

Circuit Analysis



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*Prentice-Hall, Inc.
Englewood Cliffs, N.J. 07632*

Library of Congress Cataloging in Publication Data

O'MALLEY, John R.
Circuit analysis.

Includes index.

I. Electric circuit analysis. I. Title.
TK454.O42 621.319'2 79-15570
ISBN 0-13-133827-7

Editorial/production supervision and interior
design by Gary Samartino
Cover design by Saiki/Sprung Design
Manufacturing buyer: Gordon Osbourne

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Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

PRENTICE-HALL INTERNATIONAL, INC., London
PRENTICE-HALL OF AUSTRALIA PTY. LIMITED, Sydney
PRENTICE-HALL OF CANADA, LTD., Toronto
PRENTICE-HALL OF INDIA PRIVATE LIMITED, New Delhi
PRENTICE-HALL OF JAPAN, INC., Tokyo
PRENTICE-HALL OF SOUTHEAST ASIA PTE. LTD., Singapore
WHITEHALL BOOKS LIMITED, Wellington, New Zealand

Preface

To understand the material in this book, a reader needs only a good knowledge of high school algebra and trigonometry. No knowledge of differential or integral calculus is necessary, even though there are derivatives in the introductions of inductors, capacitors, and transformers, as is required for accurate descriptions of the voltage-current relations of these components. These derivatives have clear physical explanations so that a reader is not penalized if he has not studied calculus. And no example or problem requires any knowledge of derivatives. Further, there is not a single integral anywhere in this book.

This book is more complete in many respects than some circuit analysis books for which a reader needs a knowledge of advanced mathematics. In addition to the standard circuit analysis topics, it has a chapter on the International System of Units. And the SI units, symbols, and prefixes appear throughout. This book has all three major analysis methods: mesh, loop, and nodal, along with explanations of the advantages and disadvantages of each, and a comparison, so that readers will have some basis on which to make an intelligent selection of the best analysis method to use on a particular circuit. Also, this is one of the very few books at any level that presents a formula for finding any voltage or current in the dc-excited RC circuits (or RL circuits) that are so important in timers. It also has an in-depth presentation of the use of a popular circuit analysis computer program: PCAP. This presentation is complete enough for a reader to learn how to use a digital computer to analyze almost any sinusoidally excited circuit that he will ever encounter—either in school or out. Also, this book has the various viewpoints

on circuits: electron flow versus conventional current flow; positive capacitive reactance versus negative capacitive reactance; and for phasors, peak value versus rms, and a sine reference versus a cosine reference; and so on.

This book has a practical and physical thrust. It has sections on color codes, nominal values, and tolerances. And it has pictures and descriptions of components and meters. Further, it includes practical uses along with the explanations to show to the readers that the topics are important and worth studying.

Because most readers will probably be majoring in electronics, the analysis of electronic circuits is emphasized. For example, there is an extensive explanation of the analysis of circuits containing dependent sources, which are so common in electronic circuits. And after the section on circuit hybrid parameters, there is a section on transistor hybrid parameters. Also explained are methods for finding input and output resistances of electronic circuits. And there is an electronically oriented chapter on two-port networks, their parameters, and their transfer functions.

Of course, due to all the added material, this book does not have some topics that are in many other circuits books of this level. One thing that is missing is a chapter on magnetic circuits. But there is material on magnetic flux, ampere-turns, and permeability, which is more than adequate enough to give an understanding of how inductors, transformers, and alternators work. Because magnetic circuits should be of concern just to those few readers interested in electrical machine and transformer design, the circuits material with which it is replaced should be of more general interest. Also missing is an extensive presentation of analog electrical measuring instruments. This material is omitted in favor of circuit analysis topics because of the fact that electrical instruments are becoming more and more digital. And there is no way of explaining digital instruments in a general circuit analysis book. Also omitted are the mathematical tables found at the ends of some books. These tables are unnecessary because the readers, it is assumed, have pocket calculators.

Finally, some pedagogical features of this book are perhaps as important as the contents. Throughout this book, the explanation of each topic includes its advantages and disadvantages, so that a reader can put the topic in a good perspective. Also, the readers are warned about making those mistakes that, as experience has shown, students commonly make. Further, this book is written such that the readers can read and understand the material efficiently—in as short a time as possible. The English used is easy to understand: common words, short words, short sentences, and short paragraphs. Also, examples follow the introduction of all major topics and most minor ones. It is hoped that these will give a better understanding and motivation than added words can. Also important are the answers to almost all odd-numbered problems. And a solutions manual is available for teachers.

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International System of Units

1

INTRODUCTION

1-1

This chapter presents the *International System of Units*, abbreviated SI, the international measurement language. This system differs considerably from the more familiar British system in which we measure lengths in yards, feet, and inches; power in horsepower; and temperatures in degrees Fahrenheit. With the International System we measure lengths in meters, power in watts, and temperature in kelvins or degrees Celsius. There are other differences, but we need not consider them now. One common unit, the second, is the same in both systems.

One important advantage of SI is that there is one and only one unit for each physical quantity: meter for length, kilogram for mass, second for time, ampere for current, kelvin for temperature, mole for amount of substance, and candela for luminous intensity. Units of all other physical quantities can be derived from these *base units*. Incidentally, some American scientists and scientific committees use the British spelling *metre* for the length base unit.

Another advantage of SI is a unique and well-defined set of symbols. For the base units the symbols are m for meter, kg for kilogram, s for second, A for ampere, K for kelvin, mol for mole, and cd for candela.

Another advantage is the decimal relation between multiples and submultiples of the base units for each physical quantity. For example, the kilometer is 1000 meters and the centimeter is one one-hundredth of a meter. Each is a power of 10 times the base unit meter. In contrast, the British system inch is one-twelfth of the

foot and the yard is 3 feet; these lengths have no power-of-10 relation. This relation is desirable because our common decimal number system has 10 as its base. In Sec. 1-4 we will study the SI decimal relations.

Another advantage is that six of the seven base SI units are accurately defined in terms of physical measurements that can be made in a laboratory. The kilogram is the only exception. It is a particular mass preserved by the International Bureau of Weights and Measures in a vault at Sèvres, France. Except for the ampere, defined in Sec. 2-2, we will not consider the definitions of the base units because of the extensive scientific knowledge required to understand them.

Another advantage is the international agreement on using SI. Those with foreign cars know some of the problems caused by different systems of units. Once SI is completely adopted, there will be no problems with having the wrong-size wrenches.

There are other advantages. But we will not consider them because an appreciation of them requires more scientific background than we have.

The electrical SI units are of most interest to us. Only one of these, the ampere, is a base unit. The others are *derived units*, which means they can be expressed in terms of base units. A good many of them, though, have their own names and symbols. In the following chapters we will study the ampere base unit as well as the electrical derived units.

GROUPING OF DIGITS

1-2

Some of the international committees recommending adoption of the SI also recommend using a space instead of a comma to separate digits into groups of three, as in 4 231 762 instead of 4,231,762. In four-digit numbers the space may be omitted, and preferably is. For example, instead of 7,452, either 7452 or 7 452 is accepted, but 7452 is preferred. The numbers in this book agree with this recommendation.

The reason for not using the comma to separate digits is to avoid confusion in those countries in which the comma is the decimal point. In this book we will keep the period for the decimal point, as that should cause no confusion.

To aid our reading of numbers, we will also use spaces to group digits into groups of three to the right of the decimal point. For instance, instead of 0.67452 we will write 0.674 52.

POWERS OF 10

1-3

Physical quantities vary greatly in size—especially in electric circuits. Capacitor values may be numbers like 0.000 002. In contrast, frequencies may have numbers like 1 600 000 or even much greater. Writing all the leading and trailing zeros is inconvenient and wasteful of space. We can avoid this by using powers of 10 and SI unit prefixes based on them. We will review powers of 10 before studying SI

unit prefixes. In the following discussion the term *power of 10* means the number 10 with an integer exponent.

Some examples of powers of 10 are

$$\begin{array}{ll} 1 = 10^0 & \frac{1}{10} = 0.1 = 10^{-1} \\ 10 = 10^1 & \frac{1}{100} = 0.01 = 10^{-2} \\ 100 = 10^2 & \frac{1}{1000} = 0.001 = 10^{-3} \\ 1000 = 10^3 & \frac{1}{10\,000} = 0.0001 = 10^{-4} \end{array}$$

In these examples and in general, a power of 10 equal to 1 or greater has an exponent equal to the number of zeros. For instance, the number 1 has no zeros. So, $1 = 10^0$. There is one zero in 10. Consequently, $10 = 10^1$, and so on. For powers of 10 less than 1, each exponent equals the negative of one more than the number of zeros to the right of the decimal point. For example, 0.001 has two zeros to the right of the decimal point. So, $0.001 = 10^{-3}$.

To place a number in powers-of-10 form, we usually must shift the decimal point. To keep from changing the value in this shifting, we must add 1 to the exponent of 10 for each shift left and subtract 1 for each shift right.

Example. Put 0.000 000 002 34 in powers-of-10 form.

Solution. The result depends on where we decide to put the decimal point. If we put it between the 2 and the 3, we must shift the decimal point nine places to the right. This makes -9 the exponent of 10: $0.000\,000\,002\,34 = 2.34 \times 10^{-9}$.

Throughout this book the symbol ■, as above, shows the end of an example.

There is a special name, *scientific notation*, for the powers-of-10 form with one nonzero digit to the left of the decimal point as in the answer of the example above. Sometimes we will use it and sometimes not, depending on convenience. We will not make it a point to use it.

Example. Put 43 629 000 in powers-of-10 form.

Solution. Again the answer is not unique. It depends on where we place the decimal point. If we arbitrarily select to put it between the 3 and 6, we must shift the decimal point six places to the left. (This decimal point, although not usually shown for an integer, is, of course, understood to be just to the right of the rightmost digit.) The shift of six places to the left means that 6 is the exponent of 10: $43\,629\,000 = 43.629 \times 10^6$.

We will now briefly study some mathematical operations with numbers in powers-of-10 form. For addition and subtraction the exponents must be the same. The result has this same power of 10 and it has the sum or difference of the other parts of the numbers.

Example. Add 3.63×10^3 and 47.32×10^4 .

Solution. For addition the exponents must be the same. We will arbitrarily decrease the exponent 4 of the larger number to the 3 of the smaller number. We can do this and not change the numerical value by shifting the decimal point one place to the right: $47.32 \times 10^4 = 473.2 \times 10^3$. Adding:

$$3.63 \times 10^3 + 473.2 \times 10^3 = (3.63 + 473.2) \times 10^3 = 476.83 \times 10^3$$

In the multiplication of numbers in powers-of-10 form, the exponents add. In division, they subtract. The other parts of the numbers multiply or divide in the usual fashion.

Example. Divide 3.2×10^3 into 22.72×10^6 .

Solution.

$$\frac{22.72 \times 10^6}{3.2 \times 10^3} = \frac{22.72}{3.2} \times 10^{(6-3)} = 7.1 \times 10^3$$

SI PREFIXES 1-4

An *SI prefix* is a name attached to the beginning of an SI unit to form either a decimal multiple or submultiple of the SI unit. For example, kilo is a prefix corresponding to one thousand. So, a kilometer is 1000 m. The SI prefix centi corresponds to one one-hundredth: a centimeter is 0.01 m.

The SI prefixes have symbols just as the SI units do. Following is a list of the SI prefixes and symbols along with the corresponding powers of 10.

Multiplier	Prefix	Symbol
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^2	hecto	h
10^1	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

For circuit analysis, only some of these prefixes are important: mega, kilo, milli, micro, nano, and pico. Incidentally, the symbol for micro is the Greek lowercase letter μ , pronounced "mew" as associated with cats.