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Ivan Němec

# **Dynamics of Collapse of a High-Rise Building**

Inspired by the Collapse of the Twin Towers of  
the WTC

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Ivan Němec, Martina Juráňová, Vlastislav Salajka, Tomáš Hanzlík and Šárka Sychrová

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*To Ted Belytschko in memoriam*

## Preface

The objective of this book is to study dynamics of a progressive collapse of a high-rise building. The book is inspired by the fall of the Twin Towers of the World Trade Center, but all the derivations and formulae are valid for a collapse of an arbitrary high-rise building. The computer simulations are performed on a building with the same geometry as that of the WTC Twin Towers, but no exact knowledge of a building structure was needed either for the derivation of the differential equation of a fall of a high-rise building or for the finite element method computer simulations. The analyses which have been made were based only on the facts that a building is standing and it is designed with a certain safety factor. Several additional parameters, such as the value of damping ratio, the share of the mass falling beyond the building perimeter and the average resistance of the columns to the fall have been introduced.

The book is not interested in the cause of the fall, but it studies the dynamics of the fall itself, especially the speed and the extent of the collapse. Some conclusions regarding the fall of the WTC Twin Towers collapse have been made.

The book is divided into nine chapters. In the Chapter 1 the reason why the book was written is revealed and the basic laws of mechanics are described. Also an application of the laws of conservation of energy and momentum is shown. The Chapter 2 gives brief information about the WTC Twin Towers and the history of their collapse. The Chapter 3 deals with the approach of professor Kenneth L. Kuttler to analysis of the WTC Twin Tower collapse and two modifications of his solution enabling more accuracy are suggested by the authors of this book. The Chapter 4 introduces the solution of Bažant and Verdure. They have derived a differential equation of the progressive collapse of a high-rise building and have also introduced some solutions to it. In the Chapter 5 derivation of a more general differential equation of the collapse of a high-rise building is presented. Parameters used in this differential equation are described and their magnitudes are discussed. The approach to the numerical solution of this differential equation by the Runge-Kutta method is presented. The Chapter 6 deals with results of parametric study of the differential equation derived in the Chapter 5. An influence of each parameter of solution is shown. The Chapter 7 deals with a finite elements computer simulation of the collapse of a high-rise building using the ANSYS and LS-DYNA computer programs. Also here results of a parametric study are shown and the influence of values of each parameter of solution is presented. In the Chapter 8 a comparison of the numerical solution of the differential equation and the FEM computer simulation



is shown in a table for various combinations of the parameters. The Chapter 9 introduces some conclusions of this book. Impossibility of the mechanism of the collapse presented in the final report of NIST is proved which imply the need of an additional investigation of the collapse of the Twin Towers of the World Trade Center.

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# 1 Introduction

The fall of the Twin Towers of the World Trade Center has been the subject of a series of expert discussions on the mechanism by which the fall took place, and especially its speed. The official National Institute of Standards and Technology (NIST) report states the following: “The release of potential energy due to downward movement of building mass above the buckled columns exceeded the strain energy that could be absorbed by the structure. Global collapse ensued.” But how, then, is the collapse itself supposed to occur? With what acceleration? Is the fall of the upper floors supposed to accelerate or should it decelerate and subsequently stop due to the resistance provided by the columns below, and other factors? These are issues that the NIST report does not deal with but which are discussed in this book. The authors were indeed inspired by the fall of the WTC’s Twin Towers, but the book is devoted to the dynamics of the collapse of high-rise buildings in general. In line with the conclusions of the NIST report, the authors assume that collapse was initiated by the loss of stability of the columns in one or more floors of the high-rise building due to fire or another triggering factor. These reasons are not dealt with in the book, which is concerned exclusively with the dynamics of the fall after it has, for any reason, already begun.

The dynamics governing the collapse of a high-rise building are a relatively complex problem. As with any mechanical problem, the solution must be based on the application of the fundamental laws of mechanics, these being the law of conservation of mass, the law of conservation of energy and the law of conservation of momentum. These laws are unquestionable. There are also some other equations that are needed for the solution. These are primarily required for the definition of the properties of the continuum, namely strain measures and the relations between the stress tensor and the strain tensor (so-called constitutive relations). These equations, however, do not have the nature of physical laws - they are only our simplified model of nature.

Let us demonstrate the use of the above-mentioned basic laws of mechanics via the simple problem of a billiard ball moving at speed  $v$  hitting a stationary billiard ball. Let us assume that both balls have the same mass  $m$ , and that the impact is centric. Further, let us first assume that the balls are perfectly elastic, i.e. no dissipation of energy or plastic deformation occurs during the crash.



Then, the law of conservation of momentum can be written in the following form:

$$mv = mv_1 + mv_2$$

where  $v_1$  and  $v_2$  are the speeds of the two balls after the collision. Let us write the law of conservation of energy in a similar way. Given the zero dissipation, it only expresses the equality of the kinetic energy before and after the collision:

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

The result of the solution of this system of equations is that the moving ball stops after the collision and the ball which was at rest before the collision will move at speed  $v$  after the collision. Now let us consider a case in which the balls are not elastic but perfectly plastic. This means that they will not rebound from each other upon the collision but will move together, i.e.  $v_2 = v_1$ . Then, an equation expressing the law of conservation of momentum can be written for this case as follows:

$$mv = mv_1 + mv_1 \rightarrow v_1 = v/2$$

The solution is thus obtained from a single equation and the result is that after the collision, both balls will move at half the speed of the ball moving before the collision. When substituting into an expression for the kinetic energy, we can see that the kinetic energy after the collision will be only half of the kinetic energy before the collision. The remainder of the kinetic energy is converted into heat, which is radiated into space (so-called dissipation). This particular case is of interest from the viewpoint of the mechanism of the fall of a high-rise building.

The dynamics of the collapse of a high-rise building is a rather more complex problem but the theoretically possible limit speed of the fall can be determined by simple application of the law of conservation of energy, introducing considerably simplifying assumptions. Let us suppose that the mass of each of the floors is the same and thus the mass is evenly distributed along the height of the building. The following solution is based on the assumption that the building begins to fall from the upper floors, which then impact the lower floors, after which they all fall together. When the front of the collapse (the so-called 'crushing front') arrives at the location of coordinate  $x$ , all the matter above this point will be travelling at the same speed  $v(x)$ . This speed depends on many factors but its upper limit may be