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Economic Growth

Theory and Numerical Solution Methods

Second Edition

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Preface to the Second Edition

Central Banks and Economic Offices advising governments increasingly rely on the use of dynamic, stochastic, general equilibrium growth models based on an optimizing behavior of the different agents in the economy, to obtain predictions on the effects of alternative economic policies on the main economic aggregates. This book was written to provide an understanding of theoretical concepts underlying such models, as well as the methods to obtain the numerical solutions that are needed to discuss the effects of alternative economic policies.

The book deals with modeling, simulation, and policy evaluation at an intermediate level. It was conceived to provide a comprehensive review of exogenous and endogenous, nonmonetary and monetary stochastic growth models. In all the models considered, agents solve dynamic optimization problems under uncertainty. They can be general equilibrium models where markets clear, or incorporate frictions in the form of price rigidity under monopolistic competition, as in the models pertaining to the New Keynesian Macroeconomics. All the models presented in the book assume the existence of a representative agent, with no heterogeneity.

Models of this type generally lack an analytical solution, and it has become customary in Macroeconomics to characterize their implications through the statistical properties of their numerical solutions, like correlation matrices among the main variables, estimated regressions, or impulse response functions. The contribution of the book is twofold: to present in a single volume a variety of models that are disperse through the literature, and to provide procedures to obtain numerical solutions to the different models considered. The book does not discuss estimation methods, which have seen many recent developments, either from a frequentist or from a Bayesian approach, whose discussion would require a separate volume.

For each model, we present the structure of the economy, paying special attention to the information set of each agent, their objective functions, and the restrictions they face. Analytical equilibrium conditions are then obtained, and the approach to compute numerical solutions is carefully described. In some cases, an EXCEL spreadsheet is presented to compute a single solution realization, so that the reader can grasp the specific details of the numerical solution strategy. In all cases, a MATLAB file is used to compute an arbitrary number of solution realizations.

Numerical solutions are used to illustrate theoretical properties of each model as well as to discuss the effects of economic policy interventions. Along the different chapters, 'Numerical Exercise'-type sections are included to discuss the characteristics of a given model or the effects of a particular policy. Programs to compute numerical solutions become relatively more complex as we move from exogenous to endogenous growth models and from nonmonetary to monetary models. The reader can take the opportunity to learn how to write this type of programs. No initial background on mathematical programming is needed.

We have received many comments from readers of the first edition, most of them from academic institutions, but also from some central banks. They generally point out the increasing relevance of the topics treated in the book, as reflected in many prestigious economic journals as well as in reports from Central Banks or international economic institutions like the OECD, FMI, or the World Bank. Users tend to believe that the presentation of the models and their solution methods is simple enough to allow them to address some policy questions in the models we discuss, as well as to advance in solving more complex models. Presenting the second edition of our book, we want to thank all of our readers for their confidence and their feedback.

The second edition corrects a few typographical errors of the first edition, which seem to appear no matter how many revisions were made before publication, and improves some notation. For technical reasons, the Internet address to the programs used in the book has changed. Excel spreadsheets and Matlab programs can now be downloaded from https://www.ucm.es/fundamentos-analisis-economico2/growth-textbook.

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Preface

Integrating Growth Theory and Numerical Solutions

Dynamic, stochastic models with optimizing agents have become a standard tool for policy design and evaluation at central banks and governments around the world. They are also increasingly used as the main reference for forecasting purposes. Such models can incorporate general equilibrium assumptions, as it was the case with Modern Business Cycle Theory, or different types of market frictions, in the form of price rigidity or monopolistic competition, as in the New Keynesian Macroeconomics. These models can all be considered as special cases of models of economic growth, and the theoretical and computational methods contained in this book are a first step to get started in this area.

The book combines detailed discussions on theoretical issues on deterministic and stochastic, exogenous and endogenous growth models, together with the computational methods needed to produce numerical solutions. A detailed description of the analytical and numerical approach to solving each of the different models covered in the book is provided, and the solution algorithms are implemented in EXCEL and MATLAB files. These files are provided to illustrate theoretical results as well as to simulate the effects of economic policy interventions. Theoretical discussions covered in the book relate to issues such as the inefficiency of the competitive equilibrium, the Ricardian doctrine, dynamic Laffer curves, the welfare cost of inflation or the nominal indeterminacy of the price level, and local indeterminacy in endogenous growth models, among many others. This integration of theoretical discussions at the analytical level, whenever possible, and numerical solution methods that allow for addressing a variety of additional issues that could not possibly be discussed analytically, is a novel feature of this book.

The Audience

This textbook has been conceived for advanced undergraduate and graduate students in economics, as well as for researchers planning to work with stochastic dynamic growth models of different kinds. As described above, some of the applications included in the book may be appealing to many young researchers. Analytical discussions are presented in full detail and the reader does not need to have a specific previous background on Growth theory. The accompanying software has been written using the same notation as in the textbook, which allows for an easy understanding of how each program file addresses a particular theoretical issue. Programs increase in complexity as the book covers more complex models, but the reader can progress easily from the simpler programs in the first chapters to the more complex programs in endogenous growth models or programs for analyzing monetary economies. No initial background on programming is assumed.

The book is self-contained and it has been designed so that the student advances in the theoretical and the computational issues in parallel. The structure of program files is described in numerical exercise-type of sections, where their output is also interpreted. These sections should be considered an essential part of the learning process, since the provided program files can be easily changed following our indications so that the reader can formulate and analyze his/her own questions.

Main Ideas

Exogenous and endogenous growth models are thoroughly reviewed throughout the book, and special attention is paid to the use of these models for fiscal and monetary policy analysis. The structure of each model is first presented, and the equilibrium conditions are analytically characterized. Equilibrium conditions are interpreted in detail, with special emphasis on the role of the transversality condition in guaranteeing the stability of the implied solution. Stability is a major issue throughout the book, and a central ingredient in the construction of the solution algorithms for the different models.

Even though this is not a book on economic policy, most of the models considered incorporate a variety of distortionary and non-distortionary taxes, which allow us to address a number of policy issues. Fiscal policy in non-monetary growth economies is considered in Chaps. 2–4 (exogenous growth) and Chaps. 6 and 7 (endogenous growth). Characterizing possible dynamic Laffer effects in endogenous growth models, or the effects of fiscal policy interventions in models with human capital accumulation are some of the issues considered in this first part of the book. Chapters 8 and 9 are devoted to the analysis of monetary economies that incorporate fiscal policy variables and parameters. This allows for a detailed discussion of the interaction between fiscal and monetary policy and their coordinate design. The analysis of each model starts with the characterization of steady state and a description of the long-run effects of different policy interventions. Stability conditions are then characterized on either linear or log-linear approximations, and the general solution approach is particularized in each case to compute the numerical time series solution to the model under the specific type of policy considered. We are particularly interested in characterizing the effects of a given policy intervention along the transition between steady states. Most models are presented and analyzed in continuous and discrete time so that the reader can become familiar with both formulations. Sometimes, a given model is solved under two different approaches, so that the reader can get an even better understanding of the solution techniques.

The illustrations used in the 'Numerical Exercise'-type sections throughout the book discuss a variety of characteristics of the numerical solution to each specific model, including the evaluation of some policy experiments. Most issues considered in these sections, like the details of the numerical simulation of models with technological diffusion or Schumpeterian models under uncertainty, are presented for the first time in a textbook, having appeared so far only in research papers.

Brief Description of Contents

The use of rational expectations growth models for policy analysis is discussed in the Introductory chapter, where the need to produce numerical solutions is explained. Chapter 2 presents the neoclassical Solow–Swan growth model with constant savings, in continuous and discrete time formulations. Chapter 3 is devoted to the optimal growth model in continuous time. The existence of an optimal steady state is shown and stability conditions are characterized. The relationship between the resource allocations emerging from the benevolent planner's problem and from the competitive equilibrium mechanism is shown. The role of the government is explained, fiscal policy is introduced, and the competitive equilibrium in an economy with taxes is characterized. Finally, the Ricardian doctrine is analyzed. Chapter 4 addresses the same issues in discrete time formulation, allowing for numerical solutions to be introduced and used for policy evaluation. Deterministic and stochastic versions of the model are successively considered.

Chapter 5 is devoted to solution methods and their application to solving the optimal growth model of an economy subject to distortionary and non-distortionary taxes. The chapter covers some linear solution methods, implemented on linear and log-linear approximations: the linear-quadratic approximation, the undetermined coefficients method, the state-space approach, the method based on eigenvalue–eigenvector decompositions of the approximation to the model, and also some nonlinear methods, like the parameterized expectations model and a class of projection methods. Special emphasis is placed on the conditions needed to guarantee stability of the implied solutions.

Chapter 6 introduces some endogenous growth models, in continuous and discrete time formulations. The AK model incorporating fiscal policy instruments is taken as a basis for analysis, both in deterministic and stochastic versions. The possibility of dynamic Laffer curves is discussed. A more general model with nontrivial transition, that includes the AK model as a special case, is also presented. Chapter 7 presents additional endogenous growth models. Stochastic economies with a variety of products, technological diffusion, Schumpeterian growth, and human capital accumulation are all presented in detail and the appropriate solution methods are explained. Chapters 8 and 9 are devoted to growth in monetary economies. Chapter 8 introduces the basic Sidrauski model and discusses some modelling issues that arise in practical research in these models. The interrelation between monetary and fiscal policy in steady state is also discussed. Special attention is paid to characterize the feasible combinations of fiscal and monetary policies and to the appropriate choice of policy targets. The concept of optimal rate of inflation is introduced. The possibilities for the design of a mix of fiscal and monetary policy in economies with and without distorting taxation are discussed. Conditions for the non-neutrality of monetary policy under endogenous labor supply are examined. The chapter closes with a description of the Ramsey problem that describes the choice of optimal monetary policy. Chapter 9 characterizes the transitional dynamics in deterministic and stochastic monetary economies and presents numerical solution methods for deterministic and stochastic monetary economies. Specific details are provided depending on whether the monetary authority uses nominal interest rates or the rate of growth of money supply as a control variable for monetary policy implementation. Special attention is paid to the possibility of nominal indeterminacy arising as a consequence of the specific design followed for monetary policy. The chapter closes with a presentation of Keynesian monetary models, which are increasingly used for actual policy making. After characterizing equilibrium conditions, a numerical solution approach is discussed in detail.

A more detailed synopsis of the book is provided in Sect. 1.5.

Software

As explained above, MATLAB and EXCEL files are provided to analyze a variety of theoretical issues. EXCEL files are used to compute a single realization of the solution to a given model. That is enough in deterministic economies. There are also MATLAB programs that perform the same analysis. In stochastic economies, however, characterizing the probability distribution of a given statistic through a large number of realizations becomes impossible in a spreadsheet, and it is done in MATLAB programs. All MATLAB and EXCEL files are downloadable from our Web page: https://www.ucm.es/fundamentos-analisis-economico2/growth-textbook.

Antecedents and Acknowledgments

Over the years, we have benefited from working through textbooks on Economic Growth and Dynamic General Equilibrium Economies [Barro and Sala-i-Martin (2003), Aghion and Howitt (1999), Stokey and Lucas (1989), Blanchard and Fisher (1998), Lucas (1987), Sargent (1987), Ljunquist and Sargent (2004), Hansen y Sargent (2005), Cooley (1995), Turnovsky (2000), Walsh (1998)], who obviously should not be held accountable for any misconception that might arise in this volume.

We hope to contribute to the huge literature on Economic Growth by the integration of theoretical and computational aspects in the analysis of non-monetary and monetary models of exogenous and endogenous growth. Even though we provide

a detailed discussion of a variety of different solution approaches in Chap. 5, we have emphasized the use of variations of the Blanchard and Kahn (1980) approach, in some cases following the applications by Ireland (2004) [see also his Web page: http://www2.bc.edu/~irelandp/programs.html]. Recent textbooks on Computational Methods for Dynamic Economies [Judd (1998), Heer and Maussner (2005), Marimon and Scott eds. (1997), de Jong and Dave (2007), Miranda and Fackler (2002), McCandless (2008)] provide additional reading, in some cases with alternative approaches to model solution.

The idea that any dynamic model has time series implications that can be put to test with actual data has traditionally been a central premise in the graduate programs in Economics at University of Minnesota and has clearly influenced the conception of this book. Specially important to us were the teachings of Stephen Turnovsky, Tom Sargent, and Christopher Sims. In that context, it was easy to understand that advances in Economics should come from iterating between theoretical models and actual data and from there, the need to obtain statistical implications from any model economy.

Previous versions of parts of this book have been used in advanced undergraduate and graduate courses in Economics and Quantitative Finance at Universidad Complutense (Madrid, Spain), City University of Yokohama (Yokohama, Japan), and Keio University (Tokyo, Japan). We appreciate the patience of students working out details of previous drafts. We thank Yoshikiyo Sakai and Yatsuo Maeda for the opportunity to discuss this material while still in process. We are greatly indebted to our friends and colleagues Emilio Domínguez, Javier Pérez, and Gustavo Marrero for many useful and illuminating discussions. Finally, our deepest gratitude to our families for their understanding through the long and demanding process of producing this book.

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Contents

Introduction

1

This is a book on Growth Theory and on the numerical methods needed to fully characterize the properties of most Growth models. In this introductory chapter, we describe the main characteristics of different families of Growth models and their relevance for policy analysis, which is moving leading economic and financial institutions throughout the world to increasingly rely on their use for forecasting as well as for policy evaluation. In particular, we emphasize how the richer structure provided to Growth models by their Microeconomic foundations allows us to address a much broader set of policy issues than in more traditional structural dynamic models. The book gradually builds on by increasing the degree of generality of the models being considered, as explained below. We cover: (a) neoclassical growth under a constant savings rate, (b) optimal growth, (c) numerical solution methods, (d) endogenous growth, and (e) monetary growth. Theoretical discussions on each model are presented, with special attention to characterizing the properties of equilibrium solutions and their use for fiscal policy considerations, while a specific chapter deals with monetary policy issues. Algorithms to solve all models considered are presented, together with EXCEL spreadsheets and MATLAB programs that implement them. Results obtained by these programs are commented in "*Numerical exercise*"-type sections, where some indications are provided on possible modifications of the enclosed programs. The book has been written with the intention that it may be accessible to students without an initial background on Growth Theory or mathematical software. Maintaining the same notation used in the analytical presentations in the book should allow the reader to follow easily the structure of the programs and quickly learn how to adapt them to alternative specifications or theoretical assumptions.

Growth models incorporate very specific assumptions on the structure of preferences, technology, the sources of randomness, and the policy rules followed by the economic authority, and characterize the relationship implied by such a structure between the decisions made by the different agents at each point in time and the information they have available when making their decisions. Under uncertainty,

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agents' perceptions on the future are an explicit determinant of their actions. Growth models do not make ad-hoc assumptions on the way how expectations influence agents' decisions. Rather, the solution to the optimization problems posed for each agent leads to decision rules for the different agents that incorporate expectations of functions of future variables in a very specific manner. If expectations are assumed to be rational, expectations in the model become endogenous variables, they are fully consistent with the structure of the model, and incorporate agents' perceptions of possible future changes in policy. Doing that, these models are safe from a strong criticism made on a traditional approach to economic policy evaluation by Nobel laureate R.E. Lucas that has been very influential in the last decades. This is the reason why, as we describe below, these models are increasingly being used in the research departments of Central Banks and main international economic institutions to forecast as well as to evaluate the consequences of alternative policy choices.

The counterpart comes from the fact that the type of stochastic control problems that are integrated into a Growth model lack an analytical solution, so they need to be solved following a numerical approach, accompanied by Monte Carlo simulation in the case of stochastic Growth models. The numerical solution to the model then comes in the form of artificial time series that can be analyzed using standard statistical and econometric tools, and the results compared to those obtained in corresponding time series data from actual economies. These are the main issues introduced in this chapter, which are later gradually developed throughout the book. Section 1.1 reviews some statistical concepts using simple time series models, Sect. 1.2 considers some simple dynamic macroeconomic models in which we introduce additional concepts, as well as the fundamentals of the simulation methods that will be used through the book. Section 1.3 introduces the main characteristics of Growth models, in comparison with more traditional dynamic macroeconomic models. This section motivates the convenience to work with Growth models and describes their different types, paying attention to the way they deal with the criticism to more traditional policy evaluation. Section 1.4 explains the need to obtain numerical solutions to Growth models, their potential use, and how this approach has led to changing the type of policy questions we ask and the type of answers we get. This introductory chapter ends up with a synopsis of the book, where a reference is made to the treatment of the issues mentioned along this Introduction.

1.1 A Few Time Series Concepts

Economics is full of statements relating the dynamic properties of key variables. For instance, we may say that inflation is very persistent, that aggregate consumption and GNP experience cyclical fluctuations, or that hours worked and productivity move independently from each other. These statements have direct implications in terms of the time series representations of these variables. Sometimes we are more specific, as when we state that stock exchange returns are white noise, thereby justifying the usual belief that they are *unpredictable*. The unpredictability statement comes from the fact that the forecast of a white noise process, no matter how far into the future, is always the same. That forecast is equal to the mean of the white noise process, which would likely be assumed to be zero in the case of asset returns. If returns are logarithmic, i.e., the first difference of logged market prices, then prices themselves would follow a random walk structure. These properties cannot be argued separately from each other, since they are just two different forms of making the same statement on stock market prices. We may also say at some point that the economy is likely to repeat next year its growth performance from the previous year, which incorporates the belief that annual GNP growth follows a random walk, its best one-step ahead prediction being the last observed value. A high persistence in real wages or in inflation could be consistent with first order autoregressive models with an autoregressive parameter close to 1. We briefly review in this section some concepts regarding basic stochastic processes, of the type that are often used to represent the behavior of economic variables.

1.1.1 Some Simple Stochastic Processes

A stochastic process is a sequence of random variables indexed by time. Each of the random variables in a stochastic process, corresponding to a given time index t, has its own probability distribution. These distributions can be different, and any two of the random variables in a stochastic process may either exhibit dependence of some type or be independent from each other.

A *white noise* process is,

$$y_t = \varepsilon_t, \quad t = 1, 2, 3, \ldots$$

where ε_t, $t = 1, 2, \ldots$ is a sequence of independent, identically distributed zero-mean random variables, known as the *innovation* to the process. A white noise is sometimes defined by adding the assumption that ε_t has a Normal distribution. The mathematical expectation of a white noise is zero, and its variance is constant: $Var(y_t) = \sigma_\varepsilon^2$. More generally, we could consider a *white noise with constant*, by incorporating a constant term in the process,

$$y_t = a + \varepsilon_t, \quad t = 1, 2, 3, \ldots$$

with mathematical expectation $E(y_t) = a$, and variance: $Var(y_t) = \sigma_\varepsilon^2$.

The future value of a white noise with drift obeys,

$$y_{t+s} = a + \varepsilon_{t+s},$$

so that, if we try to forecast any future value of a white noise on the basis of the information available[1] at time t, we would have:

$$E_t y_{t+s} = a + E_t \varepsilon_{t+s} = a,$$

because of the properties of the ε_t-process. That is, the prediction of a future value of a white noise is given by the mean of the process. In that sense, a white noise process is *unpredictable*. The prediction of such process is given by the mean of the process, with no effect from previously observed values. Because of that, the history of a white noise process is irrelevant to forecast its future values. No matter how many data points we have, we will not use them to forecast a white noise.

A *random walk with drift* is a process,

$$y_t = a + y_{t-1} + \varepsilon_t, \quad t = 1, 2, 3, \ldots \tag{1.1}$$

so that its first differences are white noise. If $y_t = \ln(P_t)$ is the log of some market price, then its return $r_t = \ln(P_t) - \ln(P_{t-1})$, will be a white noise, as we already mentioned. A random walk does not have a well defined mean or variance.

In the case of a *random walk without drift*, we have,

$$y_{t+s} = y_{t+s-1} + \varepsilon_{t+s}, \quad s \geq 1$$

so that we have the sequence of forecasts:

$$E_t y_{t+1} = E_t y_t + E_t \varepsilon_{t+1} = y_t,$$
$$E_t y_{t+2} = E_t y_{t+1} + E_t \varepsilon_{t+2} = E_t y_{t+1} = y_t,$$

and the same for all future variables. In this case, the history of a random walk process is relevant to forecast its future values, but only through the last observation. All data points other than the last one are ignored when forecasting a random walk process.

First order autoregressive processes, AR(1), are of the form,

$$y_t = \rho y_{t-1} + \varepsilon_t, \ | \rho | < 1,$$

and can be represented by,

$$y_t = \sum_{s=0}^{\infty} \rho^s \varepsilon_{t-s},$$

[1]That amounts to constructing the forecast by application of the conditional expectation operator to the analytical representation of the future value being predicted, where the conditional expectation is formed with respect to the sigma algebra of events known at time t.

the right hand side having a finite variance under the assumption that $Var(\varepsilon_t) = \sigma_\varepsilon^2$ only if $|\rho| < 1$. In that case, we would have:

$$E(y_t) = 0; \;\; Var(y_t) = \frac{\sigma_\varepsilon^2}{1-\rho^2}.$$

Predictions from a first order autoregression can be obtained by,

$$E_t y_{t+1} = \rho E_t y_t + E_t \varepsilon_{t+1} = \rho y_t,$$
$$E_t y_{t+2} = E_t (\rho y_{t+1}) + E_t \varepsilon_{t+2} = \rho^2 E_t y_{t+1} = \rho^2 y_t,$$

and, in general,

$$E_t y_{t+s} = \rho^s y_t, \;\; s \geq 1$$

which is the reason to impose the constraint $| \rho |< 1$. The parameter ρ is sometimes known as the *persistence* of the process. As the previous expression shows, an increase or decrease in y_t will show up in any future y_{t+s}, although the influence of that y_t-value will gradually disappear over time, according to the value of ρ. A value of ρ close to 1 will therefore introduce high persistence in the process, the opposite being true for ρ close to zero.

The covariance between the values of the first order autoregressive process at two points in time is:

$$Cov(y_t, y_{t+s}) = \rho^s Var(y_t), \; s \gtreqless 0,$$

so that the linear correlation is:

$$Corr(y_t, y_{t+s}) = \frac{Cov(y_t, y_{t+s})}{Var(y_t)} = \rho^s,$$

which dies away at a rate of ρ. In an autoregressive process with a value of ρ close to 1, the correlation of y_t with past values will be sizeable for a number of periods.

A first order autoregressive process with constant has the representation,

$$y_t = a + \rho y_{t-1} + \varepsilon_t, \; | \rho |< 1.$$

Let us assume by now that the mathematical expectation exists and is finite. Under that assumption, $Ey_t = Ey_{t-1}$, and we have:

$$Ey_t = a + E(\rho y_{t-1}) + E\varepsilon_t = a + \rho E y_t,$$