

# Slope Stability Analysis by the Limit Equilibrium Method

Fundamentals and Methods



Yang H. Huang, Sc.D., P.E.

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Limit Equilibrium Method*

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# *Slope Stability Analysis by the Limit Equilibrium Method*

## Companion Title

*LEAME Software and User's Manual: Analyzing Slope Stability by the Limit Equilibrium Method*, by Yang H. Huang, Sc.D., P.E. (ASCE Press, 2014). Offers a PC-based software program for performing two- and three-dimensional slope stability analyses, accompanied by supporting documentation and worked examples. (ISBN 978-0-7844-7799-1; online at <http://dx.doi.org/10.1061/9780784477991>)

## Other Titles of Interest

*Geotechnical Testing, Observation, and Documentation, Second Edition*, by Tim Davis. (ASCE Press, 2008). Presents an in-depth field manual for soil technicians and geotechnical engineers for use during the investigation, grading, and construction phases of geotechnical projects. (ISBN 978-0-7844-0949-7)

*Slope Stability and Earth Retaining Walls*, edited by Hanlong Liu, Ph.D.; Robert Y. Liang, Ph.D., P.E.; Ke Yang, Ph.D., P.E.; and Jamal Nusairat, Ph.D., P.E. (ASCE Geotechnical Special Publication No. 216, 2011). Addresses contemporary geotechnical subjects including slope stability evaluations; slope and landslide movement monitoring techniques; and recent developments of earth retaining structures. (Online only; <http://dx.doi.org/10.1061/9780784476277>)

*Slope Stability, Retaining Walls, and Foundations: Selected Papers*, edited by Louis Ge, Ph.D., P.E.; Jinyuan Liu, Ph.D.; James-C. Ni, Ph.D., P.E.; and Zhaoyi He, Ph.D. (ASCE Geotechnical Special Publication No. 197, 2009). Showcases recent developments and advancements in geotechnical, focusing soil stabilization, dynamic behavior of soils and foundations, earth retaining walls, and slope stability. (ISBN 978-0-7844-1049-3)

*Slopes and Retaining Structures under Seismic and Static Conditions*, edited by Mohamed A. Gabr, Ph.D., P.E.; John J. Bowders, Ph.D., P.E.; David Elton, Ph.D., P.E.; and Jorge G. Zornberg, Ph.D., P.E. (ASCE Geotechnical Special Publication No. 140, 2005). Examines the design of slopes and retaining structures that pose challenges to geotechnical engineers. (Online only; <http://dx.doi.org/10.1061/9780784407875>)

# Preface

During the past 40 years, I have been engaged in a study on the stability analysis of earth slopes. The study was initiated in 1973 when I received a research grant from the Institute for Mining and Minerals Research, University of Kentucky. When the research project was completed in 1977, the U.S. Congress passed the Surface Mining Control and Reclamation Act, which requires stability analysis for refuse dams, hollow fills, and spoil banks created by surface mining, thus putting the research into practical use. The results of the research were published in several journals and reports and also were presented in a number of short courses. Both the simplified and the computerized methods of stability analysis, as developed from this research, have been widely used by practicing engineers throughout Kentucky for the application of mining permits. The large number of out-of-state participants in the short courses indicates that the methods developed have widespread applications.

In 1983, my book *Stability Analysis of Earth Slopes* was published by the Van Nostrand Reinhold Company. The book was well received by the engineering profession and was reprinted quite a few times. It was recommended by the Professional Civil Engineering Book Club as a feature selection and was translated into Chinese and Russian by foreign publishers. Two computer programs, one called SWASE (Sliding Wedge Analysis of Sidehill Embankments) for analyzing plane or noncircular failure surfaces, and the other called REAME (Rotational Equilibrium Analysis of Multilayered Embankments) for cylindrical failure surfaces, written in both Fortran and Basic languages, were listed in the book. In 1994, the SWASE program was incorporated into the REAME program, and a separate program for three-dimensional analysis, named REAME3D, was developed. In 1996, the first Windows version of REAME for both two- and three-dimensional analyses was released and used widely by the mining industries. The program has been continuously improved, and a new version has been released every four years, culminating in the latest REAME2012. Because the name REAME is a misnomer and the computer software associated with this book is very similar to REAME2012 with only some minor changes, the name REAME has been changed to LEAME (Limit Equilibrium Analysis of Multilayered Earthworks) in this book to reflect the capabilities of the software better.

Further evolutions of the book and software have prompted their separation into two separate products. *Slope Stability Analysis by the Limit Equilibrium Method: Fundamentals and Methods* presents the basic principles at work in the analysis of slope stability and common methods for analyzing slope stability in two and

three dimensions. A companion product, *LEAME Software and User's Manual: Analyzing Slope Stability by the Limit Equilibrium Method* provides both the software program and the supporting documentation for its use. The software can be obtained at <http://dx.doi.org/10.1061/9780784477991>.

Although some of the materials presented in this book, such as the five chapters in Part 1 and Chapter 7 in Part 2, are essentially the same as the 1983 book, this revised and updated volume is dramatically different in the following aspects:

1. Many new sections have been added, such as the back-calculation of shear strength, undrained shear strength varying linearly with depth, granular materials with curved strength envelope, unsteady-state seepage, and external and internal forces.
2. Some new stability charts have been added and some others have been deleted, because they are too cumbersome for hand calculations. With the availability of the LEAME software, no one likes to resort to stability charts for preliminary analysis unless they are very simple to use.
3. Only the limit equilibrium method is covered here, and the section on finite element method is eliminated. Also, only the methods incorporated in LEAME are presented in detail, while the sections on Janbu's method and Morgenstern's and Price's method are eliminated. The section on the probabilistic method has been expanded greatly, and a new chapter on reliability is presented.
4. The three-dimensional analysis, which was not even mentioned in the previous book, is presented here in a full chapter. It covers the theoretical aspect by showing how the failure mass is divided into columns and deriving the equations used for LEAME.
5. Spreadsheets have been added to solve many of the examples, and the results are compared with the LEAME computer program. It is amazing that many problems involving iterations or using trial-and-error can be solved easily by spreadsheets. Although spreadsheets can be used to check the correctness of a computer program, they cannot serve as a substitute, because they involve only a single failure surface; to determine the minimum factor of safety, hundreds of failure surfaces need to be analyzed.
6. Homework problems and more examples have been added so the book can serve as a college text for senior and graduate courses in soil mechanics and geotechnical engineering.

This volume is divided into two parts. Part 1 presents the fundamentals of slope stability and consists of five chapters. Chapter 1 describes slope movements and discusses some of the more well-known methods for stability analysis. Chapter 2 explains the mechanics of slope failures and defines the factor of safety for both cylindrical and plane failures. Chapter 3 discusses both the laboratory and the field methods for determining the shear strength of soils used for

stability analysis. Chapter 4 illustrates some methods for estimating the location of the phreatic surface and determining the pore pressure ratio. Chapter 5 outlines remedial measures for correcting slides.

Part 2 presents methods of stability analysis and also consists of five chapters. Chapter 6 derives some simple formulas for determining the factor of safety for plane failures. Chapter 7 presents a number of stability charts for determining the factor of safety for cylindrical failures, as well as the well-known friction circle and logarithmic spiral methods. Chapter 8 discusses methods of slices for two-dimensional analysis and derives the equations used in developing LEAME. Chapter 9 discusses the three-dimensional analysis with both ellipsoidal and planar ends and derives the equations used in LEAME. Chapter 10 discusses various methods to determine the reliability of slope design, including the use of Taylor's expansion for closed-form solutions and the mean-value first order second moment (MFOSM) method for computer solutions.

The Appendix provides an introduction to the LEAME software to encourage readers to obtain the software. The LEAME computer software is completely different from the REAME program listed in the 1983 book. It is an excellent and well-tested software program to determine the factors of safety for both two- and three-dimensional slopes and contains many new features not available elsewhere. It can be downloaded and used right away to solve practical problems in slope stability.

Finally, I want to thank ASCE Press for giving me the opportunity to publish these books. It is my sincere hope that the books, especially the LEAME computer software, will be helpful to civil engineers in their engineering practice and to college professors in teaching courses in slope stability.

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## Part 1

# Fundamentals of Slope Stability



# Chapter 1

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## Introduction

Problems associated with failures of natural and artificial slopes often pose formidable challenges in geotechnical engineering. In general, an exposed inclined ground surface that is unrestrained may be prone to mass movement due to gravitational forces. The resulting shear stresses, induced along a potential or known failure surface, could exceed the shear strength of the soil and cause slope failure. The ratio of available shear strength to induced shear stress on a potential failure surface most commonly is referred to as the factor of safety. The intent of any limit equilibrium stability analysis is to determine this factor, by which the soil strength is divided or reduced, to bring the slope to the threshold of instability. The types of slope movements and the use of various limit equilibrium methods to determine the factor of safety for the sliding types of mass movements are reviewed in this chapter.

---

### 1.1 Slope Movements

The stability analysis of slopes plays a very important role in civil engineering. Stability analysis is used in the construction of transportation facilities such as highways, railroads, airports, and canals; the development of natural resources such as surface mining, refuse disposal, and earth dams; as well as many other

human activities involving building construction and excavation. Failures of slopes in these applications may be caused by movements within the human-created cut or fill, in the natural slope, or a combination of both. These movement phenomena usually are studied from two different points of view. Geologists consider the movement phenomena as a natural process and study the crux of their origin, their courses, and the resulting surface characteristics. Engineers, however, investigate the safety of slopes based on the principles of soil mechanics and develop methods for a reliable assessment of the stability of slopes, as well as the controlling and corrective measures needed. The most viable stability studies can be achieved only by the combination of both approaches. The quantitative assessment of the stability of slopes by the methods of soil mechanics must be based on knowledge of the geological structure of the area, the detailed composition and orientation of strata, and the geomorphological history of the land surface. On the other hand, geologists may obtain a clearer picture of the origin and character of the movement process by checking their considerations against the results of engineering analyses based on soil mechanics. For example, it is well known that one of the most favorable settings for landslides is where permeable or soluble beds overlie or are interbedded with relatively impervious beds. This geological phenomenon was explained by Henkel (1967) using the principles of soil mechanics.

Slopes failures involve such a variety of processes and causative factors that they afford unlimited possibilities of classification. For instance, they can be divided according to the form of failures, the type of materials moved, the age, the stage of development, or the cause of movements.

One of the most comprehensive references on landslides or slope failures is a special report published by the Transportation Research Board (Turner and Schuster 1996). According to this report, the type of slope movements is divided into five main groups: falls, topples, slides, spreads, and flows (Varnes 1978). A sixth group, complex slope movements, includes the combination of two or more of these five types. The type of materials is divided into two classes: rock and soil. Soil is further divided into debris and earth. Table 1-1 shows the classification of slope movements.

Recognizing the types of slope movements is important because they dictate the method of stability analysis and the remedial measures to be employed. Varnes (1978) described the types of movements as follows:

- In falls, a mass of any size is detached from a steep slope or cliff, along a surface on which little or no shear displacement takes place, and descends mostly through the air by free fall, leaping, bouncing, or rolling. Movements are very rapid and may or may not be preceded by minor movements leading to progressive separation of the mass from its source.
- In topples, one or more units of mass rotate forward about some pivot point, below or low in the unit, under the action of gravity and forces exerted by adjacent units or by fluids in cracks. In fact, it is tilting without collapse.

**Table 1-1.** *Classification of Slope Movements*

TYPE OF MOVEMENT			TYPE OF MATERIAL		
			Engineering Soils		
			Bedrock	Predominantly Coarse	Predominantly Fine
Falls			Rock fall	Debris fall	Earth fall
Topples			Rock topple	Debris topple	Earth topple
Slides	Rotational	Few units	Rock slump	Debris slump	Earth slump
	Translational		Rock block slide	Debris block slide	Earth block slide
		Many units	Rock slide	Debris slide	Earth slide
Lateral spreads			Rock spread	Debris spread	Earth spread
Flows			Rock flow (deep creep)	Debris flow (soil creep)	Earth flow (soil creep)
Complex			combination of two or more principal types of movement		
(Varnes 1978, © National Academy of Sciences, Washington, DC. Reproduced with permission of the Transportation Research Board, Washington, DC)					

- In slides, the movement consists of shear strain and displacement along one or several surfaces that are visible or may reasonably be inferred, or within a relatively narrow zone. The movement may be progressive; that is, shear failure may not initially occur simultaneously over what eventually becomes a defined surface of rupture, but, rather, it may propagate from an area of local failure. This displaced mass may slide beyond the original surface of rupture onto what had been the original ground surface, which then becomes a surface of separation. Slides are divided into rotational slides and translational slides. This distinction is important because it affects the methods of analysis and control. Furthermore, the presence of a weak sublayer or boundary between weathered and unweathered strata reveals another type of slide known as the compound slide (Cruden and Varnes 1996). These geological anomalies dictate the location of the surface of rupture (Hutchinson 1988).
- In spreads, the dominant form of movement is lateral extension accompanied by shear or tensile fractures. Movements may involve fracturing and extension of coherent material, either bedrock or soil, owing to liquefaction or plastic flow of subjacent material. The coherent upper units may subside, translate, rotate, or disintegrate, or they may liquefy and flow. The mechanism of failure can involve elements not only of rotation and translation but also of flows; hence, some lateral spreading failures may be regarded as complex. The sudden spreading of clay slopes was discussed by Terzaghi and Peck (1967).



- In flows, slope movements may appear in several different forms. In unconsolidated materials, they generally take the form of fairly obvious flows, fast or slow, wet or dry. In saturated soils, they are triggered when the ground becomes completely saturated by infiltration, as a result of intense rainfall or by the rise of the groundwater table. Thus, water is the primary transport agent and the saturated soils resembling a viscous fluid possess no strength and result in shallow slope failures. In bedrock, the movements are extremely slow and are distributed among many closely spaced, noninterconnected fractures that result in folding, bending, or bulging.

According to age, slope movements are divided into contemporary, dormant, and fossil movements. Contemporary movements are generally active and are relatively easily recognizable by their configuration, because the surface forms produced by the mass movements are expressive and not affected by rainwash and erosion. Dormant movements usually are covered by vegetation or are disturbed by erosion so that the traces of their last movements are not easily discernible. However, the causes of their origin still remain and the movement may be renewed. Fossil movements generally developed in the Pleistocene or earlier periods, under different morphological and climate conditions, and cannot repeat themselves at present.

According to stage, slope movements can be divided into initial, advanced, and exhausted movements. At the initial stage, the first signs of the disturbance of equilibrium appear and cracks in the upper part of the slope develop. In the advanced stage, the loosened mass is propelled into motion and slides downslope. In the exhausted stage, the accumulation of slide mass creates temporary equilibrium conditions.

Chowdhury (1980) classified slides according to their causes: (1) exceptional landslides arising from earthquake, intense precipitation, severe flooding, accelerated erosion from wave action, and liquefaction; (2) ordinary landslides resulting from known or usual causes that can be explained by traditional theories; and (3) unexplainable landslides that occur without any apparent cause.

It should be evident from this discussion that the stability of slopes is a complex problem, which may defy any theoretical analysis. In this book only the slide type of mass movements will be discussed, not only because it is more amenable to theoretical analysis but also because it is the predominant type of failures, particularly in human-created slopes.

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## 1.2 Limit Plastic Equilibrium

The primary purpose of most stability analyses is to determine the factor of safety of the slope based on the concept of limit plastic equilibrium. First, a failure surface is assumed. A state of limit equilibrium is said to exist when the shear stress along the failure surface is expressed as