mathematics

without

apologies

portrait of a problematic vocation

michael harris

mathematics without apologies

portrait
of a
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vocation

with a new preface
by the author

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To Béatrice,
who didn't want to be thanked

preface to the paperback edition

The publication of a paperback edition of *Mathematics without Apologies* provides a welcome opportunity to correct some of the misunderstandings of my intentions. I've encountered these misunderstandings in published reviews and online comments, as well as in conversations. It's my responsibility as an author, of course, to be understood. Parts of the book were written in a deliberately elusive style, playing on the ambiguity of vocabulary and (for the most part) avoiding explicit value judgments. I rationalized this as an attempt to let the material speak for itself, rather than to impose my own necessarily partial perspective.

It is only to be expected, however, that when an author's intentions are obscure, readers attempt to fill in the gaps; and I should not have been surprised that some readers chose to do so with material from just the kinds of unexamined preconceptions I was hoping the book would challenge. So, for the space of a few pages, I will attempt to clarify my original intentions in writing this book. I have organized this new preface into four parts, with each one addressing one of the four most common misunderstandings.

Mathematics without Apologies is Not a Memoir

Although both Amazon and Wikipedia seem to think that *MWA* is autobiographical, Chapter 9 is the only part of the book that is really based on my life. It is also the only part framed as a conventional linear narrative—with a pair of significant flashbacks at the end. I included the story because I think it's a pretty good one. But it also happens to be my own story, and I'm the only one in a position to tell it properly. The purpose, however, was not to talk about myself—why should the reader care what happened to me?—but rather to make specific points about mathematical discovery.

Autobiographical fragments are included elsewhere in the book (all the champagne receptions, for example) in the service of journalistic authenticity, or just to provide a semblance of narrative structure. The

events that altered the life of the ideal-typical protagonist of the socio-logical *bildungsroman* that is Chapter 2 were mainly taken from my own life, not because they are exceptionally interesting but precisely because they are typical. It would have made no difference if I had made them up.

ROUTINIZED CHARISMA AND THE MATHEMATICAL HIERARCHY

The title of Chapter 2, "How I Acquired Charisma," is a deliberate provocation, chosen to grab the reader's attention. The chapter does contain a "how" narrative, constructed out of incidents from a typical career that just happens to be my own: a more accurate title for this material would have been "Stages in the Career of a Typical Pure Math-ematician from the Awakening of Interest to Tenure." Such a title would have been less catchy, of course, but it would have made it immediately clear that the charisma of the title is *routinized charisma*. Having it is a source of satisfaction, but the point of the chapter is that it's not an un-usual distinction; it belongs to everyone who is fortunate enough to make a career as a pure mathematician.

In retrospect, though, I do regret not having had more material about the importance of cooperation in mathematics. It's more fun to talk about the compulsive craving of some mathematicians for recognition, as in the Weil anecdotes early in the chapter, but the fact is that I know no other branch of academic scholarship that is more deeply cooperative—even though the actual work of mathematical research is generally carried out individually. Once, when I was seated next to an experimental biologist at an academic dinner. I explained to her the at-tention given to comprehensive bibliographic references in mathemati-cal papers, some of them quite old, as well as the convention of ac-knowledging debts to earlier work. I asked whether there was anything similar in the literature of her field. Certainly not, she answered, they only cite their predecessors' work to point out what they got wrong. True or not, it's not the sort of thing a mathematician would ever say.

How Mathematics Is or Is Not Useful

This was the subject of the most serious misunderstandings, as expressed in at least two widely-read reviews, and in this case I don't think I'm entirely to blame. Any discussion of the utility of pure mathematics, especially of number theory, inevitably comes back to G. H. Hardy's declaration which is quoted at the beginning of Chapter 10, "I have never done anything 'useful.'"[1] Many readers seem to have decided that my unapologetic intention was to amplify Hardy's claim and to argue in favor of mathematics for its own sake.. Some worried in print that this was exactly the wrong message to send to elected officials and the general public, who would be all too happy to cut off our funding (the "external goods" to which I refer in the book).

The text anticipates this misunderstanding and addresses it repeatedly, but the message clearly didn't get through to everyone. So let me just say here that my goals were rather different. These three were perhaps the most important:

> To draw attention to the increasing pressure on scholarship to prove its utility in narrowly economic terms. The humanities feel this pressure most strongly, for obvious reasons—Chapter 3 quotes an attempt at a response to this pressure by a British philosopher—but pure mathematics is not exempt. Paradoxically, precisely because much mathematics *is* conventionally useful—for creating gadgets, enhancing competitivity and the gross national product, and so on—mathematicians find it easy to justify our discipline by what the book, following Steven Shapin, calls the *Golden Goose Argument*. Examples of this argument from three European countries are analyzed in Chapter 10. And it's not wrong. However, it misses the second point—
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> Namely, that the desire to contribute to the GNP or to create a startup, laudable though it might be, is *almost never* the reason anyone chooses to go into pure mathematics in the first place. Some of the real reasons are introduced, haphazardly, throughout the book, but I may not have insisted strongly enough that I don't think keeping decision-makers and the public in the dark about our true motivations as pure mathematicians is a viable long-term strategy.

Most importantly of all, the utility argument is itself specious because there is no general agreement as to what is and is not useful. The problematic utility of financial mathematics is the main topic of Chapter 4, but other examples would have served just as well. The mathematical methods that protect privacy can be used to undermine privacy; the differential equations that maximize extraction of fossil fuels can be used to develop alternatives. Utility in mathematics, in other words, is of profoundly *political* import—as it is (perhaps more transparently) everywhere else. This leads me to my last topic:

ETHICS

We choose to become mathematicians, of course, not for the sake of truth, beauty, or utility, but because it's *what we want to do.* If this strikes many people as unforgivable self-indulgence, it can only be because *most people don't get to do what they want to do*—or, not to put too fine a point on it, *what they get paid for is not something they particularly want to do.*[2] The freedom mathematicians enjoy to get paid to do what we want is indeed a rare privilege, and precisely because we can't claim to be more deserving of this privilege than anyone else, this position of freedom does confer a real responsibility, and not (or not exclusively) to the "powerful beings" who shadow Chapter 3.

I see this responsibility as an obligation to think through the ethical implications of all aspects of our work. I have in mind not only the implications of our work's potentially dangerous applications but also the compromises we make in order to enjoy our privileged professional situation, as well as the commitments we accept along with the "external goods" on which our professional freedom is based. If this book attempts to illuminate the circumstances of our freedom and the pressures our freedom faces, it is because I see these pressures as symptomatic of an attack on everyone's freedom, in the spirit of Margaret Thatcher's notorious and disempowering slogan TINA—There Is No Alternative. By the same token, the existence of pure mathematics—of its "relaxed field," in the vocabulary of Chapter 3—can serve as the beginning of a reminder that there is always an alternative.

NOTES

1. Especially in a book written by a number theorist, like Hardy, with a title that all too obviously alludes to Hardy's *A Mathematician's Apology*. Let me therefore make it clear that what was originally intended as a subtitle was promoted to title only after months of deliberation, with a rapidly approaching deadline, as the least bad of the available options, and was intended neither as an homage to Hardy nor as a challenge to his (often brilliant but no less often appalling) beliefs about mathematics. In this sense, the title was not "intended" at all.

2. Statistics may actually show the opposite, which would only make the accusation of self-indulgence that much more puzzling.

*Wer das Buch schriebe, hätte die Vorrede Schritt für Schritt
zurückzunehmen, aber sie ist das Beste daran, das Einzige
was wir können, wir Modernen...*

*Ich will aus solchen Vorworten zu ungeschriebenen Büchern
ein Buch machen, ein modernes Buch. Und ich schrieb
eben—das Vorwort dazu.*

—Paul Mongré, *Sant' Ilario**

When this book was nearly done and my colleagues started asking me
what it is about, I found it simplest to answer that it's about how hard
it is to write a book about mathematics. That's the short answer; the
unabridged version involves a few pages of explanation. Here are
those pages.

Of course people are writing books about mathematics all the time—
and not only for expert audiences. The most effective of these books
strip away the technical jargon to convey the magical sense that pure
thought can conjure a second life, a virtual world of shapes and num-
bers and order and rules where not only do we know that everything is
as it should be but we are also satisfied that we know *why*. Knowing
why is the specialty of mathematical reasoning, but the virtual world of
pure mathematics, not designed for any practical application, is remote
from our first and authentic life; those of us dedicated to that world feel
(or are made to feel) obliged to justify our indulging in an activity that
is charming and engrossing but that appears to bring no benefit beyond
the pleasure of knowing why.

These attempts at justification are the "apologies" of the title. They
usually take one of three forms. Pure research in mathematics as in
other fields is *good* because it often leads to useful practical conse-

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* "Whoever would write the book would have to undo its preface step by step, but it's the
best, the only thing we can do, we Moderns. . . . I want to make a book of such prefaces to un-
written books, a modern book. And thus I wrote—the preface to this book" (Hausdorff 2004).

quences (Steven Shapin calls this the *Golden Goose* argument); it is *true* because it offers a privileged access to certain truths; it is *beautiful*, an art form. To claim that these virtues are present in mathematics is not wrong, but it sheds little light on what is distinctively *mathematical* and even less about pure mathematicians' *intentions*. Intentions lie at the core of this book. I want to give the reader a sense of the mathematical life—what it feels like to be a mathematician in a society of mathematicians where first and second lives overlap. But during this guided tour of what I want to call the *pathos* of mathematics, we will repeatedly see our intentions misrepresented, and we will be reminded how hard it is to explain what impels us along this peculiar path.

Rather than rely on apologies, this book pieces together fragments found in libraries, in the arts, in popular culture, and in the media, to create a composite portrait of the mathematical vocation. The sequence of chapters very roughly follows the trajectory from the vocation's awakening, through struggles with various kinds of temptation, to its consolidation, followed by a conclusion consisting of inconclusive reflections on what we know when we "know why" and what it all means. Although I have consulted actual transcripts or recordings of mathematicians talking, my sources consist mainly of writings about mathematics, especially by participants—so the portrait is largely a self-portrait, though not of the author, of course!—but also by (usually, but not always, sympathetic) observers. Preconceptions and misrepresentations are fair game but are usually identified as such. I have paid special attention to writing or speaking by mathematicians whose manifest content may concern truth or utility or beauty, but which exhibit an aiming at something else, the values and emotional investment—the pathos, in other words—involved in pursuing the mathematical life.

Writing the book was a process of assembling and organizing this material in connection with selected themes and unifying perspectives. The process of assembling suggested virtues rather different from those usually invoked. Alternatives I explore in this book include the sense of contributing to a coherent and meaningful *tradition*, which entails both an attention to past achievements and an orientation to the future that is particularly pronounced in the areas of number theory to which my work is devoted; the participation in what has been described, in other settings, as a *relaxed field*, not subject to the pressures of material gain

and productivity; and the pursuit of *pleasure* of an elusive, but nevertheless specific, kind.

The alert reader will have noted, correctly, that these alternatives are no more distinctively mathematical than *good, true,* and *beautiful.* I certainly don't think they offer definitive solutions to the riddles of mathematical pathos; but they did make it possible for me to hint at a vision of the mathematical good life that I find more reliable than the standard account. Another author, presented with the same material, would assemble it in a different way and would likely reveal a different set of habits, virtues, and goals. This is only natural; I try to make the diversity of the community of pure mathematicians visible by recording their distinctive opinions, and there's no reason to assume they come to the field sharing identical motivations. At most, I hope that the reader will see coherence in my personal assemblage.

The "problematic" subtitle alludes to the problems that define the intellectual landscape where the mathematical life makes its home. It's conventional to classify mathematicians as "problem solvers" or "theory builders," depending on temperament. My experience and the sources I consulted in writing this book convince me that curiosity about problems guides the growth of theories, rather than the other way around. Alexander Grothendieck and Robert Langlands, two elusive costars of the present book's narrative, count among the most ambitious of all builders of mathematical theories, but everything they built was addressed to specific problems with ancient roots. Entering the mathematical life is largely a matter of seeking an orientation among such outstanding problems. In this way, as in every other way, the mathematical life is a running dialogue with human history.

The mathematical life is problematic in other ways. Trade books about mathematics typically follow a quest narrative. They share with the currently dominant model of science writing an attachment to a simple moral economy in which the forces of light and darkness are clearly delineated. The quest may be embodied in the protagonist's need to overcome external obstacles or to meet an intellectual challenge; its happy ending takes the form of a triumph over a hostile or unpromising environment or the rewarding of the protagonist's unique talents—or both at once. The reality is not so simple. The most interesting obstacles are less straightforward and more problematic. These can

never be overcome: they are inseparable from the practice of mathematics itself. There is the need to guarantee a constant supply of the material underpinnings of our practice, what the moral philosopher Alasdair MacIntyre calls "external goods." We might feel deeply ambivalent about what we provide in exchange. We set aside our ethical compass and contract Faustian bargains (Faust is a recurring figure in this book). We promise Golden Geese, immutable truths, ineffable beauty. We collude in the misrepresentation of our values and our intentions, in "the alienation from oneself that is experienced by those who are forced to describe their activities in misleading terms."*

But any burdens left on our conscience when we contract bargains, Faustian or otherwise, can be separated from mathematics, at least conceptually. It's not the least of the paradoxes explored in this book that the pathos of mathematics grows darkest and most problematic at its moments of greatest success. Satisfaction in solving a problem can be intense, but it is short-lived; our pathos is driven by what we have not yet understood. André Weil, one of the twentieth century's dominant mathematicians, described this as "achiev[ing] knowledge and indifference at the same time." We never understand more than a finite amount of the limitlessness of what mathematics potentially offers to the understanding. If anything, the situation is even more frustrating: the more we learn, the more we realize how much more we have yet to understand. This is also a kind of Faustian bargain—Goethe's Faust got to keep his soul until he reported to the Devil that he was satisfied with what he had seen. The mathematical soul, embodied in a historical tradition oriented to a limitless future, can rest secure in the knowledge that its dissatisfaction is guaranteed.

And yet there are enough of us who find the attractions of the mathematical vocation irresistible to fill any number of books this size. Is this because the vocation is so problematic, or in spite of that? In fact, this book narrates a quest after all, an abstract quest, to explain how being a pure mathematician is possible and, no less importantly, to explain why the reader should wonder why an explanation is needed—all

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* The quotation is from (Collini 2013), about the subordination of higher education to market priorities—thus primarily concerned with the "good"; but it applies equally well to the two other ways of describing mathematics to which my title refers.