-Bar Linkage Could Trace 54
4.3 Ackermann-Type Steering Mechanism 55
4.3.1 Requirements for an Ackermann-Type Steering Mechanism 56
4.3.2 Synthesis of the Ackermann-Type Steering Mechanism 57
4.3.3 Sensitivity Analysis 60
4.3.4 Noncircular Profile Design 62
4.4 Parametric Optimization and Numerical Simulation 68
4.4.1 The Optimization Variables and Constraint Conditions .. 69
4.4.2 Object Function and the Optimization Analysis 69
4.5 Conclusions 71
References 74

5 **Workspace and Singularity of Planar Parallel Linkage** 77
5.1 Workspace of a Mechanism 77
5.2 Theoretical Foundation for the Workspace of Parallel Mechanism Under Rotational Actuations 78
5.3 Workspace of a Planar 3-RRR Parallel Mechanism 80
5.4 Distribution of Singularities of a Planar 3-RRR Parallel Mechanism Within Its Reachable Workspace 84
5.5 Conclusions 92
References 93

6 **Statics Analysis of Special Planar Linkages** 95
6.1 Rigid Statics of the Foldable Stair 95
6.2 Inner Force Analysis of the Foldable Stair Mechanism 102
6.3 Numerical Simulations on the Foldable Stair Mechanism 106
6.4 Conclusions 109
References 109

7 **Kinetostatics of Double Planar Linkages** 111
7.1 Introduction 112
7.2 Foldable Frame of a Morphing Wing 114
7.2.1 Primary Requirements for the Deployable Frame of a Morphing Wing 114

7.2.2 Synthesis of the Deployable Frame of the Morphing Wing 117
7.3 Aeroelastics of the Deployable Frame for the Morphing Wing 122
7.3.1 Statics of the Deployable Frame 124
7.3.2 Primary Theory of Interval Analysis 131
7.4 Lifting Guidance Mechanism for Forklift Truck 135
7.5 Dynamics Analysis for Lifting Guidance Mechanism Forklift Truck 138
7.5.1 Dynamics of a Lift Mechanism 138
7.5.2 Interval Analysis of Dynamic Stiffness 147
7.6 Conclusions 155
References 156

8 Structural Dynamics of Planar Linkages 159
8.1 Structural Dynamics Analysis of Deployable Structure 159
8.2 Mechanism of the Foldable Staircase 161
8.3 Structural Dynamics of the Staircase 162
8.3.1 Structural Dynamics of the Driving Mechanism 163
8.3.2 Structural Dynamics of the Driving Mechanism and One Staircase 174
8.4 Structural Dynamics of the Foldable Stairs 178
8.4.1 Transfer of the State Vectors Through the Scissorlike Elements 180
8.4.2 Structural Dynamics of the Foldable Stair 182
8.5 Structural Dynamics of a Unit for the Foldable Wing Frame 193
8.5.1 Structural Dynamics of a Link 194
8.5.2 Structural Dynamics of a Unit 203
8.6 Conclusions 212
References 213

9 Foldable Tower with Multi-planar Linkages 217
9.1 Foldable Structures for Wind Turbine Tower 217
9.2 Geometry Design of Foldable Towers 219
9.2.1 Fundamental Structure Analysis 219
9.2.2 Constraint and Equivalent Stiffness Analysis of the Triple Rectilinear Planar Linkage 224
9.2.3 Structures for Foldable Towers 231
9.2.4 Calculation of Foldable Ratio 233
9.3 Stiffness Analysis of the Foldable Tower 235
9.3.1 Statics of the Foldable Tower 235
9.3.2 Equivalent Stiffness of the Foldable Tower 237
9.4 Foldable Tower Consisting of Multi-planar Linkages 252
9.5 Concept Design of Topology Changeable Linkages 254
9.5.1 Synthesis of a Unit Reconfigurable Mechanism 254
9.5.2 Structure Synthesis of Completely Foldable Lift Mechanism 261

9.6 Conclusions 265
References 266

10 Extended Applications of Double Planar Linkages 267
10.1 Foldable Structures for Solar Cell Panels 267
10.2 Synthesis of Photovoltaic Mounting Bracket for Solar Panels 268
10.3 Geometry Design of the Photovoltaic Mounting Bracket 272
10.4 Stiffness Analysis of the Photovoltaic Mounting Bracket 276
10.4.1 Statics Analysis of the Photovoltaic Mounting Bracket .. 276
10.4.2 Effect of Links with Different Cross Sections 284
10.5 Numerical Simulations on Photovoltaic Mounting Bracket 287
10.5.1 Numerical Simulations of Rectangular Cross Section 288
10.5.2 Numerical Simulations of Section of the Angle Steel 288
10.5.3 Simulation Analysis 292
10.6 Conclusions 293
References 294

Chapter 1
Design Requirements of Planar Four-Bar Linkages

Generally, a mechanism is a device that transforms motion and force, while a machine typically contains one or more mechanisms that are designed to provide significant forces and transmit significant power [1]. A mechanical system, or machine, generally consists of a power source and a mechanism for the controlled use of this power [2]. Mechanical design is a complex process, requiring many skills. It is an iterative process with many interactive phases [3]. In general, mechanical design often starts from mechanism design. In the design process, there is no clear-cut dividing line between mechanisms and machines. A number of links connected in sequence via joints form a kinematic chain. It is called a mechanism when the kinematic chain has definite motion under one or more actuations. Among the mechanisms, planar linkages play a very important role in mechanical engineering. Simply the best has become the nowadays design principle. Planar four-bar linkages as the simplest closed-chain mechanism are widely used from mechanical engineering to civil engineering and aerospace engineering. This book therefore focuses on some special planar mechanisms which typically consist of a number of four-bar linkages and discusses the position design, kinematic synthesis, workspace and singularity of planar parallel linkages, stiffness of the linkage structure, kinetostatics, and structural dynamics. Possible applications of these special planar linkages in mechanical engineering and civil engineering are illustrated via examples.

1.1 Analysis of Planar Linkages

A linkage is a collection of interconnected links; the physical connection between two of which is called a joint [2]. The analysis of planar linkages mainly focuses on the motion generation, position, velocity, and acceleration. Because planar linkages are the simplest but widely used mechanism form, the objective of motion generation of a planar linkage is to calculate the mechanism parameters required to

J. Zhao et al., *Design of Special Planar Linkages*, Springer Tracts in Mechanical Engineering, DOI 10.1007/978-3-642-38448-6_1,

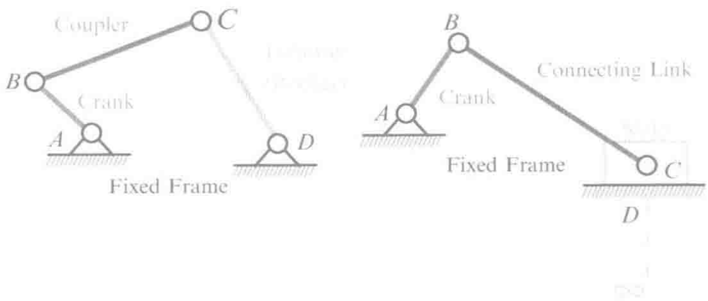

Fig. 1.1 Two typical planar kinematic chains

achieve a set of specified poses which include the positions and orientations of a link [2]; the velocity and acceleration analysis can be accomplished with geometry as well as analytical methods. This section will briefly introduce some terms that will be used in what follows.

1.1.1 Planar Kinematic Chain

A system of rigid bodies connected together in sequence by joints is called a kinematic chain. A kinematic chain is called closed if it forms a loop; otherwise it is called open if it does not form any loop. A kinematic chain is called serial kinematic chain if each link within the chain, except the first one and the last one, is only connected to two other links. A kinematic chain is a planar kinematic chain if all links of the chain are constrained to move in or parallel to a same plane. In planar kinematic chain, the mainly used joints are revolute joint (*R* joint) and prismatic joint (*P* joint). The revolute joint is often called rotary or pin joint and the prismatic joint is also called sliding joint. An *R* joint allows the two links connected to have a relative rotation. The joint used to attach a door to the frame is one of such examples. A *P* joint permits the two links connected to make a relative translation. The best example of this is the connection between a piston and a cylinder in an internal combustion engine. Figure 1.1 shows the two typical planar kinematic chains.

As shown in Fig. 1.1, a side link which can fully revolve relative to the frame is called a crank; correspondingly any link which does not fully revolve is called a rocker.

1.1.2 Planar Linkages

When a rotary actuator is assigned to any joint of the left kinematic chain shown in Fig. 1.1, it forms a four-bar linkage as there is a determined motion for the link

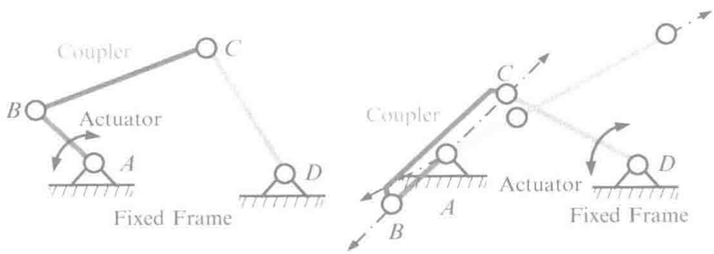

Fig. 1.2 A same planar four-bar kinematic chain but with different actuators

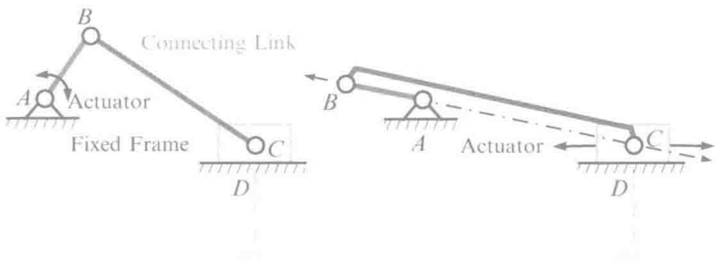

Fig. 1.3 Identical slider-crank kinematic chains with different actuators

system. In a four-bar linkage, if the shortest side link revolves and the other side link oscillates, it is called a crank-rocker mechanism. In a four-bar linkage, if both of the side links revolve, it is called a double-crank mechanism. In a four-bar linkage, if neither of the side links can make a full rotation, it is called a double-rocker mechanism.

The linkage has continuous and smooth motion when the actuator is set on revolute joint A, shown in Fig. 1.2 (left). However, it has two positions where the force or motion from rocker BC could not be transformed to the crank AB as AB and BC are collinear when the actuator is set on revolute joint D which are shown in Fig. 1.2 (right). These two positions are called dead points. Although the actuator could not transfer any force or motion to the follower at any dead point, it could be overcome through inertia of the linkages in applications.

The same is true for the second slider-crank kinematic chain. The linkage has continuous smooth motion when the actuator is set on revolute joint A, shown in Fig. 1.3 (left). However, it also has two dead points when the actuator is set on slider D, one of which is shown in Fig. 1.3 (right). The dead points can only be passed through inertia of the linkages in applications.

Therefore, avoiding or taking measures to conquer the effect of the dead points is one of the design tasks. There are some techniques introduced in many textbooks [1, 4, 5]; this book does not repeat these problems.

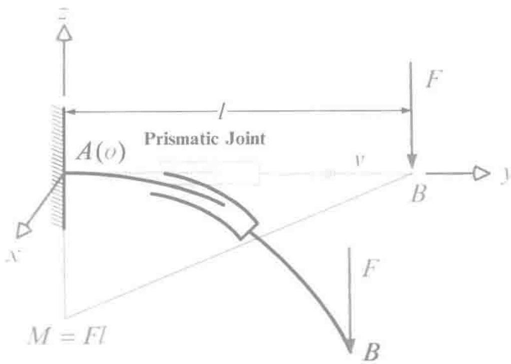

Fig. 1.4 Vertically loaded cantilever with one prismatic joint

Prismatic joint sometimes should not be used as passive joint. A passive prismatic joint in a cantilevered beam whose free end is being exerted one transversal load is discussed. As shown in Fig. 1.4, the free end part connected with the fixed wall through a prismatic joint is sliding rightward at a speed of v under a vertical load F.

The deflection of every point on the cantilever can be obtained in line with energy method:

$$z(y) = -\int_0^y \frac{1}{EI_x(y)} M_x(y) \frac{\partial M_x(y)}{\partial F} \, \mathrm{d}y \tag{1.1}$$

where the sign "–" indicates that the deflection is along the negative direction of z-axis, E represents the Young's modulus of elasticity of the cantilevered beam, $I_x(y)$ stands for the second inertia moment of the beam about the x-axis at y, $M_x(y) = F(l - y)$ which denotes the bending moment of the beam at position of y, and l is the whole distance of the cantilevered beam at the instant.

Equation (1.1) can be expanded as

$$z(y) = -\frac{Fl^3}{3EI_x} + \frac{F(l - y)^3}{3EI_x} \tag{1.2}$$

where $I_x = I_x(y)$ is assumed to be a constant for the beam.

Equation (1.2) shows that the deflection of every position on the beam depends on the parameter of I_x if all of the other parameters are the same for both segments of the beam. Therefore, the deflections of the prismatic joint on the two segments are often different in most cases. And this will surely induce a great resistance for the sliding motion. Therefore, the prismatic joint shown in Fig. 1.4 is not fit for passive connection under the transversally loaded conditions. However, there are a lot of missions in engineering that need such passive prismatic joints. So, it is urgently necessary to synthesize a simple but robust structure to fulfill this requirement. Chapter 9 will discuss this topic.

1.1.3 Workspace and Singularity

The reachable workspace of a planar linkage is the set of positions, consisting of both a reference point and the orientation about this point, that are reachable by its end effector. An ideal design is that the end effector of the linkage has unconstrained free motion within its reachable workspace. However, the workspace does have boundaries, defined in part via the extreme reach allowed by the chain, and sometimes contain the singularities. The singularity distribution and the shape and size of the workspace for a planar linkage are primary considerations in the design.

For the analysis of position, velocity, and acceleration of a linkage, there are a lot of methods, such as geometry method, complex number method, and vector method which all suit the planar linkages.

1.2 Design of Planar Linkages

Linkage design is often divided into three major categories of tasks, which are called motion generation, function generation, and point-path generation [1, 2, 5]. A good planar linkage should keep a continuous smooth movement while transmitting forces or torques. Therefore, another problem for the design of planar linkages is that the actuator could remain in continuous motion and the transmitting angle should keep within a reasonable range.

If a linkage is to be used in a continuous operation, the input crank should be able to fully rotate so that it can be driven by a rotating power source. A study of the configurations of a 4R linkage led Grashof to conclude that, for the shortest link of length s and the longest link of length l, the shortest link will fully rotate if

$$s + l \leq a + b \tag{1.3}$$

where a and b are the lengths of the other two links.

Equation (1.3) is known as Grashof's criterion, and linkages that have a rotatable crank are called Grashof linkages. There are four linkage types that satisfy Grashof's criterion shown in Fig. 1.5. If the input link is the shortest, then the crank-rocker is obtained (subfigure (a) in Fig. 1.5). If the ground link is the shortest, then both the input and output links will fully rotate relative to the ground; this is the double-crank linkage (subfigure (b) in Fig. 1.5). Finally, if the floating link is the shortest link, then the input and output links are rockers (subfigure (c) in Fig. 1.5); this is the Grashof double-rocker [2]. The planar four-bar linkage is a non-Grashof double-rocker (subfigure (d) in Fig. 1.5) if the lengths of the links do not satisfy Eq. (1.3).

For a Grashof linkage, it is also a very important requirement to keep a good transmission angle. This is a check problem after geometry design.

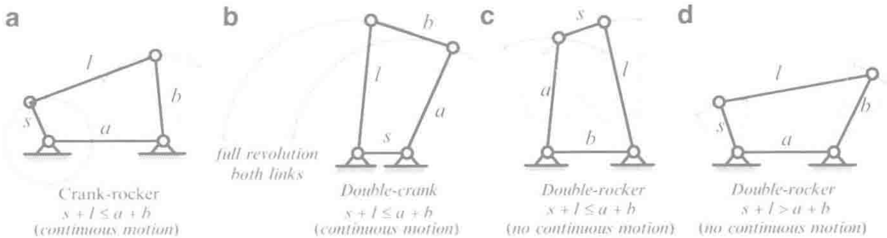

Fig. 1.5 Grashof's criterion for a planar four-bar linkage

1.2.1 Position Design

The design of any planar linkages is always available to be transformed to the design of a planar four-bar linkage. Position design, which is also called motion generation, only focuses on planar four-bar linkages. The objective of planar four-bar motion generation is to calculate the mechanism parameters required to achieve a set of prescribed coupler poses which include the positions and orientations of a link [6]. According to Burmester theory, a general planar four-bar linkage could trace five arbitrary accurate positions at most. However, the modern design often requires tracing more positions. This book proposes a uniform design theory for planar four-bar linkages. It can search the accurate results when there are such solutions for the specified n precise positions; otherwise, the best approximate solutions will be found. The merit of the method proposed in this book is that the engineers could directly obtain the planar four-bar linkages when the specified positions are known.

1.2.2 Kinetostatic Design

Kinetostatic design mainly focuses on the synthesis of kinematics and load capacity of a mechanism. For example, an ideal mechanical steering system used in the automobiles should satisfy the pure rotation criterion when turning, which is called Ackermann turning geometry. Figure 1.6 indicates that the four wheels must rotate about a pivot on the rear axle to keep a pure rotation so that the slip could be eliminated.

Figure 1.7 presents a foldable stair. This stair remains the walking conversions of human being and all the advantages of a concrete stair in civil engineering but allows the users to completely fold it after use. Therefore, both the kinematics and structural dynamics of the stair in application should be analyzed and properly designed. For the kinetostatics, the foldable stair consists of a number of identical planar four-bar linkages; therefore, it should first meet the motion requirement in folding and unfolding, and then, the whole structure should have enough strength and stiffness, but the self-weight should not be too high so that it can be easily folded.

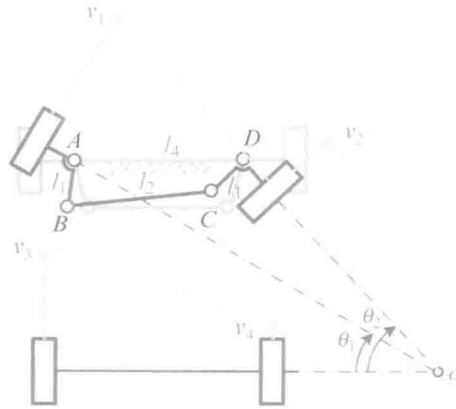

Fig. 1.6 A planar linkage satisfies the kinematic geometry of Ackermann criterion

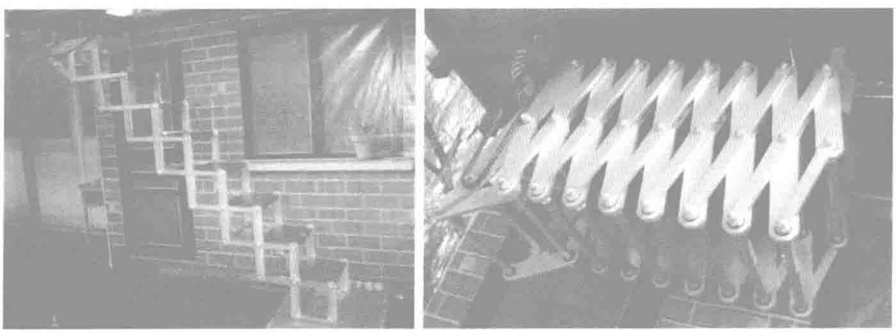

Fig. 1.7 Foldable stair. (**a**) Deployed stair. (**b**) Folded stair

Kinetostatic analysis could provide the theoretical evaluation for the structure design on kinematics and statics aspects. To reduce the self-weight of a mechanism, the theoretical strength and stiffness of the structure should be considered in structure and geometry design. After manufacturing and assembly, experiment is another means to appraise the design. Figure 1.8 shows the load test for the foldable stair. Identical strength theory was adopted in the design of the foldable stair. But some approximations were also used for the sake of manufacturing. So the kinetostatic experiment must be made after design.

What is worth pointing out is that only the kinetostatic analysis and design is not enough for the operation of a linkage.

1.2.3 Dynamics Design

For some complex planar linkages, they often form a truss structure under the working conditions. The dynamics stability and vibration mode of the structure have great effects on a mechanism. To research the dynamic effect on complex