TSHILIDZI MARWALA
ILYES BOULKAIBET
SONDIPON ADHIKARI

PROBABILISTIC FINITE ELEMENT MODEL UPDATING USING BAYESIAN STATISTICS

APPLICATIONS TO AERONAUTICAL AND MECHANICAL ENGINEERING

WILEY

PROBABILISTIC FINITE ELEMENT MODEL UPDATING USING BAYESIAN STATISTICS

APPLICATIONS TO AERONAUTICAL AND MECHANICAL ENGINEERING

Tshilidzi Marwala and Ilyes Boulkaibet

University of Johannesburg, South Africa

Sondipon Adhikari

Swansea University, UK

WILEY

This edition first published 2017

Registered Office
John Wiley & Sons, Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, United Kingdom

For details of our global editorial offices, for customer services and for information about how to apply for permission to reuse the copyright material in this book please see our website at www.wiley.com.

Library of Congress Cataloging-in-Publication Data

Names: Marwala, Tshilidzi, 1971– author. | Boulkaibet, Ilyes, 1981– author. | Adhikari, Sondipon, author.
Title: Probabilistic finite element model updating using Bayesian statistics: applications to aeronautical and mechanical engineering / Tshilidzi Marwala, Ilyes Boulkaibet and Sondipon Adhikari.
Description: Chichester, UK ; Hoboken, NJ : John Wiley & Sons, 2017. | Includes bibliographical references and index.
Identifiers: LCCN 2016019278| ISBN 9781119153030 (cloth) | ISBN 9781119153016 (epub)
Subjects: LCSH: Finite element method. | Bayesian statistical decision theory. | Engineering–Mathematical models.
Classification: LCC TA347.F5 M3823 2016 | DDC 620.001/51825–dc23
LC record available at https://lccn.loc.gov/2016019278

A catalogue record for this book is available from the British Library.

Cover image: Godruma/Gettyimages

Set in 10/12pt Times by SPi Global, Pondicherry, India
Printed and bound in Malaysia by Vivar Printing Sdn Bhd

10 9 8 7 6 5 4 3 2 1

Acknowledgements

We would like to thank the University of Johannesburg and the University of Swansea for contributing towards the writing of this book. We also would like to thank Michael Friswell, Linda Mthembu, Niel Joubert and Ishmael Msiza for contributing towards the writing of this book.

We dedicate this book to the schools that gave us the foundation to always seek excellence in everything we do: the University of Cambridge and the University of Johannesburg.

Tshilidzi Marwala, PhD
Johannesburg
1 February 2016

Ilyes Boulkaibet, PhD
Johannesburg
1 February 2016

Sondipon Adhikari, PhD
Swansea
1 February 2016

Nomenclature

AI	Artificial intelligence
AIC	Akaike information criterion
APEPCS	Adaptive pruned-enriched population control scheme
AR	Acceptance rate
BFGS	Quasi-Newton Broyden–Fletcher–Goldfarb–Shanno
BIC	Bayesian information criterion
CG	Conjugate gradient
c.o.v.	Coefficient of variation
DIC	Deviance information criterion
DOF	Degree of freedom
DWIS	Dynamically weighted importance sampling
FEM	Finite element model
FRF	Frequency response function
GA	Genetic algorithm
GS	Gibbs sampling
HMC	Hybrid Monte Carlo
MC	Markov chain
MCDWIS	Monte Carlo dynamically weighted importance sampling
MD	Molecular dynamics
MCMC	Markov chain Monte Carlo
M-H	Metropolis–Hastings
ML	Maximum likelihood
MAP	Maximum a posteriori
NS	Nested sampling
PDF	Probability distribution function
PSO	Particle swarm optimisation
SA	Simulated annealing
SHMC	Shadow hybrid Monte Carlo
S2HMC	Separable shadow hybrid Monte Carlo

SS	Slice sampling
VV	Velocity verlet
N	Number of degrees of freedom
$\boldsymbol{Z_X}$	Experimental data vector
$\boldsymbol{Z_i}$	Analytical data vector
$\boldsymbol{\theta}$	Uncertain parameter vector
$Dev(\boldsymbol{\theta})$	Deviance of $\boldsymbol{\theta}$
P_D	Posterior mean deviance parameter
$\boldsymbol{S}$	Structure's sensitivity matrix
J	Objective function
$\boldsymbol{Z}$	Evidence
$\ddot{x}$	Acceleration
$\boldsymbol{W}$	Weighting matrix
$\boldsymbol{H}$	Hessian matrix
$\boldsymbol{I}$	Unit matrix
η	Step size used by the conjugate gradient technique
$\boldsymbol{V}$	Variance matrix
$\boldsymbol{\Omega}$	Diagonal matrix with diagonal elements of the natural frequencies
x_i	Chromosome vector or position vector
p_i	Best position
v_i	Velocity
d	Dimension of the updated vector
$\mathbb{R}$	One-dimensional real domain
$\mathbb{R}^n$	n-dimensional real domain
$\mathbb{R}^{m\times n}$	$m\times n$-dimensional real domain
$\boldsymbol{T}$	Transformation matrix
$\boldsymbol{V}_{\theta_j}$	Covariance matrix of the updated vector $\boldsymbol{\theta}$ at the jth iteration
$\boldsymbol{V}_{Z_X}$	Covariance of the measured data
P	Probability function
$\mathcal{D}$	Experimental model data
$P(\boldsymbol{\theta}\|\mathcal{D})$	The posterior probability distribution function
$q(\cdot\|\cdot)$	Proposed probability distribution function
$T_n(\cdot\|\cdot)$	Transition matrix
$\mathcal{N}(\mu, \sigma)$	Normal distribution with mean μ and variance σ
$P(\cdot\|\cdot)$	Joint density
μ_f	Expectation value of the function f
f_i^m	ith measured natural frequency
ω_i^m	ith measured circular natural frequency
N_m	Number of measured modes
f_i	ith analytical frequency obtained from the finite element model
j	Imaginary unit of a complex number
$\|\|\boldsymbol{\Lambda}\|\|$	Euclidean norm of $\boldsymbol{\Lambda}$
λ	Lagrange multiplier
K	Bayes factor
$\boldsymbol{E}_i$	Error vector

$E(\cdot)$	Mean value
$E(\mathbf{z}\mathbf{z}^T)$	Variance matrix of $\mathbf{z}$
R_t	Normalisation constant ratio
$\boldsymbol{X}^m$	The Fourier-transformed displacement
$\boldsymbol{F}^m$	Force matrix
W	Kinetic energy
V	Potential energy
∇V	Gradient of V
H	Hamiltonian function
$H_{[2k]}$	Shadow Hamiltonian function of order $2k$
$\mathbf{p}$	Momentum vector
$\nabla(\cdot)$	Gradient
K_B	Boltzmann constant
T	Temperature
δt	Time step
$\{\cdot,\cdot\}$	Poisson bracket of two functions

Contents

1

Introduction to Finite Element Model Updating

1.1 Introduction

Finite element model updating methods are intended to correct and improve a numerical model to match the dynamic behaviour of real structures (Marwala, 2010). Modern computers, with their ability to process large matrices at high speed, have facilitated the formulation of many large and complicated numerical models, including the boundary element method, the finite difference method and the finite element models. This book deals with the finite element model that was first applied in solving complex elasticity and structural analysis problems in aeronautical, mechanical and civil engineering. Finite element modelling was proposed by Hrennikoff (1941) and Courant and Robbins (1941). Courant applied the Ritz technique and variational calculus to solve vibration problems in structures (Hastings *et al.*, 1985). Despite the fact that the approaches used by these researchers were different from conventional formulations, some important lessons are still relevant. These differences include mesh discretisation into elements (Babuska *et al.*, 2004).

The Cooley–Turkey algorithms, which are used to speedily obtain Fourier transformations, have facilitated the development of complex techniques in vibration and experimental modal analysis. Conversely, the finite element model ordinarily predicts results that are different from the results obtained from experimental investigation. Among reasons for the discrepancy between finite element model prediction and experimentally measured data are as the following (Friswell and Mottershead, 1995; Marwala, 2010; Dhandole and Modak, 2011):

- model structure errors resulting from the difficulty in modelling damping and complex shapes such as joints, welds and edges;
- model order errors resulting from the difficulty in modelling non-linearity and often assuming linearity;

Probabilistic Finite Element Model Updating Using Bayesian Statistics: Applications to Aeronautical and Mechanical Engineering, First Edition. Tshilidzi Marwala, Ilyes Boulkaibet and Sondipon Adhikari.

- model parameter errors resulting in difficulty in identifying the correct material properties;
- errors in measurements and signal processing.

In finite element model updating, it is assumed that the measurements are correct within certain limits of uncertainty and, for that reason, a finite element model under consideration will need to be updated to better reflect the measured data. Additionally, finite element model updating assumes that the difficulty in modelling joints and other complicated boundary conditions can be compensated for by adjusting the material properties of the relevant elements. In this book, it is also assumed that a finite element model is linear and that damping is sufficiently low not to warrant complex modelling (Mottershead and Friswell, 1993; Friswell and Mottershead, 1995). Using data from experimental measurements, the initial finite element model is updated by correcting uncertain parameters so that the model is close to the measured data. Alternatively, finite element model updating is an inverse problem and the goal is to identify the system that generated the measured data (Brincker *et al.*, 2001; Dhandole and Modak, 2010; Zhang *et al.*, 2011; Boulkaibet, 2014; Fuellekrug *et al.*, 2008; Cheung and Beck, 2009; Mottershead *et al.*, 2000).

There are two main approaches to finite element model updating, namely, maximum likelihood and Bayesian methods (Marwala, 2010; Mottershead *et al.*, 2011). In this book, we apply a Bayesian approach to finite element model updating.

1.2 Finite Element Modelling

Finite element models have been applied to aerospace, electrical, civil and mechanical engineering in designing and developing products such as aircraft wings and turbo-machinery. Some of the applications of finite element modelling are (Marwala, 2010): thermal problems, electromagnetic problems, fluid problems and structural modelling. Finite element modelling typically entails choosing elements and basis functions (Chandrupatla and Belegudu, 2002; Marwala, 2010). Generally, there are two types of finite element analysis that are used: two-dimensional and three-dimensional modelling (Solin *et al.*, 2004; Marwala, 2010).

Two-dimensional modelling is simple and computationally efficient. Three-dimensional modelling, on the other hand, is more accurate, though computationally expensive. Finite element analysis can be formulated in a linear or non-linear fashion. Linear formulation is simple and usually does not consider plastic deformation, which non-linear formulation does consider. This book only deals with linear finite element modelling, in the form of a second-order ordinary differential equation of relations between mass, damping and stiffness matrices. A finite element model has *nodes*, with a grid called a *mesh*, as shown in Figure 1.1 (Marwala, 2001). The mesh has material and structural properties with particular loading and boundary conditions. Figure 1.1 shows the dynamics of a cylinder, and the mode shape of the first natural frequency occurring at 433 Hz.

These loaded nodes are assigned a specific density all over the material, in accordance with the expected stress levels of that area (Baran, 1988). Sections which undergo more stress will then have a higher node density than those which experience less or no stress. Points of stress concentration may have fracture points of previously tested materials, joints, welds and high-stress areas. The mesh may be imagined as a spider's web so that, from each node, a mesh

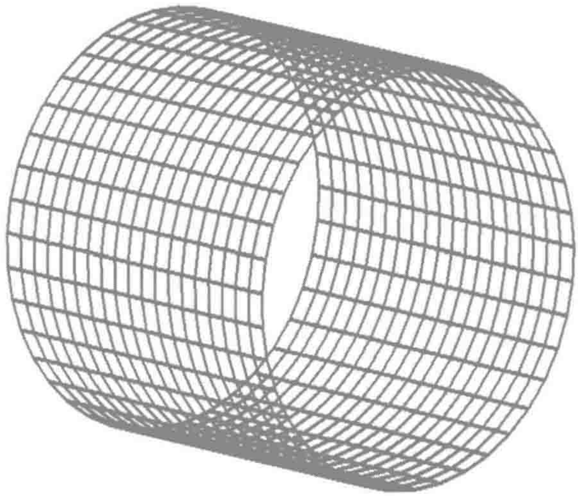

Figure 1.1 A finite element model of a cylindrical shell

element extends to each of the neighbouring nodes. This web of vectors has the material properties of the object, resulting in a study of many elements.

On implementing finite element modelling, a choice of elements needs to be made and these include beam, plate, shell elements or solid elements. A question that needs to be answered when applying finite element analysis is whether the material is isotropic (identical throughout the material), orthotropic (only identical at 90°) or anisotropic (different throughout the material) (Irons and Shrive, 1983; Zienkiewicz, 1986; Marwala, 2010).

Finite element analysis has been applied to model the following problems (Zienkiewicz, 1986; Marwala, 2010):

- vibration analysis for testing a structure for random vibrations, impact and shock;
- fatigue analysis to approximate the life cycle of a material or a structure due to cyclical loading;
- heat transfer analysis to model conductivity or thermal fluid dynamics of the material or structure.

Hlilou *et al.* (2009) successfully applied finite element analysis in softening material behaviour, while Zhang and Teo (2008) successfully applied it in the treatment of a lumbar degenerative disc disease. White *et al.* (2008) successfully applied finite element analysis for shallow-water modelling, while Pepper and Wang (2007) successfully applied it in wind energy assessment of renewable energy in Nevada. Miao *et al.* (2009) successfully applied a three-dimensional finite element analysis model in the simulation of shot peening. Bürg and Nazarov (2015) successfully applied goal-oriented adaptive finite element methods in elliptic problems, while Amini *et al.* (2015) successfully applied finite element modelling in functionally graded piezoelectric harvesters. Haldar *et al.* (2015) successfully applied finite element modelling in the study of the flexural behaviour of singly curved sandwich composite structures, while Millar and Mora (2015) successfully applied finite element methods to study the buckling in simply supported Kirchhoff plates. Jung *et al.* (2015) successfully used finite element models and computed tomography to estimate cross-sectional constants of composite blades, while Evans and Miller (2015) successfully applied a finite element model to predict the failure of pressure vessels. Other successful applications of finite element analysis are in the areas of metal powder compaction processing (Rahman *et al.*, 2009), ferroelectric materials (Schrade *et al.*, 2007), rock mechanics (Chen *et al.*, 2009), orthopaedics (Easley *et al.*, 2007), carbon nanotubes (Zuberi and Esat, 2015), nuclear reactors (Wadsworth *et al.*, 2015) and elastic wave propagation (Gao *et al.*, 2015; Gravenkamp *et al.*, 2015).

1.3 Vibration Analysis

An important aspect to consider when implementing finite element analysis is the kind of data that the model is supposed to predict. It can predict data in many domains, such as the time, modal, frequency and time–frequency domains (Marwala, 2001, 2010). This book is concerned with constructing finite element models to predict measured data more accurately. Ideally, a finite element model is supposed to predict measured data irrespective of the domain in which the data are presented. However, this is not necessarily the case because models updated in the time domain will not necessarily predict data in the modal domain as accurately as they will for data in the time domain. To deal with this issue, Marwala and Heyns (1998) used data in the modal and frequency domains simultaneously to update the finite element model in a multi-criteria optimisation fashion. Again, whichever domain is used, the updated model performs less well on data in a different domain than those used in the updating process. In this book, we use data in the modal domain. Raw data are measured in the time domain and Fourier analysis techniques transform the data into the frequency domain. Modal analysis is applied to transform the data from the frequency domain to the modal domain. All of these domains include similar information, but each domain reveals different data representations.

1.3.1 Modal Domain Data

The modal domain expresses data as natural frequencies, damping ratios and mode shapes. The technique used for extracting the modal properties is a process called *modal analysis* (Ewins, 1995). Natural frequencies are basic characteristics of a system and can be extracted by exciting the structure and analysing the vibration response. Cawley and Adams (1979) used changes in the natural frequencies to identify damage in composite materials. Farrar *et al.* (1994) successfully used the shifts in natural frequencies to identify damage on an I-40 bridge. Other successful applications of natural frequencies include damage detection in tabular steel offshore platforms (Messina *et al.*, 1996, 1998), spot welding (Wang *et al.*, 2008) and beam-like structures (Zhong and Oyadiji, 2008; Zhong *et al.*, 2008).

A mode shape represents the curvature of a system vibrating at a given mode and a particular natural frequency. West (1982) successfully applied the modal assurance criterion for damage on a Space Shuttle orbiter body flap, while Kim *et al.* (1992) successfully used the coordinate modal assurance criterion of Lieven and Ewins (1988) for damage detection in structures. Further applications of mode shapes include composite laminated plates (Araújo dos Santos *et al.*, 2006; Qiao *et al.*, 2007), linear structures (Fang and Perera, 2009), beam-type structures (Qiao and Cao, 2008; Sazonov and Klinkhachorn, 2005), optical sensor configuration (Chang and Pakzad, 2015), multishell quantum dots (Vanmaekelbergh *et al.*, 2015) and creep characterisation (Hao *et al.*, 2015).

1.3.2 Frequency Domain Data

The measured excitation and response of the structure are converted into the frequency domain using Fourier transforms (Ewins, 1995; Maia and Silva, 1997), and from these the *frequency response function* is extracted. Frequency response functions have, in general, been used to identify faults (Sestieri and D'Ambrogio, 1989; Faverjon and Sinou, 2009). D'Ambrogio and Zobel (1994) used frequency response functions to identify the presence of faults in a truss structure, while Imregun *et al.* (1995) used frequency response functions for damage detection. Lyon (1995) and Schultz *et al.* (1996) used measured frequency response functions for structural diagnostics. Other direct applications of the frequency response functions include the work of Shone *et al.* (2009), Ni *et al.* (2006), X. Liu *et al.* (2009), White *et al.* (2009) and Todorovska and Trifunac (2008). Additional applications include missing-data estimation (Ugryumova *et al.*, 2015), identification of a non-commensurable fractional transfer (Valério and Tejado, 2015), as well as damage detection (Link and Zimmerman, 2015).

1.4 Finite Element Model Updating

In real life, it turns out that the predictions of the finite element model are quite different from the measurements. As an example, for a finite element model of a simply suspended beam, the differences between the model-predicticted natural frequencies and the measured frequencies are shown in Table 1.1 (Marwala and Sibisi, 2005; Marwala, 2010). These results are for a fairly easy structure to model, and they demonstrate that the finite element model's data are different from the measured data. Finite element model updating has been studied quite extensively (Friswell and Mottershead, 1995; Mottershead and Friswell, 1993; Maia and Silva, 1997; Marwala, 2010). There are three approaches used in finite element model updating: direct methods, iterative deterministic and uncertainty quantification methods. Direct approaches are computationally inexpensive, but reproduce modal properties that are physically unrealistic.

Although the finite element model can predict measured quantities, the updated model is limited in that it loses the connectivity of nodes, results in populated matrices and in loss of

Table 1.1 Comparison of finite element model and real measurements

Mode number	Finite element frequencies (Hz)	Measured frequencies (Hz)
1	42.30	41.50
2	117.0	114.5
3	227.3	224.5
4	376.9	371.6