

Dynamic of Civil Engineering and Transport Structures and Wind Engineering

Edited by
Kamila Kotrasova and Jozef Melcer



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Dynamic of Civil Engineering and Transport Structures and Wind Engineering

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6th International Scientific Conference on
Dynamic of Civil Engineering and Transport Structures
and Wind Engineering
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May 25-29, 2014, Donovaly, Slovak Republic

Edited by

Kamila Kotrasova and Jozef Melcer



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PREFACE

Ladies and Gentleman, dear Readers,

the publication summarizes conference papers from the 6th international scientific conference on Dynamic of Civil Engineering and Transport Structures and Wind Engineering (DYN-WIND 2014). The conference was held from the 25th to 29th of May 2014 at Donovaly in Slovak Republic. The conference was organized by University of Žilina, Faculty of Civil Engineering, Department of Structural Mechanics, Slovak Scientific - Technical Transport Society in Zvolen, Doprastav, a.s. Bratislava.

In the submitted publication, which is divided into two chapters, are summarized selected conference papers from researchers of eminent slovak-czech-polish-russian-ukrainian and other educational universities, research institutes and industrial practice.

The publication includes following issues

- Effects of transport means on structures and environment
- Quasi-static and dynamic problems of roads, pavements and railroad tracks, bridges, incline and ground solids stability, buildings and other civil engineering and transport structures
- Problems of fatigue, life-time and reliability of structural materials and constructions
- Numerical and experimental methods and computer simulations
- Mathematical applications
- Earthquake engineering
- Transport noise and environment
- Wind characteristics
- Wind loads on building and structures and their aero-elastic response
- Experimental methods, wind tunnels, field and full-scale measurements
- Industrial and architectonical aerodynamics, cross wind on transport means and other problems
- Computational methods in wind engineering
- Risk and social impact

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CHAPTER 1:

Dynamics of Civil Engineering and Transport Structures

Experimental Modal Analysis of the Footbridge across Vltava River in Prague Damaged by Flood in June 2013

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Keywords: damage, flood, footbridge, experimental modal analysis, natural frequencies, natural modes

Abstract. The investigated footbridge across Vltava River in Prague was built in 1984. Almost the entire footbridge structure was overflowed by the extreme flood in August 2002 and the substantial part of the footbridge deck was overflowed afterwards by the great flood in June 2013 again. A significant damage on the footbridge deck was found out by a visual inspection of the load-bearing structure of the footbridge after the second flood. The basic objective of the performed experimental modal analysis was the initial experimental reliability verification of the obviously damaged footbridge structure. The measured natural frequencies and modes were compared with results of the several dynamic tests which were performed regularly during the lifetime of the footbridge. The localization of potential invisible damages was carried out. The recommendations for the range and the arrangement of the static load test, which was performed afterwards for fundamental reliability assessment of a footbridge structure, were made on the basis of the evaluated results of the experimental modal analysis.

Introduction

In the last two decades, the whole territory of the Czech Republic has been hit by a series of great floods, in which many bridges has been damaged significantly. The important question was solved in many cases whether the bridge structure has to be demolished or under which operating conditions the bridge operation is acceptable.

The investigated footbridge across Vltava River in Prague was built in 1984. The footbridge has been a very important way for pedestrians and cyclists from its commissioning because the nearest possible river crossing has been at a distance of about 2 km therefore the eventual closure of the footbridge always has been not only technical but also “political” problem.

The five hundred year flood hit Prague in August 2002 almost the entire footbridge structure was overflowed by this extreme flood. The parts of the footbridge upon the central pillars remained above the water level only. However no damage was found out on the footbridge load-bearing structure subsequently.

The substantial part of the footbridge deck was overflowed afterwards by the fifty year flood in June 2013 again and a large object floating down the river crashed into the footbridge deck, afterward a significant damage on the footbridge deck was found out by a visual inspection of the footbridge load-bearing structure after the end of the flood. It was determined immediately to investigate experimentally the footbridge structure more in detail. It was decided to realize an experimental modal analysis and a static load test on the footbridge.

The basic objective of the performed experimental modal analysis was the primary experimental reliability verification of the evidently damaged footbridge structure. The basic goal was divided to the several partial goals:

- a) Determination of the important natural frequencies and mode shapes.

b) Comparison of the measured characteristics of the natural vibration with the characteristics measured by three dynamic tests which were performed regularly during the lifetime of the footbridge in 1984, in September 1997 [1] and in November 2010 [2].

c) Damage detection and localization of the footbridge [3].

d) The data obtaining for determination of the range and arrangement of the static load test. The static load test had to check the load carrying capacity more deeply.

Authors solved a very similar problem in 2010 [4]. The footbridge across Bečva River, which had the same structural system practically, was damaged by the great flood in May 2010.

Description of the Investigated Footbridge

The footbridge is made from the prestressed concrete. The bearing structure is the prestressed suspension deck with three spans 85.5 m, 96.0 m and 67.5 m (Fig. 1). The prestressed deck is composed from precast segments DS – L and DS – Lv made from concrete B500 and from the monolithic saddle designed upon both of two intermediate piers. The dimensions of segments are $3.8 \times 0.3 \times 3.0$ m. The concrete hinge joints were made in the lower part of the piers.

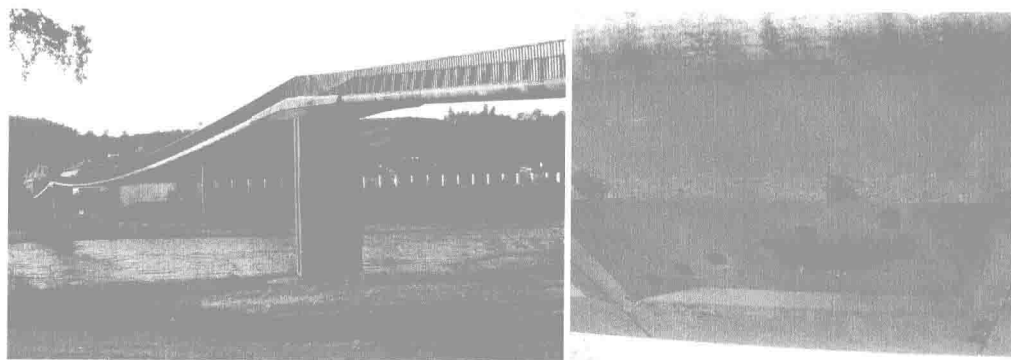


Fig. 1 The overview on the investigated footbridge (left) and the view of the damage (right), which was found out on the footbridge deck by a visual inspection after the end of the flood.

Description of the Experimental Modal Analysis Arrangement

Random wind excitation and the ambient vibration technique were used for determination of fundamental natural frequencies and modes of the footbridge. The footbridge vibration was measured by three seismic piezoelectric accelerometers Type 8344 Brüel&Kjaer. The measurement system Pulse and Front-end 3050-B-040 Brüel&Kjaer were used for data acquisition and data analysis. The response of the footbridge to wind excitation was measured in vertical direction in a chosen net of points (48 points – 24 cross sections and 2 points in each one) on the upper face of the footbridge deck. Two sensors were placed in the points of the net successively during the experiment. Third sensor was located throughout the experiment in the reference point, which was situated about one quarter of the central span of the footbridge.

The Results of the Experiment

The 11 natural frequencies were evaluated in the frequency range 0.4 to 3.0 Hz during the experimental modal analysis of the investigated footbridge. The mode shapes corresponding to the natural frequencies were evaluated too. The comparison of the chosen natural frequencies with corresponding ones measured by the dynamic tests performed in 1984, 1997 and 2010 is mentioned in Table 1. The selected measured mode shapes are compared in Fig. 2, Fig. 3 and Fig. 4.

Table 1 The comparison of the selected evaluated natural frequencies of the footbridge with ones measured by the dynamic tests performed in September 1997 [1] and in November 2010 [2].

Measured natural frequencies in 2013			Measured natural frequencies in 2010 [2]			Measured natural frequencies in 1997 [1]		Deviation between frequencies from 2010 and 2013	
No.	$f_{(j)}$	Expanded uncertainty $U_{k=2}$	No.	$f_{(j)}$	Expanded uncertainty $U_{k=2}$	No.	$f_{(j)}$	$\Delta_{(j)}$	
(j)	[Hz]	[Hz]	(j)	[Hz]	[Hz]	(j)	[Hz]	[%]	
(1)	0.47	+/- 0.02	(1)	0.51	+/- 0.02	(1)	0.525	-8.3	+/- 5.3
(2)	0.63	+/- 0.02	(2)	0.66	+/- 0.02	(2)	0.650	-5.2	+/- 4.2
(3)	-	-	(3)	0.92	+/- 0.02	(3)	0.925		
(4)	0.97	+/- 0.02	(4)	1.04	+/- 0.02	-	-	-6.7	+/- 2.6

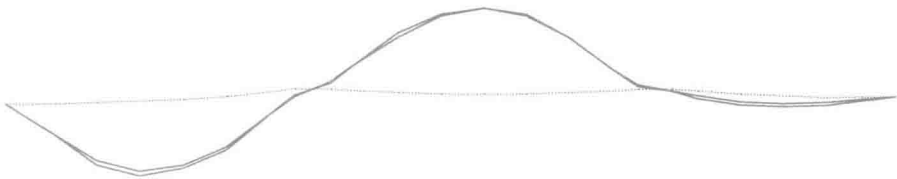


Fig. 2 The comparison of the 1st natural mode shapes of the footbridge measured in 1997 and 2010.

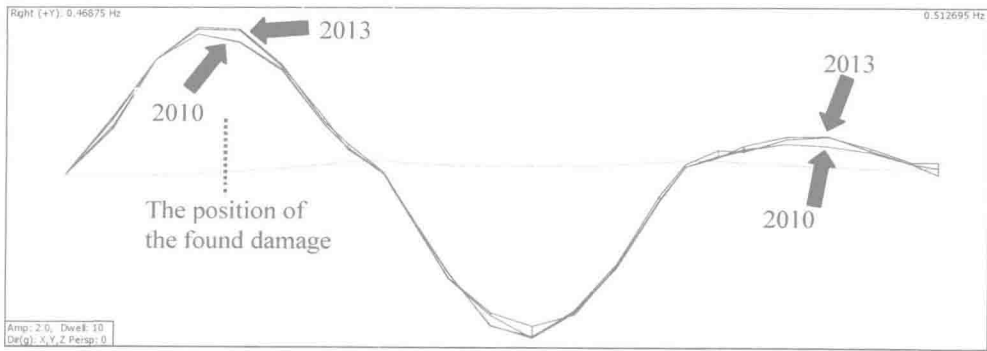


Fig. 3 The comparison of the 1st natural mode shapes of the footbridge measured in 2010 and 2013.

Summary

The results of the experimental modal analysis performed on the footbridge after the flood in the June 2013 were compared with the results of three dynamic tests which were performed regularly during the lifetime of the footbridge [1,2] especially the fundamental natural modes determined in November 2010 [2] were used.

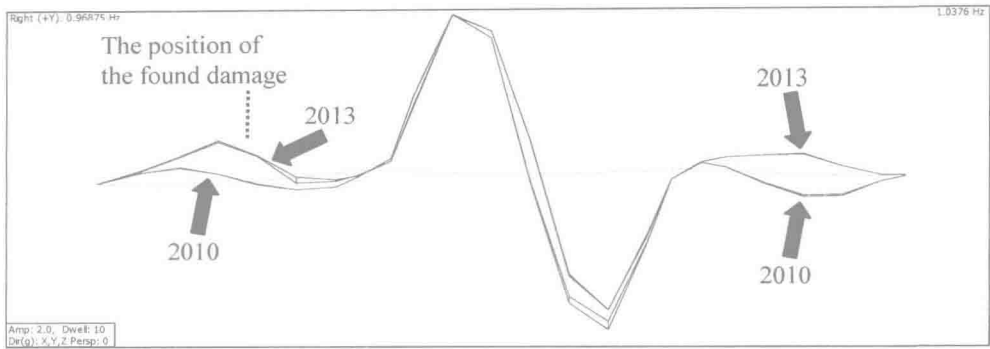


Fig. 4 The comparison of the 4th natural mode shapes of the footbridge measured in 2010 and 2013.

All compared frequencies decreased between November 2010 and June 2013 (some examples are shown in Table 1). It was caused evidently by the deterioration of the structural condition of the footbridge.

The similarity of the natural mode shapes measured in 2010 and in 1997 was high (see Fig. 2). On the other hand, some differences were found out between the natural mode shapes from 2013 and 2010 (see Fig. 3 and Fig. 4). Based on the detailed comparison of the natural mode shapes, several areas were determined, where the dynamic behaviour of the footbridge was changed evidently (see Fig. 3 and Fig. 4) and where some hidden fault could be at these places. The damage shown in Fig. 1, which was found before the start of the modal analysis, was located in one of these areas. It was recommended to investigate these areas in more detail especially during the static load test.

Acknowledgements

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Human-Induced Vertical Vibration of the Footbridge across Opatovicka Street

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Keywords: forced vibration, human – structure interaction, human – induced vibration, vandalism, ordinary pedestrian traffic, Monte Carlo simulation

Abstract. The presented article is focused on a theoretical dynamic analysis of the footbridge across the Opatovicka street in Prague, which acts as a simply supported beam. The structure was loaded by an ordinary pedestrian traffic, synchronous runners and vandals. The ordinary traffic was simulated by the Monte Carlo method as a stream of moving periodic forces with stochastic parameters. The synchronous runners and vandals were modeled as a combination of biomechanical models of human body, which influenced the structure vibration only passively, and driving forces, which loaded the structure in the contact points between the human body models and the structure. The driving force was the time – dependent function based on decomposition to the Fourier's Series and periodic triangle function. The obtained theoretical results are compared with experimental data.

Introduction

Mathematical description of the dynamic loading of structures, caused by a human walking, is quite complex process, which is mostly simplified by a deterministic loading model, also called the DLF (Dynamic Loading Factor) approach [2,3]. This model has been derived from the theory of Fourier's Series and could be expressed by the Eq. 4. The computed response of the structure loaded as a combination of the DLF model and biomechanical models of vandals and runners is described in this paper. The response caused by an ordinary pedestrian traffic was determined too.

Solved Structure

The footbridge chosen for the theoretical and experimental dynamic analysis is located in Prague across Opatovicka Street. It is a slab-on-girder bridge. The load-bearing structure consists of 6 steel girders and a reinforced concrete deck. The shape of the steel girders is box section with height $h = 520$ mm. Horizontal axial distance of steel beams is $b = 1100$ mm. The structure acts as a simply supported beam with the length of a span $L = 25.1$ m.

Theoretical Modal Analysis

Two models of investigated structure were created for a theoretical modal analysis, 3D solid model – ADINA and 2D beam model MATLAB, which has been used for the forced vibration as well. The problem of natural vibration is described by the equation

$$[\mathbf{M}]\{\ddot{\mathbf{w}}(t)\} + [\mathbf{K}]\{\mathbf{w}(t)\} = \{\mathbf{0}\} \quad (1)$$

where $[\mathbf{M}]$ denotes the mass matrix of the structure, $[\mathbf{K}]$ is the stiffness matrix, $\{\ddot{\mathbf{w}}(t)\}$ $\{\mathbf{w}(t)\}$ are the column vectors of acceleration and deflection respectively and $\{\mathbf{0}\}$ is the zero column vector. The computed natural frequencies are summarized in Table 1. Note that in the case of the 2D beam model the natural frequencies were computed via the Inverse Iteration Method. The Subspace

Iteration Method was implemented for the 3D model in the ADINA software. The 3D model was created for estimation of torsional natural modes of vibration, which are not calculable with using a 2D beam model.

Table 1 The computed natural frequencies.

	$f_{(1)}$ [Hz]	$f_{(2)}$ [Hz]	$f_{(3)}$ [Hz]	$f_{(4)}$ [Hz]
ADINA 3D model	2.47	5.13	9.74	13.18
2D beam model MATLAB	2.14	x	8.56	x

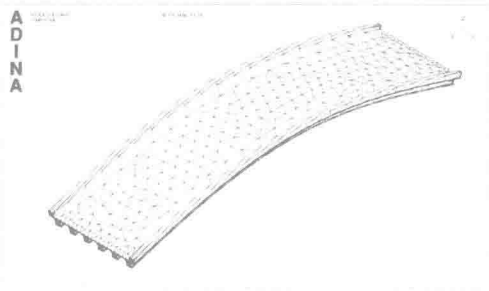


Fig. 1 The first natural mode $f_{(1)}=2.47$ Hz computed on the 3D model.

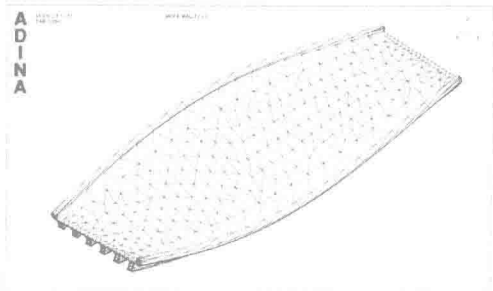


Fig. 2 The second natural mode $f_{(2)}=5.13$ Hz computed on the 3D model.

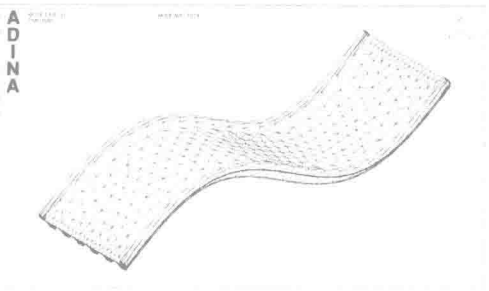


Fig. 3 The third natural mode $f_{(3)}=9.74$ Hz computed on the 3D model.

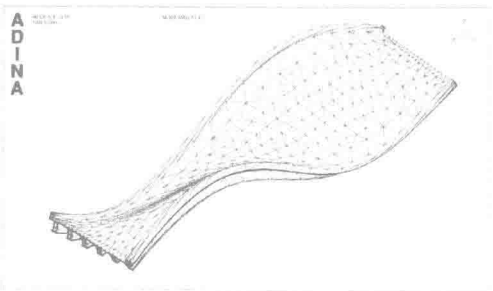


Fig. 4 The fourth natural mode $f_{(4)}=13.18$ Hz computed on the 3D model.

Modeling of Acting Pedestrians and Vandals – Forced Vibration

The 2D beam model was used for a study of the forced vibration. The bending stiffness and continuously distributed mass of the model were taken from the static design with values $EI_y = 3.83 \times 10^9 \text{ Nm}^2$ and $\mu = 5300 \text{ kg}$. The forced vibration of 2D model, discretized into N nodes with N degrees of freedom, is defined by the equation

$$[M]\{\ddot{w}(t)\} + [C]\{\dot{w}(t)\} + [K]\{w(t)\} = \{P(t)\} \tag{2}$$

Note that only vertical parts of deflection in the beam nodes were considered in the present study. The meaning of symbols in the Eq. 2 is following, $[M]$ is the mass matrix, $[K]$ is the stiffness