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MODERN PHYSICS

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PREFACE

This textbook was written for an introductory course in modern physics, including relativity, quantum physics, and applications. Such a course would normally follow immediately the standard introductory course in calculus-based classical physics. Special efforts have been made to keep the text at an introductory level, thereby easing the transition from classical to modern physics.

The major goal of this text, and of the course for which it is intended, is to instill in the student an appreciation of the concepts and methods of twentieth-century physics. The text has grown from a course at Oregon State University, taken generally during the sophomore year, which serves two functions for two different audiences. (1) Physics majors, who will later take a more rigorous course in quantum physics, find an introductory modern course helpful to broaden their perspective before undertaking the rigors of the traditional junior-year studies in classical mechanics and electromagnetism. (2) Nonmajors, who likely will take no further physics, are increasingly finding need for modern physics in their disciplines; an introductory classical course is hardly sufficient for chemists, computer scientists, nuclear and electrical engineers, and so forth.

Necessary prerequisities for undertaking the text include any standard comprehensive introductory calculus-based course covering mechanics, electromagnetism, thermal physics, and optics. Calculus is used extensively in this text; no previous training in differential equations is assumed. Brief mention is made of complex variables and partial derivatives, but no previous knowledge is assumed.

For a course length of one academic quarter, Chapters 1 to 11 (covering special relativity, quantum theory, atomic, nuclear, and particle physics) are recommended; sections marked with an asterisk can be omitted without loss of continuity. For a semester-long course, additional topics from Chapters 12 to 16 may be included. Statistical physics (Chapter 12) is a necessary introduction to the remaining chapters; although most of Chapter 13 (molecular structure) can be attempted without a background in statistics (only the last section on molecular spectroscopy uses Boltzmann statistics), Chapter 14 (solids) should not be undertaken without an understanding of Fermi-Dirac statistics. Chapters 15 and 16 use statistical physics extensively.

The ordering of topics within the text is standard, with possibly one exception—the Bohr atomic theory is grouped with Rutherford scattering under introductory atomic physics, rather than under introductory quantum physics. Although the Bohr theory perhaps belongs with the latter group by historical association, it is not essential to the traditional logical development of quantum theory (Planck-Einstein-Millikan-Compton-deBroglie-Davisson-Heisenberg-Schrödinger), and it therefore is delayed until Chapter 6, which treats introductory atomic physics. (Although deBroglie did use his "matter wave" theory to derive the Sommerfeld quantization condition for stable atomic orbits, it is not at all pivotal to deBroglie's work and does not occupy a prominent place in his writings.) Instructors who feel strongly otherwise may wish to cover Chapter 6 (Rutherford-Bohr) before Chapters 4 (deBroglie waves) and 5 (Schrödinger theory); since Chapter 6 refers to wave mechanics only in the discussion of the last section and in the questions and problems, this inversion can be done without serious loss of continuity.

In order to keep the length of the text within reasonable limits (and to preserve its introductory nature) many topics have been omitted. Although a complete course in modern physics must cover these topics, the essentials of modern physics can be appreciated without them, keeping in mind that the purpose of this text is *not* to train potential relativists, quantum mechanics, or particle physicists. Such topics as 4-vectors, spacetime diagrams, parity, symmetric and antisymmetric wave functions, total angular momentum, isotopic spin, hyperchange, and nuclear models have been eliminated; although these topics appear in other "introductory" modern physics texts, their inclusion can add to the already great potential for confusion and cannot be justified at the elementary level to which this text aspires. That is, if a student "understands" atomic structure based on l and s, is it necessary to introduce j and does its introduction contribute to enlightenment? I think not. With the student already reeling from the introduction of lepton number, baryon number, and strangeness, do we create enlightenment or confusion by introducing isospin and hyperchange? Can a student be expected to gain *any* insight into nuclear structure from a few descriptive paragraphs on the shell and collective models?

On the other hand, real insight into modern physics requires such concepts as degeneracy and the quark model, and these are treated extensively, even at the elementary level.

Throughout the text, the empirical basis of modern physics is repeatedly emphasized. This emphasis takes two forms. (1) The *experimental tests* of the theories of modern physics are presented and discussed. Examples include tests of special relativity and the experiments that support quantum theory. The usual experimental evidence for atomic shell structure (radii and ionization energies) is shown, but in addition the electrical conductivity and magnetic susceptibility are discussed. In other areas the experimental tests are also emphasized. (2) *Applications* of all basic phenomena are presented, including barrier penetration (wave mechanics), lasers (atomic physics), radioactive dating, transuranic and superheavy elements, neutron activation analysis (nuclear physics), liquid helium (statistics), molecular spectroscopy (molecular physics), semiconductor devices, ferromagnetism, and electrical conductivity (solids). Other unusual references to experiments can be found throughout the text as well as in the questions and problems.

A unique feature of the present text is the inclusion of introductory material on astrophysics and cosmology (Chapters 15 and 16). This material should not be seen as a separate study, but rather as the logical culmination of the previous 14 chapters, drawing not only on relativity and quantum theory, but also on topics from atomic, nuclear, particle, molecular, statistical, and solid-state physics.

Although many of our students learn *in spite of* the efforts of instructors and authors, others clearly can benefit from their efforts. A large fraction of our students learns best by example; each chapter therefore includes many worked examples that illustrate basic techniques. At the end of each chapter are many questions and problems. The questions can be used for self-study or for class discussion; they are designed to force the student to consider the material in the chapter in a nonmathematical way and perhaps from a slightly different viewpoint. The problems are written to cover a range of abilities, including simple "plug-in" problems, from which the student gains familiarity with the important formulas, mathematical manipulations and derivations, which promote the development of mathematical skills, and advanced topics, which present unusual applications or new material and often call for special insight. The problems are *not* keyed to specific sections of the text; such keys tend to encourage compartmentalization in the mind of the student, discourage the student from adopting a synthetic approach that (along with analytic ability) is a necessary and complementary part of the scientific method, and breed laziness in both students and instructors. An important part of the learning process is recognizing the approach to use for each type of problem; when reading "light of wavelength λ falls on a metal surface. . . ." the student should recognize a photoelectric effect problem, without being told to look at Section 3.3. The problems in each chapter are ordered roughly to correspond with the ordering of subjects in the chapter.

While it has been attempted to use SI units wherever possible (nanometers, rather than angstroms, for wavelength and atomic sizes and spacings; joules, rather than calories, for heat energy), several exceptions have been made. Electron volts are used for energy as a matter of convenience. Densities of ordinary materials are expressed in grams per cubic centimeter (g/cm^3); occasionally kilograms per cubic meter (kg/m^3) are used. Conventional units are used for astrophysical quantities.

It is, I think, unfortunate that novel and text share a common classification as "books," for the former is certainly a solitary enterprise representing the outlook and creative talents of one person, while the latter is a cooperative enterprise, requiring the active participation of many people. In this sense the effort more resembles that of a motion picture, and one would therefore like to roll the credits across the screen, acknowledging the work of those who have contributed to the finished product, beginning with Producer-Director Robert McConnin and including the efficient and creative editorial, design, artistic, and production staff at John Wiley and Sons. It has been a pleasant experience to work with them, and only the fear of slighting some by omission keeps me from trying to list them all by name. Special recognition is due typist Carole Vogel, who successfully rendered marginally legible handwritten pages into a finished typescript, correcting numerous errors in the process.

I have benefitted from the comments of many reviewers who read the manuscript at various stages, and I would like especially to thank the anonymous reviewers who contributed valuable advice and suggestions, and also my colleagues at Oregon State University, particularly Peter Fontana, John Gardner, and Carl Kocher, who helped to excise some of the more embarrassingly inaccurate statements and oversimplifications. Discussions with other colleagues at Oxford University and at the Los Alamos National Laboratory, where portions of this work were conceived and executed, were also important in shaping my thoughts, and I thank those individuals for contributing their time. Needless to say, the blame for any remaining inaccuracies or misstatements rests with the author.

For well over a decade I have benefitted from close professional associations with two physicists whose influence and teachings are, perhaps in subtle ways, reflected on nearly every page of this text, and I would like to thank them for their patient instruction and for their friendship and support—Rolf Steffen, who taught me the virtues of elegance, and Bill Steyert, who taught me the equally valuable lesson that apparently simple explanations are possible only through deep understanding. The balance between theory and experiment that I have tried to achieve was accomplished primarily through the philosophical yin-and-yang inspired by these two special colleagues.

Finally, I must mention a fact that is self-evident to those who have previously written textbooks but perhaps is not so apparent to those who approach this activity, as I did, with the naïveté of inexperience—preparing a text is not an activity that can be done peripherally to other academic and research activities without long-term sacrifices by one's family. With good cheer they have endured neglect and accepted the many demands on my time, and despite all they have been enthusiastic helpers in the production process. It is to them, and to my parents for their unflagging support and encouragement, that this work is dedicated.

Corvallis, Oregon
June 1982

Kenneth S. Krane

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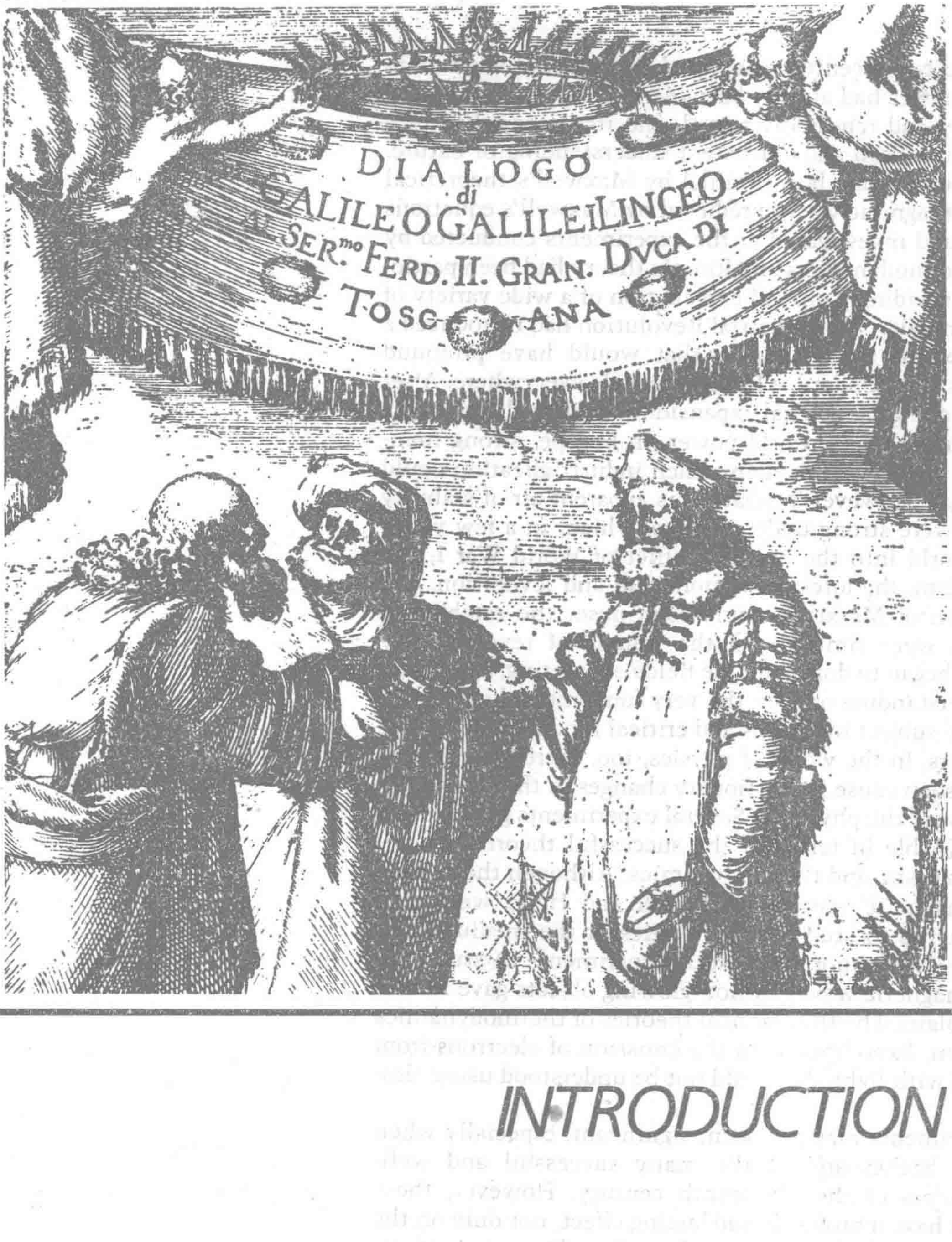

This illustration is from Galileo's Dialogue Concerning the Two Chief World Systems, *published in 1632. The dialogue was a debate between those (represented by Aristotle and Ptolemy, the figures on the left) who supported the accepted view that the Earth was fixed at the center of the universe, and those (like Copernicus, the figure on the right, and like Galileo himself) who supported the view that the Earth moved about the sun. Galileo had been prohibited by the Church from teaching such "controversial" views, but by presenting both sides of the issue he tried (unsuccessfully, as it turned out) to circumvent the Church's prohibition. Galileo hoped his readers would draw their own conclusions, based on the evidence; for this reason, Galileo is often regarded as the father of modern science.*

1

INTRODUCTION

At the end of the nineteenth century it seemed that most of what there was to know about physics had already been learned. Newton's dynamics had been carefully and repeatedly tested, and its success had provided a framework for a deep and consistent understanding of nature. Electricity and magnetism had been unified by Maxwell's theoretical work, and the electromagnetic waves predicted by Maxwell's equations had been discovered and investigated in the experiments conducted by Hertz. The laws of thermodynamics and kinetic theory had been particularly successful in providing a unified explanation of a wide variety of phenomena. More generally, the Industrial Revolution had introduced a measure of technological sophistication that would have profound influence on the lives and standard of living of people everywhere. After a period of economic and geographical expansion, the United States was beginning to assert its role as a world power. In Europe, strong monarchies had provided an environment in which industrialization could proceed at a rapid pace. However, beneath this apparent air of stability and optimism there were strong undercurrents, which, in a few years, would plunge the world into the brutal conflict of World War I; the rising tide of militarism, the forces of nationalism and revolution, and the gathering strength of Marxism would soon upset the established order. The fine arts were similarly in the middle of revolutionary change, as new ideas began to dominate the fields of painting, sculpture, and music. The understanding of even the very fundamental aspects of human behavior were subject to serious and critical modification by the Freudian psychologists. In the world of physics, too, there were undercurrents that would soon cause revolutionary changes in the apparently successful world view of the physicist. Several experiments gave results that were not explainable in terms of the successful theories of mechanics, electromagnetism, and thermodynamics. Although the properties of the electromagnetic waves of Maxwell and Hertz were well understood, experiments to study the properties of the medium that transmits those waves were not successful. Experiments to study the emission of electromagnetic waves by hot, glowing objects gave results that could not be explained by the classical theories of thermodynamics and electromagnetism. Experiments on the emission of electrons from surfaces illuminated with light also could not be understood using classical theories.

These few experiments may not seem significant, especially when viewed against the background of the many successful and well-understood experiments of the nineteenth century. However, these experiments were to have a profound and lasting effect, not only on the world of physics, but on all of science, on the political structure of our world, and on the way we view ourselves and our place in the universe. Within the short span of two decades, the results of these experiments were to lead to the special theory of relativity and to the quantum theory; soon after the revolutions inspired by these new theories came the development of atomic physics, nuclear physics, and solid-state physics, with the monumental impact that applications of research in these fields have had on our daily lives.

Modern physics

The designation *modern physics* usually refers to those developments that began with the relativity and quantum theories, and includes the applications of those modern theories to understanding the properties of the atom, of the atomic nucleus and the particles of which it is composed, of collections of atoms in molecules and solids, and, on a cosmic scale, of the origin and evolution of the universe. Our discussion of modern physics in this text touches on each of these fields. We begin with the *relativity theory*, exploring its assumptions, implications, and experimental verification. After reviewing the experiments that signaled the inadequacy of classical concepts of particles and waves, we discuss the success of the *quantum theory*, or wave mechanics, as it is sometimes known, in resolving those failures. A complete discussion of wave mechanics requires mathematical proficiency beyond the level of this text, therefore we undertake only a superficial introduction to the techniques and applications of wave mechanics. The remainder of the text deals with applications of these principles, first to the study of the structure and properties of the atom, and next to the study of the structure and properties of the atomic nucleus and the elementary particles. We then show that many of the same principles can be applied to the study of groups of atoms, both small groups in molecules and large groups in solids. Finally, we turn from the microscopic to the cosmic and discuss the applications of modern physics to the understanding of some problems of astrophysics and cosmology.

As you undertake this study, keep in mind that the details of the story of modern physics have been written only during this century, and that many of the discoveries have been made during our lifetimes. This means that the story of modern physics is not yet complete and will continue to evolve. Many of the theories that are part of the discipline of modern physics are only approximations (although sometimes very good ones). Often we find that each time we look deeper or refine our techniques we learn something new and must revise our theories to account for the new discoveries. As a result, sometimes modern physics takes on the appearance and structure of a patchwork quilt with a different explanation for each of the effects that we study. Beneath it all, however, lies the fabric of wave mechanics, holding all these diverse fields together and forming the basis on which they are constructed.

Any field of science builds on the results of previous investigations, and modern physics is no exception to this principle. The previous work in our case is classical physics and so before we begin our study of modern physics we review some of the required principles of classical physics.

1.1 REVIEW OF CLASSICAL PHYSICS

Although we will find many areas in which the concepts of modern physics differ radically from those of classical physics, we will frequently find the need to refer back to concepts of classical physics. In order to identify the more important fundamentals of classical physics and to define the notation we will use, some of the concepts of classical physics we will be using are briefly reviewed.

Mechanics An object of mass m moving with velocity v has a *kinetic energy* defined by

$$K = \tfrac{1}{2}mv^2 \tag{1.1}$$

and a *linear momentum* **p** defined by

$$\mathbf{p} = m\mathbf{v} \tag{1.2}$$

When that object collides with another object, we analyze the collision by applying two fundamental conservation laws:

Conservation laws

I. Conservation of Energy The total energy of an isolated system (on which no net external forces act) remains constant. This means (in this case) that the total energy of the two particles *before* the collision is equal to the total energy of the two particles *after* the collision.

II. Conservation of Linear Momentum The total linear momentum of an isolated system remains constant; the total linear momentum of the two particles *before* the collision is equal to the total linear momentum of the two particles *after* the collision. Since linear momentum is a vector, application of this law usually gives us two equations, one for the x components and another for the y components.

These two conservation laws are of the most basic importance to understanding and analyzing a wide variety of problems in classical physics. Problems 1 through 5 at the end of this chapter review the use of these laws.

The importance of these conservation laws is both so great and so fundamental that, even though in Chapter 2 we will learn that the special theory of relativity modifies Equations (1.1) and (1.2), the laws of conservation of energy and linear momentum will still be valid.

Another application of the principle of conservation of energy occurs when a particle moves subject to an external force F. Corresponding to that external force there is often a potential energy V, defined such that (for one-dimensional motion)

$$F = -\frac{dV}{dx} \tag{1.3}$$

The total energy E is just the sum of the potential and kinetic energies:

$$E = K + V \tag{1.4}$$

As the particle moves, K and V may change, but E remains constant. (In Chapter 2, we find that the special theory of relativity gives us a new definition of total energy.)

When an object moving with linear momentum **p** is at a displacement **r** from the origin O, the *angular momentum* **l** about the point O is defined by

$$\mathbf{l} = \mathbf{r} \times \mathbf{p} \tag{1.5}$$

There is a conservation law for angular momentum, just like that for linear momentum. In practice this has many important applications. For example, when a charged particle moves near, and is deflected by, another charged particle, the total angular momentum of the system (the two particles) remains constant if no net external torques act on the system. If the second particle is so much heavier than the first that its motion is unchanged by the influence of the first particle, the angular momentum of the first particle remains constant (because the second particle acquires no angular momentum). A similar situation occurs when a comet moves in the gravitational field of the sun. As it approaches the sun, r decreases, and so p must increase if l is to remain constant; the comet therefore accelerates as it approaches the sun.

Electricity and Magnetism The electrostatic force (Coulomb force) between two charged particles q_1 and q_2 is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \tag{1.6}$$

In the SI system of units, which we are going to use, the constant $1/4\pi\varepsilon_0$ has the value

$$\frac{1}{4\pi\varepsilon_0} = 8.988 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$$

The corresponding potential energy is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \tag{1.7}$$

In all equations derived from Equation (1.6) or (1.7) as starting points, *the quantity* $1/4\pi\varepsilon_0$ *must appear.* In some texts and reference books, you may find electrostatic quantities in which this constant does not appear. In such cases, the centimeter-gram-second (cgs) system has probably been used, in which the constant $1/4\pi\varepsilon_0$ is *defined* to be 1. You should always be very careful in making comparisons of electrostatic quantities from different references and check that the units are identical.

An electric current i causes a magnetic field $\mathbf{B}$. The case we are most concerned with is that of the circular loop of current of radius r; the magnetic field at the center of such a loop is

$$B = \frac{\mu_0 i}{2r} \tag{1.8}$$

In SI units, B is measured in teslas (one tesla (T) is one newton per ampere-meter). The constant μ_0 is

$$\mu_0 = 4\pi \times 10^{-7} \text{ N}\cdot\text{s}^2/\text{C}^2$$

Be sure to remember that i is in the direction of the conventional (*positive*) current, opposite to the actual direction of travel of the negatively charged electrons that might produce the current. The direction of $\mathbf{B}$ is chosen according to the right-hand rule: if you hold the wire in the right hand with the thumb pointing in the direction of the current, the fingers point in the direction of the magnetic field.

Magnetic moment

It is often convenient to define the *magnetic moment* $\boldsymbol{\mu}$ of a current loop:

$$|\boldsymbol{\mu}| = iA \tag{1.9}$$

where A is the geometrical area enclosed by the loop. The direction of $\boldsymbol{\mu}$ is perpendicular to the plane of the loop, according to the right-hand rule. When a current loop is placed in a uniform *external* magnetic field $\mathbf{B}_{\text{ext}}$, there is a torque $\boldsymbol{\tau}$ on the loop that tends to line up $\boldsymbol{\mu}$ with $\mathbf{B}_{\text{ext}}$:

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}_{\text{ext}} \tag{1.10}$$