DISCRETE-TIME SIGNAL PROCESSING

Alan V. Oppenheim · Ronald W. Schafer

PRENTICE HALL SIGNAL PROCESSING SERIES
ALAN V. OPPENHEIM, SERIES EDITOR

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Discrete-time signal processing continues to be a dynamic and rapidly growing field with a wide range of applications including speech and data communication, acoustics, radar, sonar, seismology, remote sensing, instrumentation, consumer electronics, and many others. *Discrete-Time Signal Processing,* by the authors of the classic text *Digital Signal Processing* (Prentice Hall, 1975), is a completely up-to-date, thorough, and coherent treatment of the fundamentals of this field. Considerable emphasis is placed on illustrative examples and intuitive interpretation. The authors include more than 400 carefully prepared problems to help the reader develop a thorough foundation in the use of this material.

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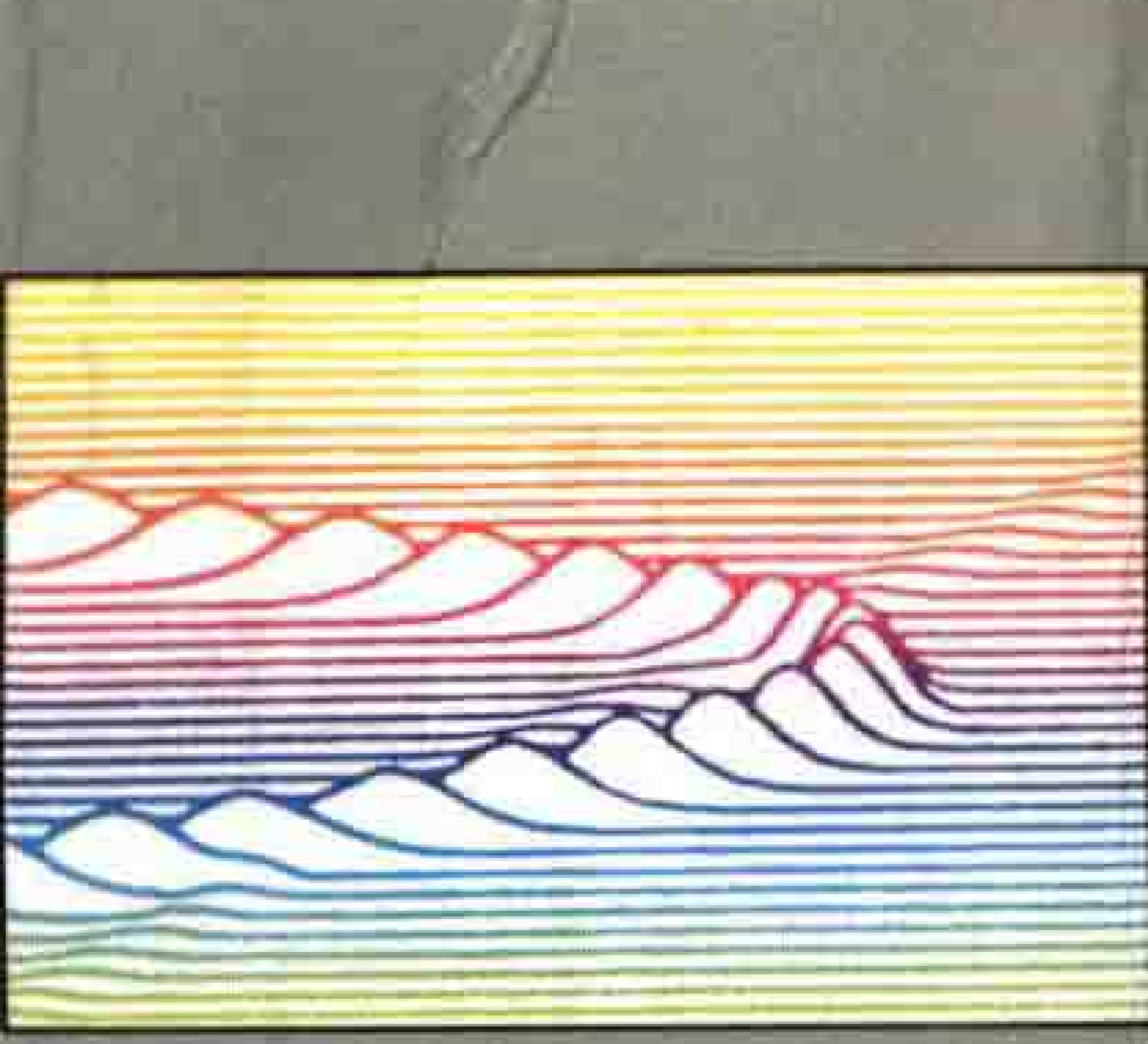

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To Phyllis, Jason, and Justine

To Dorothy, Bill, Kate, and Barbara
and in memory of John

Alan V. Oppenheim received the S.B. and S.M. degrees in 1961 and the Sc.D. degree in 1964, all in electrical engineering, from the Massachusetts Institute of Technology. In 1964 he joined the faculty at MIT, where he is currently Professor of Electrical Engineering and Computer Science. Since 1967 he has also been affiliated with MIT Lincoln Laboratory and since 1977 with Woods Hole Oceanographic Institution. His research interests are in the general area of signal processing and its applications to speech, image, and seismic data processing. He is coauthor of the widely used textbooks *Digital Signal Processing* and *Signals and Systems.* He is also editor of several advanced books on signal processing.

Dr. Oppenheim is a member of the National Academy of Engineering, a Fellow of the IEEE, and a member of Sigma Xi and Eta Kappa Nu. He has been a Guggenheim Fellow and a Sackler Fellow. He has also received a number of awards for outstanding research and teaching including the IEEE Education Medal, the IEEE Centennial Award, and the Society Award and Technical Achievement Award of the IEEE Society on Acoustics, Speech, and Signal Processing.

Ronald W. Schafer received the B.S.E.E. and M.S.E.E. degrees from the University of Nebraska in 1961 and 1962, respectively, and the Ph.D. degree from the Massachusetts Institute of Technology in 1968. From 1968 to 1974 he was a member of the Acoustics Research Department at Bell Laboratories, Murray Hill, New Jersey, and since 1974 he has been on the faculty of the Georgia Institute of Technology as Regents' Professor and holder of the John O. McCarty/Audichron Chair in the School of Electrical Engineering. His research interests are in discrete-time signal processing and its application to problems in speech communication and image analysis. He is coauthor of *Digital Signal Processing* and *Digital Processing of Speech Signals.*

Dr. Schafer is a Fellow of the IEEE and the Acoustical Society of America and a member of Sigma Xi, Eta Kappa Nu, and Phi Kappa Phi. He was awarded the Achievement Award and the Society Award of the IEEE Society on Acoustics, Speech, and Signal Processing, the IEEE Region III Outstanding Engineer Award, and the IEEE Centennial Award, and he shared the 1980 Emanuel R. Piore Award with L. R. Rabiner for their work in speech processing. He has received several awards for teaching, including the 1985 Class of 1934 Distinguished Professor Award at Georgia Tech.

Preface

This text has its origins in our initial thought several years ago of revising and updating our first text, *Digital Signal Processing*, which was published in 1975. The vitality of that book attests to the tremendous interest in and influence of signal processing, and it is clear that the field continues to grow in importance as the available technologies for implementing signal processing continue to develop. Shortly after beginning the revision, we realized that it would be more appropriate to develop a new textbook strongly based on our first one and at the same time continue to have the original text also available.

The title *Discrete-Time Signal Processing* was chosen for this new book for several reasons. In the mid-1960s digital signal processing emerged as a new branch of signal processing, driven by the potential and feasibility of implementing real-time signal processing using digital computers. The term *digital signal processing* refers specifically to processing based on digital technology, which inherently involves both time and amplitude quantization. However, the principal focus in essentially all texts on digital signal processing is on time quantization, i.e., the discrete-time nature of the signals. Furthermore, many signal processing technologies (e.g., charge transport devices and switched capacitor filters) are discrete time but not digital; i.e., signal values are clocked so that time is quantized but the signal amplitudes are represented in analog form.

In the mid-1970s, when the original text was published, courses on digital and discrete-time signal processing were available in only a few schools, and only at the graduate level. Now the basic principles are often taught at the undergraduate level, sometimes even as part of a first course on linear systems, or at a somewhat more advanced level in third-year, fourth-year, or beginning graduate subjects. Much of our thinking in planning this new text is in recognition of the importance of this material at the undergraduate level. In particular, we have considerably expanded the treatment of a number of topics, including linear systems, sampling, multirate signal processing, applications, and spectral analysis. In addition, a large number of examples are included to emphasize and illustrate important concepts. We have also removed and condensed some topics. This new text contains a rich set of more than 400 problems, and a solutions manual is available for course instructors.

It is assumed that the reader has a background of advanced calculus, including an introduction to complex variables, and an exposure to linear system theory for continuous-time signals, including Laplace and Fourier transforms, as taught in most undergraduate electrical and mechanical engineering curricula. With this background, the book is self-contained. In particular, no prior experience with discrete-time signals, z-transforms, discrete Fourier transforms, and the like is assumed. In later sections of some chapters, some topics such as quantization noise are included that assume a basic background in stochastic signals. A brief review of the background for these sections is included as Appendix A.

It has become common in many signal processing courses to include exercises to be done on a computer, and many of the homework problems in this book are easily turned into problems to be solved with the aid of a computer. With one or two exceptions, we have purposely avoided providing software to implement algorithms described in this book, for a variety of reasons. Foremost among them is that there are readily available a variety of inexpensive signal processing software packages for demonstrating and implementing signal processing on any of the popular personal computers and workstations. These packages are well documented and have excellent technical support, and many of them have excellent user interfaces that make them easily accessible to students. Furthermore, they are in a constant state of evolution, which strongly suggests that available software for classroom use should be constantly reviewed and updated. While we may have current favorites, these will no doubt change over time, and consequently our preference is for computer-based exercises to be independent of any specific software system or vendor. We have on occasion in this text illustrated points through the use of FORTRAN programs. We chose FORTRAN specifically for its general readability rather than with the implication that these specific programs are recommended for use in research or practical applications. Even though FORTRAN is often inefficient as an implementation of an algorithm, it can be a convenient language for communicating the structure of an algorithm.

The material in this book is organized in a way that provides considerable flexibility in its use at both the undergraduate and graduate level. A typical one-semester undergraduate elective might cover in depth Chapter 2, Sections 2.0–2.9; Chapter 3, Sections 3.0–3.6; Chapter 4; Chapter 5, Sections 5.0–5.3; Chapter 6, Sections 6.0–6.5; Chapter 7, Sections 7.0–7.2 and 7.4–7.5 and a brief overview of Sections 7.6–7.7. If students have studied discrete-time signals and systems in a general signals and systems course, it may be possible to move more quickly through the material of Chapters 2, 3, and 4, thus freeing time for covering Chapter 8. A first-year graduate course could augment the above topics with the remaining topics in Chapter 5, a brief exposure to the practical considerations in Section 3.7 and Sections 6.7–6.10, a discussion of optimal FIR filters as incorporated in Sections 7.6 and 7.7, and a thorough treatment of the discrete Fourier transform (Chapter 8) and its computation using the FFT (Chapter 9). The discussion of the DFT can be effectively augmented with many of the examples in Chapter 11. In a two-semester graduate course, the entire text together with a number of current advanced topics can be covered.

In Chapter 2, we introduce the basic class of discrete-time signals and systems and define basic system properties such as linearity, time invariance, stability, and causality. The primary focus of the book is on linear time-invariant systems because of the rich set of tools available for designing and analyzing this class of systems. In particular, in Chapter 2 we develop the time-domain representation of linear time-invariant systems through the convolution sum and introduce the class of linear time-invariant systems represented by linear constant-coefficient difference equations. In Chapter 6, we develop this class of systems in considerably more detail. Also in Chapter 2, we introduce the frequency-domain representation of signals and systems through the Fourier transform. The primary focus in Chapter 2 is on the representation of sequences in terms of the Fourier transform, i.e., as a linear combination of complex exponentials, and the development of the basic properties of the Fourier transform. We defer until Chapter 5 a detailed discussion of the analysis of linear time-invariant systems using the Fourier transform.

In Chapter 3, we carry out a detailed discussion of the relationship between continuous-time and discrete-time signals when the discrete-time signals are obtained through periodic sampling of continuous-time signals. This includes a development of the Nyquist sampling theorem. In addition, we discuss upsampling and downsampling of discrete-time signals, as used, for example, in multirate signal processing systems and for sampling rate conversion. The chapter concludes with a discussion of some of the practical issues encountered in conversion from continuous time to discrete time including prefiltering to avoid aliasing and modeling the effects of amplitude quantization when the discrete-time signals are represented digitally.

In Chapter 4, we develop the z-transform as a generalization of the Fourier transform. In Chapter 5, we carry out an extensive and detailed discussion of the use of the Fourier transform and the z-transform for the representation and analysis of linear time-invariant systems. In particular, in Chapter 5 we define the class of ideal, frequency-selective filters and develop the system function and pole-zero representation for systems described by linear constant-coefficient difference equations, a class of systems whose implementation is considered in detail in Chapter 6. Also in Chapter 5, we define and discuss group delay, phase response and phase distortion, and the relationships between the magnitude response and the phase response of systems, including a discussion of minimum-phase, allpass, and generalized linear phase systems.

In Chapter 6, we focus specifically on systems described by linear constant-coefficient difference equations and develop their representation in terms of block diagrams and linear signal flow graphs. Much of this chapter is concerned with developing a variety of the important system structures and comparing some of their properties. The importance of this discussion and the variety of filter structures relate to the fact that in a practical implementation of a discrete-time system, the effects of coefficient inaccuracies and arithmetic error can be very dependent on the specific structure used. While these basic issues are similar whether the technology used for implementation is digital or discrete-time analog, we illustrate them in this chapter in the context of a digital implementation through a discussion of the effects of coefficient quantization and arithmetic roundoff noise for digital filters.

While Chapter 6 is concerned with the representation and implementation of linear constant-coefficient difference equations, Chapter 7 is a discussion of the procedures for obtaining the coefficients of this class of difference equations to approximate a desired system response. The design techniques separate into those used for infinite impulse response (IIR) filters and those used for finite impulse response (FIR) filters.

In continuous-time linear system theory, the Fourier transform is primarily an analytical tool for representing signals and systems. In contrast, in the discrete-time case, many signal processing systems and algorithms involve the explicit computation of the Fourier transform. While the Fourier transform itself cannot be computed, a sampled version of it, the discrete Fourier transform (DFT), can be computed, and for finite-length signals the DFT is a complete Fourier representation of the signal. In Chapter 8, the discrete Fourier transform is introduced and its properties and relationship to the discrete-time Fourier transform are developed in detail. In Chapter 9, the rich and important variety of algorithms for computing or generating the discrete Fourier transform is introduced and discussed, including the Goertzel algorithm, the fast Fourier transform (FFT) algorithms, and the chirp transform.

In Chapter 10, we introduce the discrete Hilbert transform. This transform arises in a variety of practical applications, including inverse filtering, complex representations for real bandpass signals, single-sideband modulation techniques, and many others. It also has particular significance for the class of signal processing techniques referred to as cepstral analysis and homomorphic signal processing, as discussed in Chapter 12.

With the background developed in the earlier chapters and particularly Chapters 2, 3, 5, and 8, we focus in Chapter 11 on Fourier analysis of signals using the discrete Fourier transform. Without a careful understanding of the issues involved and the relationship between the DFT and the Fourier transform, using the DFT for practical signal analysis can often lead to confusions and misinterpretations. We address a number of these issues in Chapter 11. We also consider in some detail the Fourier analysis of signals with time-varying characteristics by means of the time-dependent Fourier transform.

In Chapter 12, we introduce a class of signal processing techniques referred to as cepstral analysis and homomorphic signal processing. This class of techniques, although nonlinear, is based on a generalization of the linear techniques that were the focus of the earlier chapters of the book.

In writing this book, we have been fortunate to receive valuable assistance, suggestions, and support from numerous colleagues, students, and friends. Over the years, a number of our colleagues at MIT and Georgia Institute of Technology have taught the material with us, and we have benefited greatly from their perspectives and input. These colleagues include Professors Jae Lim, Bruce Musicus, and Victor Zue at MIT and Professors Tom Barnwell, Mark Clements, Monty Hayes, Jim McClellan, Russ Mersereau, David Schwartz, and Mark Smith at Georgia Tech. Professors McClellan and Zue along with Jim Glass of MIT were also generous with their time in helping us to prepare several of the figures in the book.

In choosing and developing an effective and complete set of homework problems to include in this book, a number of students provided considerable help in sorting through, categorizing, and critiquing the large selection of potential homework problems that have accumulated over the years. We would particularly like to express our appreciation to Joseph Bondaryk, Dan Cobra, and Rosalind Wright for their indispensable help with this task as well as their further help with a variety of other aspects such as figure preparation and proofreading. The later stages of production of any text require the time-consuming and often tedious job of proofreading and scrutinizing the galley proofs and page proofs for errors, omissions, and last-minute improvements. We were extremely fortunate to have a long list of "volunteers" to help with this task. At MIT, Hiroshi Miyanaga and Patrick Velardo read a large portion of both the galley proofs and page proofs with exceptional care and dedication. Our sincere thanks also to MIT students Larry Candell and Avi Lele for meticulous reading of many chapters of the page proofs and to Michele Covell, Lee Hetherington, Paul Hillner, Tae Joo, Armando Rodriguez, Paul Shen, and Gregory Wornell for their help with galley proofs. Similarly, our thanks to Georgia Tech students Robert Bamberger, Jae Chung, Larry Heck, and David Pepper for careful reading of the page proofs. Cheung Auyeung, Beth Carlson, Kate Cummings, Bryan George, Lois Hertz, David Mazel, Doug Reynolds, Craig Richardson, Janet Rutledge, and Kevin Tracy also gave valuable assistance with the galley proofs. We greatly appreciate the many valuable and perceptive suggestions made by all our students.

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MIT and Georgia Tech have provided us with a stimulating environment for research and teaching throughout a major part of our technical careers and have provided significant encouragement and support for this project. In addition RWS particularly thanks the John and Mary Franklin Foundation for many years of support through the John O. McCarty Chair.

Much of the structure and content of this book was shaped during the summer of 1985 when we were both guests in the Ocean Engineering Department at the Woods Hole Oceanographic Institution, and we wish to express our gratitude for this hospitality. In addition, AVO gives special thanks to the Woods Hole Oceanographic Institution and to our friends and summer neighbors the Wares and the Voses of Gansett Point for providing an exceptionally rejuvenating, productive, and enjoyable summer environment since 1978.

We feel extremely fortunate to have worked with Prentice Hall on this project. Our relationship with Prentice Hall spans many years and many writing projects. The encouragement and support provided by Tim Bozik, Hank Kennedy, and many others at Prentice Hall enhance the enjoyment of writing and completing a project such as this one.

Alan V. Oppenheim
Ronald W. Schafer

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