I. P. NATANSON

THEORY OF FUNCTIONS OF A REAL VARIABLE

Volume II

Translated from the Russian by
LEO F. BORON

Theoretical physicists, engineers, senior and graduate students of Lebesgue integrals and related chapters of mathematics will not only find this an invaluable reference book, but will also be delighted by the clarity and elegance of diction and proofs.

I. P. NATANSON

THEORY OF FUNCTIONS OF A REAL VARIABLE

(Teoria functsiy veshchestvennoy peremennoy,
Chapters X to XVII and Appendices)

VOLUME II

Translated from the Russian by
LEO F. BORON

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TRANSLATOR'S NOTE

THIS is a translation of Professor Natanson's *Teoriya Funktsii Veshchestvennoi Peremennoi*, Chapters X-XVI, XVIII of the second revised Russian edition, 1957; Chapter XVIII forms the content of Chapter XVII of the present volume. Chapter XVII of the Russian edition was omitted as such because the author follows quite closely (except for Appendix VI) the material in the Boron-Hewitt American edition of Chapters I-IX listed as Editor's Appendices and written by Professor Dr. Edwin Hewitt, dealing with the definition and basic properties of arbitrary measurable sets on the real line, the basic properties of measurable functions defined on sets which are not necessarily bounded, the definition of the Lebesgue integral for functions defined on unbounded sets, square-summable functions defined on arbitrary measurable sets on the real line, functions of finite variation on the infinite line $(-\infty, \infty)$ and the Stieltjes integral of such functions. Appendices I, II, III are the same as in the second Russian edition. Appendix IV, Change of Variable in the Lebesgue Integral, is §5, Chapter IX, 2nd ed. Appendix V, Hausdorff's Theorem, is §5, Chapter XI, 1st ed. Appendix VI, Indefinite Integrals and Absolutely Continuous Set Functions, is §6, Chapter XVII, 2nd ed. Appendix VII, The Role of Russian and Soviet Mathematicians in the Development of the Theory of Functions of a Real Variable, is Chapter XVII, 1st ed.

In this volume, whenever reference is made to Chapters I-IX, it is understood to mean that the reader should consult the first volume of Natanson's *Theory of Functions of a Real Variable*, Chapters I-IX, Frederick Ungar, N.Y., 1955.

The enthusiastic response to the first volume of the American edition prompted the preparation of the remainder of this excellent Russian work for the English-speaking mathematical public. The translator's only hope is that he has reasonably closely approximated the reader's expectations.

LEO F. BORON

Philadelphia, 1960

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