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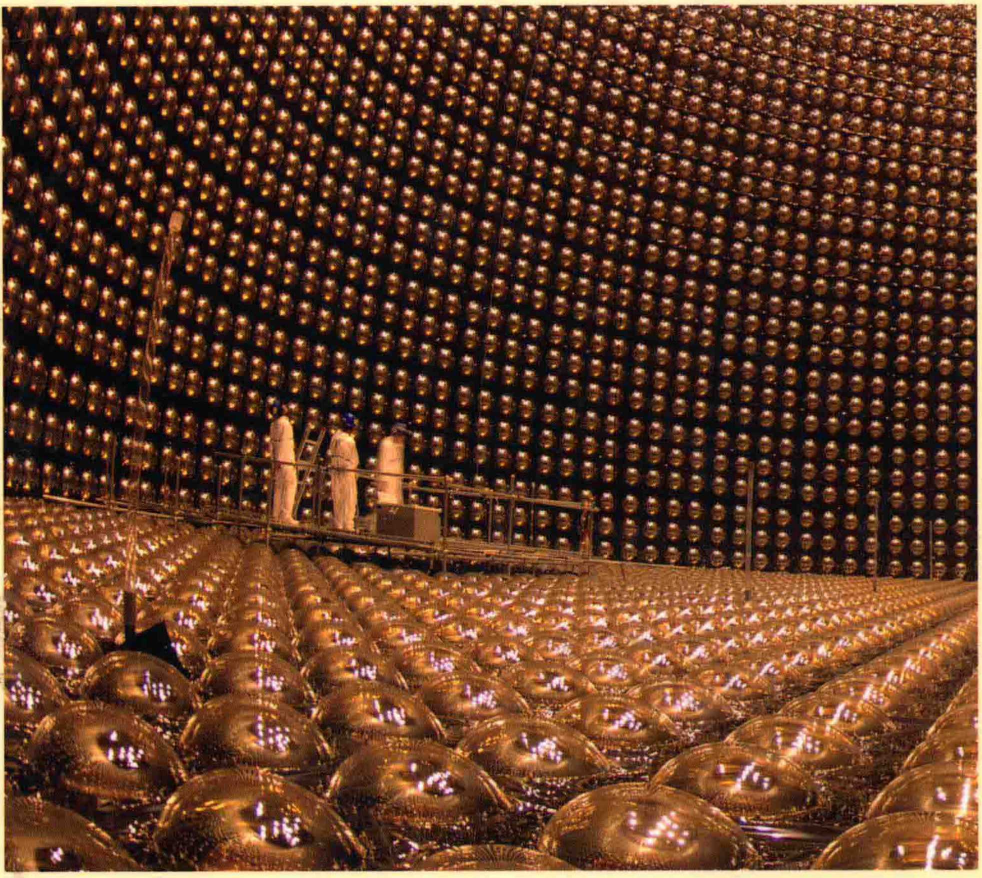
Flavor Mixing of Leptons and Neutrino Oscillations

Editor
Harald Fritzsch



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Preface

In 1911, Otto Hahn and Lise Meitner observed that in the radioactive beta decays of atomic nuclei the energy was not conserved. In the beta decay of a nucleus into another nucleus, an electron was emitted. One expected that the energy of the electron is given by the energy of the outgoing nucleus. However, the energy of the electron was not fixed, but continuous and always less than the expected energy. Apparently, energy was not conserved in the weak decays.

In 1930, Wolfgang Pauli suggested that in the beta decay a second neutral particle, which could not be observed, was emitted and that this particle carried the missing energy. This elusive particle was later called "neutrino". Pauli did not publish his idea, since he was convinced that the neutrino could not be observed in the experiments. However, in 1956, Clyde Cowan and Frederick Reines observed the neutrinos in an experiment conducted close to a big nuclear reactor. A neutrino emitted from the reactor collided with a proton and turned into a positron, while the proton changed into a neutron. Both the neutron and the positron were observed.

Later, it was discovered that for each charged lepton there is a neutrino — thus there are electron-neutrinos, muon-neutrinos and tau-neutrinos. The leptons come in three families, an electron-family, a muon-family and a tau-family, analogous to the three families of the quarks, the (u, d)-family, the (c, s)-family and the (t, b)-family.

It was assumed that the neutrinos are left-handed fermions and do not have a mass. But, already in 1957, Bruno Pontecorvo suggested that the neutrinos have a small mass and that they can oscillate. Since only the electron-neutrino was known at that time, he studied the possibility of oscillations between the neutrino and the antineutrino. Forty-one years later, the neutrino oscillations were discovered.

In the late 1960s, there began the Homestake experiment in Lead, South Dakota (United States) by astrophysicists Raymond Davis and John Bahcall. Its purpose was to collect and count the neutrinos emitted by the nuclear fusion in the Sun. Bahcall did the theoretical calculations and Davis designed the experiment. The solar neutrinos collided with the nuclei of Chlorine-37 atoms and produced nuclei of Argon-37 atoms. These atoms were collected later on. By this way Davis determined the number of solar neutrinos arriving on Earth. Bahcall had calculated the rate at which the detector should capture neutrinos, but Davis found only one third of the expected rate. This created the "solar neutrino problem".

The Homestake experiment was followed by other experiments such as Kamiokande in Kamioka, Japan and the Sudbury Neutrino Observatory (SNO) in Ontario, Canada. When cosmic rays collide with the nuclei in the upper atmosphere, many pions are created, which decay into neutrinos, in particular. These atmospheric neutrinos were investigated in Kamioka. In particular, the flux of neutrinos created in the atmosphere above Kamioka was compared to the flux of neutrinos coming from the other side of the Earth. Through this way the neutrino oscillations were discovered in 1998 and therefore neutrinos must have small masses. The atmospheric mixing angle, describing these oscillations, turned out to be rather large, close to 40°.

Oscillations of solar neutrinos were discovered in 2001 with the SNO detector, which could investigate both charged and neutral current interactions of neutrinos. The solar mixing angle was also large, about 34°. In the neutrino experiments using reactor neutrinos, the Daya Bay experiment in China, in particular, the third mixing angle was determined in 2012 to about 9°. Thus two neutrino mixing angles are large, while the flavor mixing angles for the quarks are quite small. Here the largest angle is the Cabibbo angle, about 13°. So far this phenomenon is not understood.

We still do not know if the CP symmetry is violated for leptons, analogous to the CP violation for the quarks. CP violation could be discovered by observing a difference between the oscillations of neutrinos and of antineutrinos. At the Massive Neutrinos conference, four speakers discussed the various models for CP violation.

In the neutrino oscillations, only the mass differences of the neutrinos can be measured, not the absolute masses. They are less than 0.1 eV. The masses of the neutrinos might be in the range 0.01 eV–0.1 eV. Presumably the flavor mixing angles are functions of the quark or lepton masses. The experimental data are in good agreement with the predictions of the "texture zero models", which were discussed at the conference by several speakers. In particular, there might be a connection between the large mixing angles and the very small masses of the neutrinos. These masses might be Majorana masses, implying that lepton number is not conserved. Thus the neutrino mass could be measured by the neutrinoless double beta decay experiments.

The International Conference on Massive Neutrinos was held from 9–13 February 2015 at the Nanyang Executive Centre of the Nanyang Technological University in Singapore (group photo on page vii). It was organized by Prof. Phua, Prof. Xing and by me. There were 56 speakers all together. Twelve lectures were on the recent or future results of the neutrino experiments. The various theories for the neutrinos were discussed by 26 speakers. Other talks were on neutrinoless double beta decay, flavor physics of quarks, cosmic neutrino background, cosmology, dark matter and astrophysics. We thank all speakers and participants for their contributions to this volume, which provide an overview of the present state of research, and for their participation in the discussions.

H. Fritzsch

NANYANG TECHNOLOGICAL UNIVERSITY

Institute of Advanced Studies

Contents

Birth of Lepton Flavor Mixing

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The history of the lepton flavor mixing could be traced back to the early 60s, when Maki, Nakagawa and Sakata (MNS) discussed the neutrino mixing. Their work emerged in the course of the developments of the composite model of elementary particles which was initiated by Sakata. In Sakata's model, the weak interaction of the hadrons can be described by two types of transitions among the fundamental triplet baryons. This pattern of the weak interaction of the hadrons is similar to that of leptons provided that the neutrino consists of a single species. From this similarity, Maki, Nakagawa, Ohnuki and Sakata proposed the so-called Nagoya model, in which the fundamental triplet baryons are regarded as composite states of the leptons and a hypothetical object called B-matter. Although the Nagoya model did not make a remarkable success, when existence of two kinds of neutrinos was discovered in 1962, Maki, Nakagawa and Sakata precisely formulated lepton flavor mixing to associate leptons with the fundamental baryons in the framework of the Nagoya model. To recognize their contributions, the flavor mixing matrix of the lepton sector is called the MNS matrix. See also: M. Kobayashi, "Neutrino mass and mixing — The beginning and future", *Nucl. Phys. B (Proc. Suppl.)* Vol. 235–236, (2013), pp. 4–7.

Neutrino Masses and Flavor Mixing

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We discuss the neutrino oscillations, using texture zero mass matrices for the leptons. The reactor mixing angle θ_l is calculated. The ratio of the masses of two neutrinos is determined by the solar mixing angle. We can calculate the masses of the three neutrinos: $m_1 \approx 0.003$ eV, $m_2 \approx 0.012$ eV, $m_3 \approx 0.048$ eV.

The flavor mixing of the quarks is parametrized by the CKM-matrix. There are several ways to describe the CKM-matrix in terms of three angles and one phase parameter. I prefer the parametrization, given below, which Z. Xing and I introduced years ago,[1,2] given by the angles θ_u, θ_d and θ:

$$U = \begin{pmatrix} c_u & s_u & 0 \\ -s_u & c_u & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} e^{-i\delta} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} \times \begin{pmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1}$$

where $c_{u,d} \sim \cos\theta_{u,d}$, $s_{u,d} \sim \sin\theta_{u,d}$, $c \sim \cos\theta$ and $s \sim \sin\theta$.

The angle θ_u describes the mixing between the quarks "u–c," the angle θ_d the mixing between the quarks "d–s" and the angle θ the mixing among the heavy quarks "t, c–b, s." The three angles have been determined by the experiments:

$$\theta_u \simeq 5.4^\circ, \quad \theta_d \simeq 11.7^\circ, \quad \theta \simeq 2.4^\circ.$$

Presumably the flavor mixing angles are not fixed values, but functions of the quark masses. If the masses change, the mixing angles will also change. For example, the Cabibbo angle $\theta_C \simeq 13^\circ$ could be given by the ratio of the quark masses:

$$\tan\theta_C \simeq \sqrt{m_d/m_s}\,. \tag{2}$$

This relation works very well:

$$\tan\theta_C \simeq 0.23\,, \tag{3}$$

$$\sqrt{m_d/m_s} \simeq 0.23\,. \tag{4}$$

Such a relation can be derived, if the quark mass matrices have "texture zeros," as shown by S. Weinberg and me in 1977.[3–5]

Let me discuss a simple example, using only four quarks: u, d–c, s. Their mass matrices have a zero in the $(1,1)$-position:

$$M = \begin{pmatrix} 0 & A \\ A^* & B \end{pmatrix} . \tag{5}$$

These mass matrices can be diagonalized by a rotation. The rotation angles are:

$$\theta_u \simeq \sqrt{m_u/m_c} \simeq 0.09 \,, \quad \theta_d \simeq \sqrt{m_d/m_s} \simeq 0.23 \,. \tag{6}$$

The Cabibbo angle is given by the difference:

$$\theta_C \simeq \left| \sqrt{m_d/m_s} - e^{i\phi} \sqrt{m_u/m_c} \,\right| . \tag{7}$$

In the complex plane this relation describes a triangle. The phase parameter is unknown, however it must be close to $90°$, since the Cabibbo angle is given by the ratio m_d/m_s:

$$\theta_C \simeq \sqrt{m_d/m_s} \,. \tag{8}$$

Thus the triangle is close to a rectangular triangle.

For six quarks the "texture zero" mass matrices for the quarks of charge $(2/3)$ and of charge $(-1/3)$ are:

$$M = \begin{pmatrix} 0 & A & 0 \\ A^* & B & C \\ 0 & C^* & D \end{pmatrix} . \tag{9}$$

We can calculate the angles θ_u and θ_d as functions of the mass eigenvalues:

$$\theta_u \simeq \sqrt{m_u/m_c} \,, \quad \theta_d \simeq \sqrt{m_d/m_s} \,. \tag{10}$$

Using the observed mass values for the quarks, we find:

$$\theta_d \simeq (13.0 \pm 0.4)^\circ \,, \quad \theta_u \simeq (5.0 \pm 0.7)^\circ \,.$$

The experimental values agree very well with the theoretical results:

$$\theta_d \simeq (11.7 \pm 2.6)^\circ \,, \quad \theta_u \simeq (5.4 \pm 1.1)^\circ \,.$$

Now we consider the flavor mixing of the leptons. The neutrinos, emitted in weak decays, are mixtures of different mass eigenstates. This leads to neutrino oscillations — at least two neutrinos must have finite masses.

The lepton flavor mixing is described by a 3×3 unitary matrix U, analogous to the CKM mixing matrix for the quarks. It can be parametrized in terms of three angles and three phases. I use a parametrization, introduced by Z. Xing and me:[6]

$$U = \begin{pmatrix} s_l s_\nu c + c_l c_\nu e^{-i\varphi} & s_l c_\nu c - c_l s_\nu e^{-i\varphi} & s_l s \\ c_l s_\nu c - s_l c_\nu e^{-i\varphi} & c_l c_\nu c + s_l s_\nu e^{-i\varphi} & c_l s \\ -s_\nu s & -c_\nu s & c \end{pmatrix} P_\nu \,, \tag{11}$$

where $c_{l,\nu} \sim \cos\theta_{l,\nu}$, $s_{l,\nu} \sim \sin\theta_{l,\nu}$, $c \sim \cos\theta$ and $s \sim \sin\theta$. The angle θ_ν is the solar angle $\theta_{\rm sun}$, the angle θ is the atmospheric angle $\theta_{\rm at}$, and the angle θ_l is the "reactor angle." The phase matrix $P_\nu = {\rm Diag}\{e^{i\rho}, e^{i\sigma}, 1\}$ is relevant only, if the neutrino masses are Majorana masses.

The neutrino oscillations are described by two large mixing angles:

$$\theta_{\rm sun} = \theta_\nu \simeq 34^\circ\,, \quad \theta_{\rm at} = \theta \simeq 45^\circ\,.$$

The reactor angle θ_l is much smaller: $\theta_l \simeq 13^\circ$.

We assume that the mass matrices of the leptons have "texture zeros":

$$M = \begin{pmatrix} 0 & A & 0 \\ A^* & B & C \\ 0 & C^* & D \end{pmatrix}. \tag{12}$$

In this case we can calculate two leptonic mixing angles as functions of mass ratios:

$$\tan\theta_l \simeq \sqrt{m_e/m_\mu}\,, \quad \tan\theta_\nu \simeq \sqrt{m_1/m_2}\,. \tag{13}$$

From the solar mixing angle we obtain for the neutrino mass ratio:

$$m_1/m_2 \simeq 0.42\,. \tag{14}$$

This relation and the experimental results for the mass differences of the neutrinos, measured by the neutrino oscillations, allow us to determine the three neutrino masses:

$$\begin{aligned} m_1 &\simeq 0.003 \text{ eV}\,, \\ m_2 &\simeq 0.012 \text{ eV}\,, \\ m_3 &\simeq 0.048 \text{ eV}\,. \end{aligned} \tag{15}$$

We expect that the mass matrices of the quarks and leptons are not exactly given by texture zero matrices. Radiative corrections of the order of the fine-structure constant α will contribute — the zeros will be replaced by small numbers.

The ratios of the masses of the quarks with the same electric charge seem to be universal:

$$\begin{aligned} \frac{m_u}{m_c} &\simeq \frac{m_c}{m_t} \simeq 0.005\,, \\ \frac{m_d}{m_s} &\simeq \frac{m_s}{m_b} \simeq 0.044\,. \end{aligned} \tag{16}$$

The dynamical reason for this universality is unclear. It might follow from specific properties of the texture zero mass matrices. But in the case of the charged leptons there is no universality:

$$\begin{aligned} \frac{m_e}{m_\mu} &\simeq 0.005\,, \\ \frac{m_\mu}{m_\tau} &\simeq 0.06\,. \end{aligned} \tag{17}$$

If the ratios of the charged lepton masses were universal, the mass of the electron would have to be about 6 MeV.

The universality is not expected to be exact, due to radiative corrections. A radiative correction of the order of $\pm(\alpha/\pi)m_\tau \simeq \pm 4$ MeV would have to be added to the charged lepton masses. Such a contribution is relatively small for the muon and the tauon, but large in comparison to the observed electron mass. One expects that the physical electron mass is the sum of a bare electron mass M_e, due to the texture zero mass matrix, and a radiative correction R_e:

$$m_e = M_e - R_e \,.$$

If we assume, that the bare electron mass is given by the universality, we obtain:

$$M_e = 5.51 \text{ MeV}\,, \quad R_e = 5.00 \text{ MeV}\,.$$

Radiative corrections also contribute to the muon and the tauon mass, but here the corrections are small in comparison to the bare masses and will be neglected.

We calculate the angle θ_l (see Eq. (13)):

$$\tan\theta_l \simeq \sqrt{M_e/m_\mu} \simeq 0.23\,, \quad \theta_l \simeq 13^\circ\,. \tag{18}$$

This angle agrees with the experimental result.

If we would not have taken into account the radiative corrections for the electron mass, we would obtain a value for the reactor angle, which is much smaller than the experimental value:

$$\tan\theta_l \simeq \sqrt{m_e/m_\mu} \simeq 0.07\,, \quad \theta_l \simeq 4^\circ\,. \tag{19}$$

The neutrino masses, given in Eq. (15), are very small, much smaller than the masses of the charged leptons. The ratio of the mass of the tau neutrino and of the mass of the tauon is only about 2.7×10^{-11}.

Are the neutrino masses normal Dirac masses, as the masses of the charged leptons and quarks, or are they Majorana masses? In this case the smallness of the neutrino masses can be understood by the "seesaw"-mechanism.[8–12] Here the neutrino masses are mixtures of Majorana and Dirac masses.

The mass matrix of the neutrinos is a matrix with one "texture zero" in the (1, 1)-position. The two off-diagonal terms are given by the Dirac mass term D — a large Majorana mass term appears in the (2, 2)-position:

$$M_\nu = \begin{pmatrix} 0 & D \\ D & M \end{pmatrix}. \tag{20}$$

After diagonalization one obtains a large Majorana mass M and a small neutrino mass:

$$m_\nu \simeq D^2/M\,. \tag{21}$$