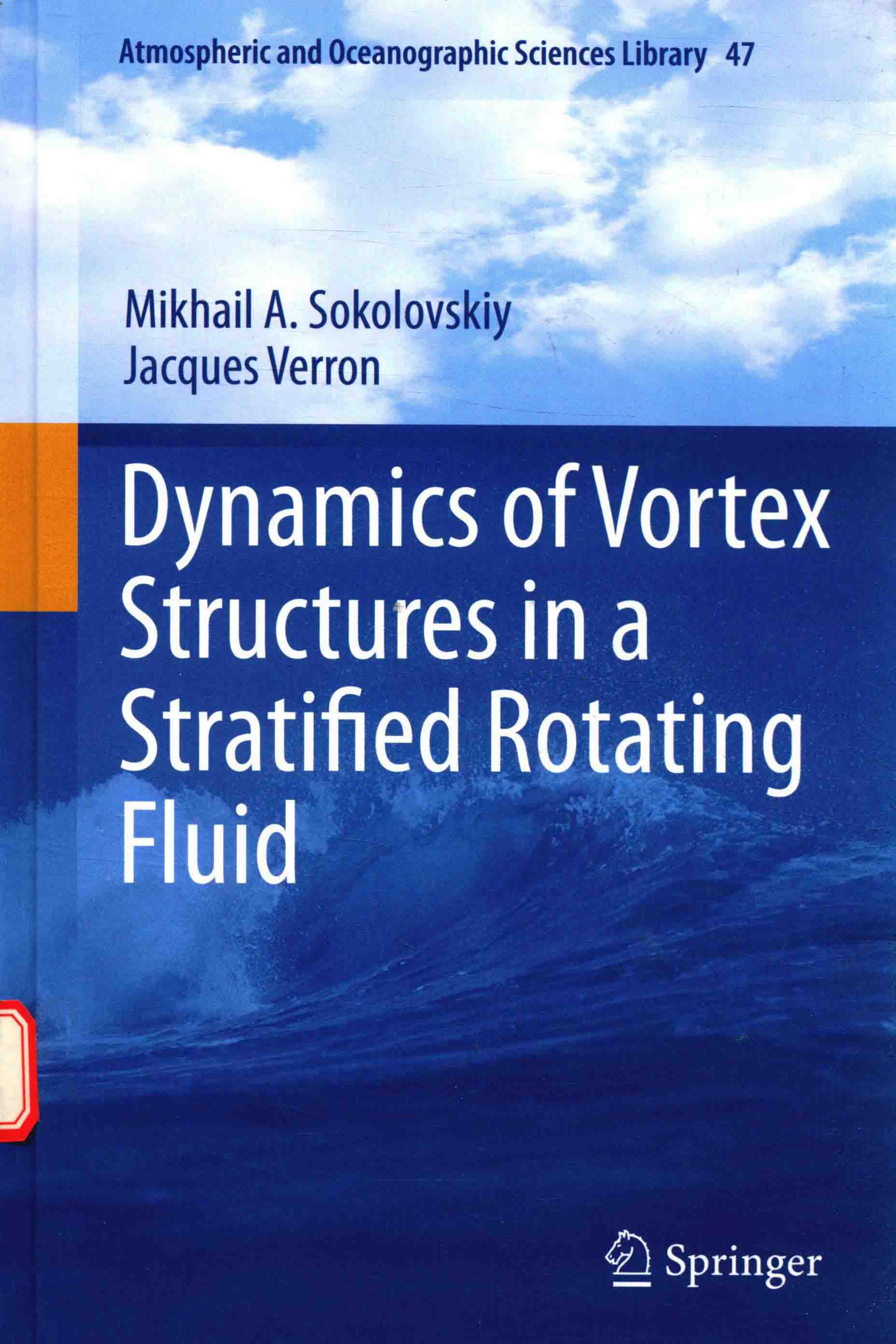

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Mikhail A. Sokolovskiy
Jacques Verron

Dynamics of Vortex Structures in a Stratified Rotating Fluid

Springer

Mikhail A. Sokolovskiy • Jacques Verron

Dynamics of Vortex Structures in a Stratified Rotating Fluid

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This book is an extended edition of the translation of the book in Russian "Dinamika vikhrevykh structur v stratifitsirovannoy vraschayuscheysya zhidkosti" by M.A. Sokolovskiy and J. Verron. The book was published originally in Russian by Publishing House of Izhevsk Institute of Computer Science, Moscow in 2011.

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Dedicated

to the blessed memory of
Professor Vadim Fedorovich Kozlov,
the Founder of Far Eastern School
of Geophysical Fluid Dynamics,
Vladivostok, Russia
(1933–2005)

and

to Emil Hopfinger,
Directeur de Recherche CNRS,
Ex-Director of Laboratoire des Ecoulements
Géophysiques et Industriels,
Grenoble, France

Foreword

This monograph is an expanded English translation of the Russian book published in 2011 by the Izhevsk Institute of Computer Science Publishing House under the same name and reviewed by Dr. G.M. Reznik.

The objective of this book is studying, in the framework of quasi-geostrophy, the role of stratification in the synoptic-scale vortex dynamics problems for atmosphere and ocean, and the effect of a bottom topography on large/meso-scale currents and vortices. The book summarizes the long-term joint studies of the authors in vortex dynamics of stratified rotating fluid.

The book is intended for experts in physical oceanography and meteorology, hydrodynamics, dynamical systems, and teachers, post-graduate students, and students in those fields.

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Moscow-Grenoble
March 2013

Preface

This book is a summary of our 18-year period of joint work in the field of vortex dynamics in a stratified rotating fluid under INTAS 94-3614 (1995–1997), INTAS/AIRBUS 04-80-7297 (2009–2011), PICS 5805 (2011–2013) as well as within the framework of Groupement de Recherche Europeen (GDRE) "Regular and chaotic hydrodynamics" (2006–2009) and during mutual scientific visits in Grenoble and Moscow (1996–2012).

This monograph is an expanded English translation of the Russian book [858] published in 2011 by the Izhevsk Institute of Computer Science Publishing House under the same name and reviewed by Dr. G. M. Reznik. The translation was made by G. N. Krichivets and O. I. Yakovenko.

We have made the following changes as compared with the Russian edition of the book: Sects. 3.2.2 and 3.5.2, Chap. 4, Appendices A and B, and Index are added; Introduction, Sects. 2.2.2.1 and 3.5.3, and the list of references are expanded, most pictures are now in color, and found misprints are corrected. Appendix A is written by Emil Hopfinger. It covers the description of laboratory experiments on heton interaction which partially fit our theoretical results. In Appendix B, Mikhail Sokolovskiy tells a short biography of his teacher, Professor Vadim F. Kozlov.

Part of the results has been obtained in collaboration with V. F. Kozlov[††], V. M. Gryanik, K. V. Koshel, V. G. Makarov, Z. Kizner, V. N. Zyryanov, X. Carton, P. A. Davies, E. Hopfinger, S. Valcke, B. N. Filyushkin, I. M. Vagina, and N. G. Kozhelupova, whose contributions to the work and kind permission to include their results in the book are much appreciated.

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[††] Deceased.

A. Yu. Gurulev, M. A. Guzev, L. Kh. Ingel', V. M. Kaistrenko, M. V. Kalashnik, M. G. Khublaryan††, R. Khvoles, T. R. Kil'matov, V. I. Klyatskin, G. K. Korotaev, M. N. Koshlyakov, A. G. Kostyanoy, V. V. Kozlov, E. A. Kulikov, L. G. Kurakin, N. P. Kuzmina, S. S. Lappo††, L. Ya. Lyubavin, I. S. Mamaev, V. V. Meleshko††, A. P. Mirabel, T. Miyazaki, S. V. Muzylev, P. K. Newton, R. I. Nigmatulin, V. V. Novotryasov, V. L. Okulov, N. Paldor, G. N. Panin, E. G. Pavia, E. N. Pelinovsky, M. S. Permyakov, X. Perrot, S. V. Prants, V. V. Pukhnachov, A. B. Rabinovich, J. N. Reinaud, G. M. Reznik, K. A. Rogachev, P. B. Rutkevich, E. A. Ryzhov, E. A. Sagomonyan, G. I. Shapiro, A. I. Shavlyugin, G. V. Shevchenko, J. Sommeria, D. V. Stepanov, G. G. Sutyrin, T. G. Talipova, M. V. Tevs, D. V. Treschev, O. O. Trusenkova, A. N. Vul'fson, O. I. Yakovenko, V. I. Yudovich††, A. G. Zatsepin, P. O. Zavialov, V. V. Zhmur and D. V. Zyryanov for useful discussions of some problems considered in this book.

We express our gratitude to V. M. Gryanik, Z. Kizner, K. V. Koshel and O. I. Yakovenko for helpful comments on the Russian version of this book.

We are extremely grateful to Emil Hopfinger who kindly agreed to write Appendix A.

The authors hope that the presented results will be of interest to experts in geophysical fluid dynamics and physical oceanography. We will appreciate critical analysis of the results of our work.

Moscow-Grenoble
March 2013

Mikhail Sokolovskiy
Jacques Verron

Contents

Chapter 1
The Introductory Chapter

Abstract In Sect. 1.1 of this chapter we explain the subject of Geophysical Fluid Dynamics, and give the description of main vortex structures that have become objects of the present book: (a) *Heton* – a two-layer vortex with opposite rotations in different layers, and (b) *Intrathermocline lens*, which is studied in this work as a vortex patch in the intermediate layer of a three-layer ocean model. In this section, we propose also a short review of works in the related topics. Section 1.2 is a mathematical introduction. It contains all main formulae which are used further in Chaps. 2 and 3.

1.1 Introduction

The theory of vortex motion of an inviscid incompressible fluid [11, 12, 66, 82, 87, 169, 285, 308, 438, 485, 490, 504, 510, 511, 541, 548, 565, 593, 595, 610, 625, 673, 713–716, 744, 745, 788, 789, 808, 890, 911, 950, 987], going back to the classical works of Helmholtz, Gröbli, Kirchhoff, Rankin, Greenhill, Taylor, Poincaré [see 37, 102, 640, 702], has developed mainly due to the need to understand the properties of atmospheric cyclones and anticyclones. Indeed, simple two-dimensional hydrodynamic models of discrete vortices provide some insight on the type of interaction occurring between elementary vortices and into the structure of the velocity field they induce.

However, many effects intrinsic to motion (especially, vortex motion) in the atmosphere or ocean cannot be explained without allowance being made for the rotation of the medium as a whole and the heterogeneity (stratification) of the density field that forms under the effect of gravity. The solution of some important hydrodynamic problems of a planetary nature has become possible with the development of *Geophysical Fluid Dynamics* (GFD) [40, 82, 196, 242, 295, 304, 403, 404, 449, 583, 584, 599, 624, 626, 689, 690, 743, 759, 798, 822, 858, 896, 922, 969, 977, 988] dealing with this class of problem (an independent branch of hydromechanics which has developed over the last three decades), the setting of domains of

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flow parameters dominated by vortex motions [143, 169, 194, 239, 318, 357, 798], and the generalization of methods of Hamiltonian dynamics to the description of geophysical processes [102, 304, 404, 822]. Thus, within the context of the quasi-geostrophic approximation, which is valid for fast rotating stably stratified fluids [404, 689], Gryanik and co-authors successively created the theories of discrete vortices for a two-layer fluid [315], a stratified medium comprising an arbitrary number of uniform-density layers [327–329], and a continuously stratified fluid [316, 317]. The contour dynamics method (CDM) has been developed by Kozlov et al. [477] for the description of finite-size vortices in a two-layer fluid and generalized later for three layers [844–846] and for the case of continuous stratification [457]. These works formed the basis for new studies, the results of which are partially reflected in this publication.

This book focuses substantially on the analysis of the dynamics of both discrete and distributed baroclinic vortices with zero total intensity in a two-layer medium (with constant density within each layer), since the two-layer model has the main features of the large-scale (mesoscale) dynamics of the atmosphere and ocean [404, 689]. Some results deal with hetons in a three-layer fluid. Unlike classical (barotropic) vortices in an ideal fluid, the baroclinic vortices possess a reserve of not only kinetic but also available potential (thermal) energy. As shown in [315], the baroclinic nature of vortices radically changes both the structure of the velocity fields they induce and the character of vortex interactions. Structures comprising two vortices with zero total intensity feature an important *self-motion* property (a two-layer pair moving without changing in shape or intensity [315]). In particular, in the case of two point vortices concentrated in different layers of a two-layer fluid and having equal intensities with opposite signs, each induces a rectilinear and uniform motion in the other.

The notion of *heton* was introduced by Hogg and Stommel [350] with the aim of emphasizing the ability of a baroclinic vortex pair to transfer heat. The word "heton" is derived from "heat". Indeed, when the geostrophic and hydrostatic approximations [404, 574, 689], conventional for GFD, are valid, any vortex of the top (bottom) layer, which has negative (positive) intensity, induces a downward local distortion of the interface between the layers. Such situation is referred to as a "warm heton". When the vorticity sign of each vortex changes, the sign of the curvature of the interface also changes, the situation turning into a "cold heton".

Since under conditions of stable stratification, the bottom layer should be denser and colder, the integral amount of heat in the domain containing vortices of this type will clearly be anomalous to that in any domain with the same volume. It is therefore obvious that the motion of hetons consisting of combinations of oppositely rotating vortices has a greater effect on the redistribution of heat (heat and salt, in the case of the ocean) than the motion of any other vortex structures. The notion of "heton" is also used to refer to vortices with finite horizontal dimensions (vertical patches, i.e., domains with constant values of potential vorticity Π_1 and Π_2 in the top and bottom layer, respectively). When the centers of the vortex patches in different layers are vertically aligned, the heton is said to have a "vertical" axis, otherwise the axis is said to be "tilted" (Fig. 1.1). The possible formation mechanisms of a warm (cold)

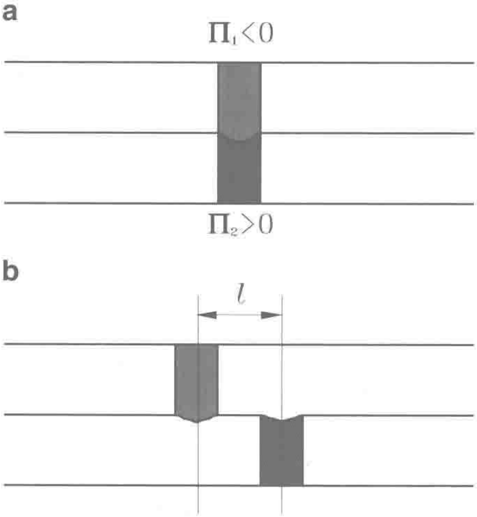

Fig. 1.1 Schematic representation of a two-layer distributed heton with a vertical (**a**) and tilted axis (**b**). Π_1 and Π_2 are the potential vorticities for the top (*red*) and bottom (*blue*) layers, respectively

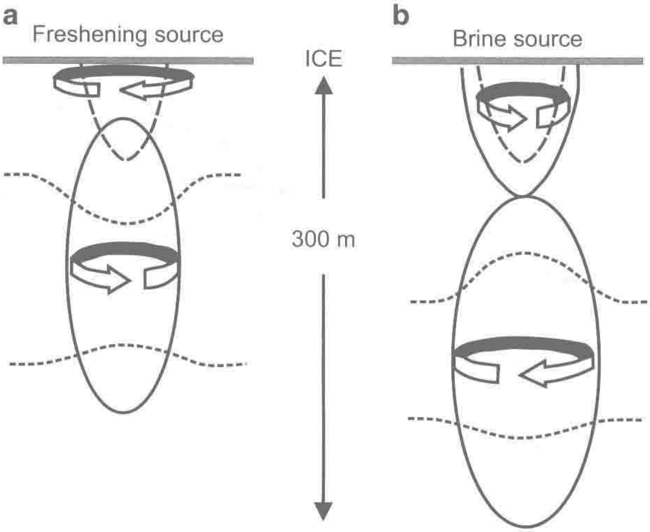

Fig. 1.2 Visualization of the formation mechanism of a warm (**a**) and cold (**b**) hetons according to [162] (Published with permission of American Geophysical Union)

heton with a vertical axis under ice in the ocean due to ice melting are given in Fig. 1.2. The formation scheme of a heton with a tilted axis due to the mechanism of baroclinic instability of the coastal current' front of potential vorticity anomaly (PVA[1] — in the terminology of the authors of [633]) is given in Fig. 1.3 (see also

---

[1] PVA concept is in wide use in geophysical applications, in particular, in the analysis of drift inertial recirculations in the ocean [160], in the studies of evolution of tropic cyclones [623], long-living mesoscale convective systems [729].

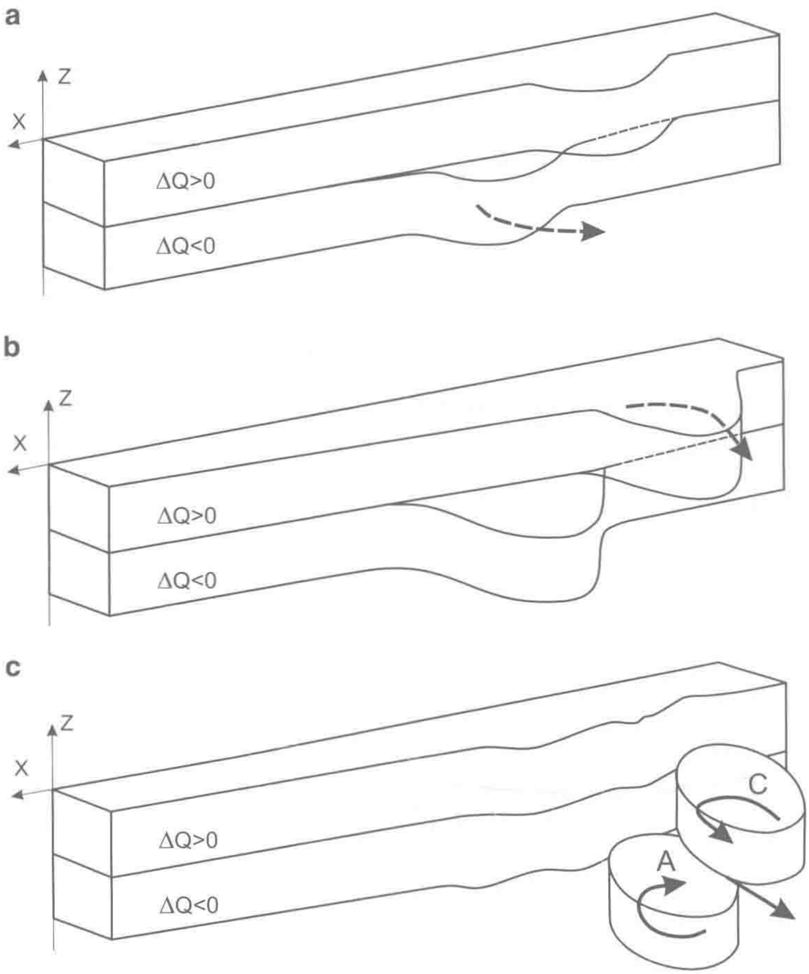

Fig. 1.3 The formation of a heton with a tilted axis due to baroclinic instability mechanism according to [633] (Published with permission of American Meteorological Society). Here the anomality of potential vorticity is denoted by ΔQ

[178]). Hetons can form also from unstable jets in the deep ocean [275,587]. Indeed, they have been observed south of the Gulf Stream; they are composed of a Gulf Stream cold-core ring associated with a Sargasso Sea water anticyclone.

The vortex structure, which has been derived from *in situ* observations south of Australia along 132° E in the Antarctic circumpolar current in January–March 1977 onboard "Professor Zubov" research vessel and which represents a heton with a tilted axis, is shown in Fig. 1.4. Naturally hetonic structures described also in [57] in relation to Agulhas eddies near the continental slope of southern Africa and in [763] in relation to tidal vortices in the neighborhood of Academy Bay in the Sea of Okhotsk. In summary, many observations of oceanic and atmospheric vortices confirm the existence of hetons.

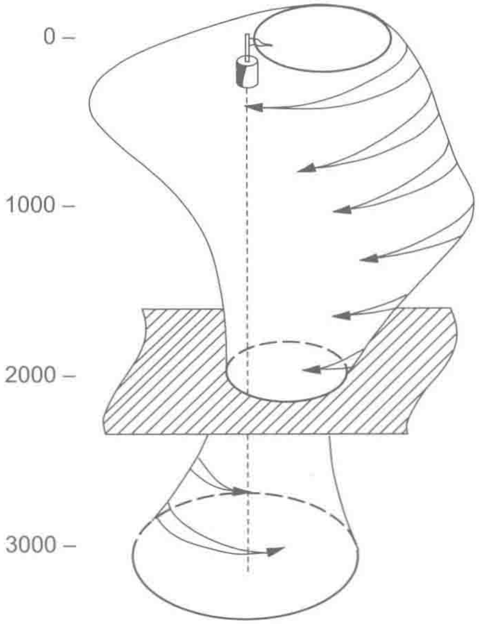

Fig. 1.4 The spatial structure of a heton with a tilted axis derived from direct instrumental observations (current meter, hydrographic and XBT data) in the Antarctic Circumpolar Current according to [803] (Published with permission of American Meteorological Society)

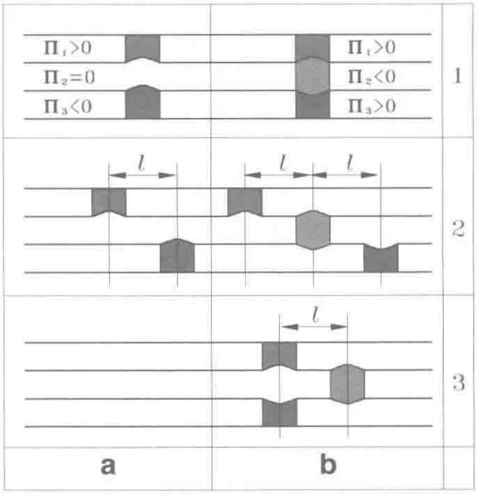

Fig. 1.5 Schematic representation of a three-layer distributed heton with equal-thickness layers (**1**) with a *vertical* axis, (**2**) with *tilted* axis and (**3**) with *arched* axis in the case of a three-layer fluid; (**a**) 1-modal heton; (**b**) 2-modal heton or S-vortex (in terminology of Morel and McWilliams [632]). *Red*, *magenta* and *blue* colors correspond to the upper, middle and lower layers, respectively

A hierarchy of hetons exists in the multilayer (in particular, three-layer) model [325–327,329]. The integral vorticity of each heton is zero. The vertical distribution of vorticity follows the structure of baroclinic modes of potential-vortex operator.

Therefore, it is reasonable to refer to such baroclinic structures as *m-modal* hetons[2] ($m = 1, 2, \ldots, M-1$). A single baroclinic mode, i. e., just a heton, exists in the case of $M = 2$, corresponding to a two-layer model. When $M > 2$, only the first among the m-*modal* hetons is most close to the two-layer one (Fig. 1.5). When

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[2] In our opinion, the term *modon*, which was used for such vortices in [327, 844], would be better. However, this term has been assigned to distributed dipole vortex structures [872].

all vortices are located along the same vertical, we will still say that such *m-modal* heton has a *vertical* axis, while when the vortices are spaced some distance apart, the axis is said to be *arc-wise*. Hetons can be generated in a laboratory by sources and sinks of mass [309, 310], by mechanical and locally spinning the top-layer fluid [899, 900], or by heat sources [342]. Such vortices naturally form when baroclinic currents, associated with the phenomenon of deep convection in the ocean, become unstable [59, 119, 132, 199, 200, 220–222, 224, 229, 321, 324, 524–528, 540, 571, 576, 612, 668, 669, 864, 951, 952, 978, 983, 984, 1017]. The heton idealization is also used when analyzing the dynamics of tropical cyclones and hurricanes in the atmosphere [271, 621, 622, 897], surface temperature anomalies [162–165], instability of boundary currents in the ocean [823], and intrathermocline vortices [352, 408].

Well known intrathermocline vortices are Mediterranean lenses (meddies).

Mediterranean anticyclonic and cyclonic lenses form over the continental slope of the Iberian Peninsula. Dynamic instability of a Mediterranean near-bottom current flowing through the Strait of Gibraltar into the Atlantic Ocean causes the appearance of such vortices. A Mediterranean Water (MW) jet is directed along the southern and western slopes of the peninsula [60, 108, 393, 556]. When the flow crosses bottom canyons and hills, there occurs both downwelling along the canyons [13, 20], and the separation of the stream in the seaward direction [262].

Meddies are characterized by high temperature and salinity with respect to surrounding waters. This fact allows us to determine their position in the ocean and size of MW with increased content of heat and salt, as well as to study their evolution during all the stages of their lifetime up to their decay. As a rule, all these vortices are located in the intermediate ocean layer.

Figure 1.6 shows the vertical distributions of temperature, salinity, and density in the center of the MESOPOLYGON lens and in the background region [379]. The differences between the lens core and the surrounding waters can vary from 1° to 4 °C in temperature and from 0.3 to 1.0 PSU in salinity depending on the distance of the lens from the region of its formation [258]. The temperature and salinity anomalies nearly compensate to form a homogeneous density. Indeed, the vertical density profiles in the surrounding waters and in the lens center are close, while the core is distinguished for density homogeneity. However, the absolute values of specific density in the lens cores vary in the range from 27.5 to 28.0, and the lens cores are located at depths from 800 to 1,400 m [261].

Figure 1.7 shows a T/S-diagram for the maximum values of temperature and salinity in the meddy cores based on the data catalogue for the period 1968–2007 [261]. The variability range for these characteristics is related to the different regions and mechanisms of the meddy formation. The long-term measurements show that the main regions of lens formation are the canyons of the Gulf of Cadiz [13, 46, 176, 393, 883], the region of Cape Saint Vincent [752, 806, 813], canyons of the continental slope and western coast of the Iberian Peninsula [107, 111] as well as the region near Cape Ortega [681, 682].

According to long-term observations, it appears that the Cadiz Bay canyons, the region of the Saint-Vincent Cape [13, 19, 46, 108, 813] as well as canyons along the western coast of Portugal [408, 681] are the main areas of meddies' formation.