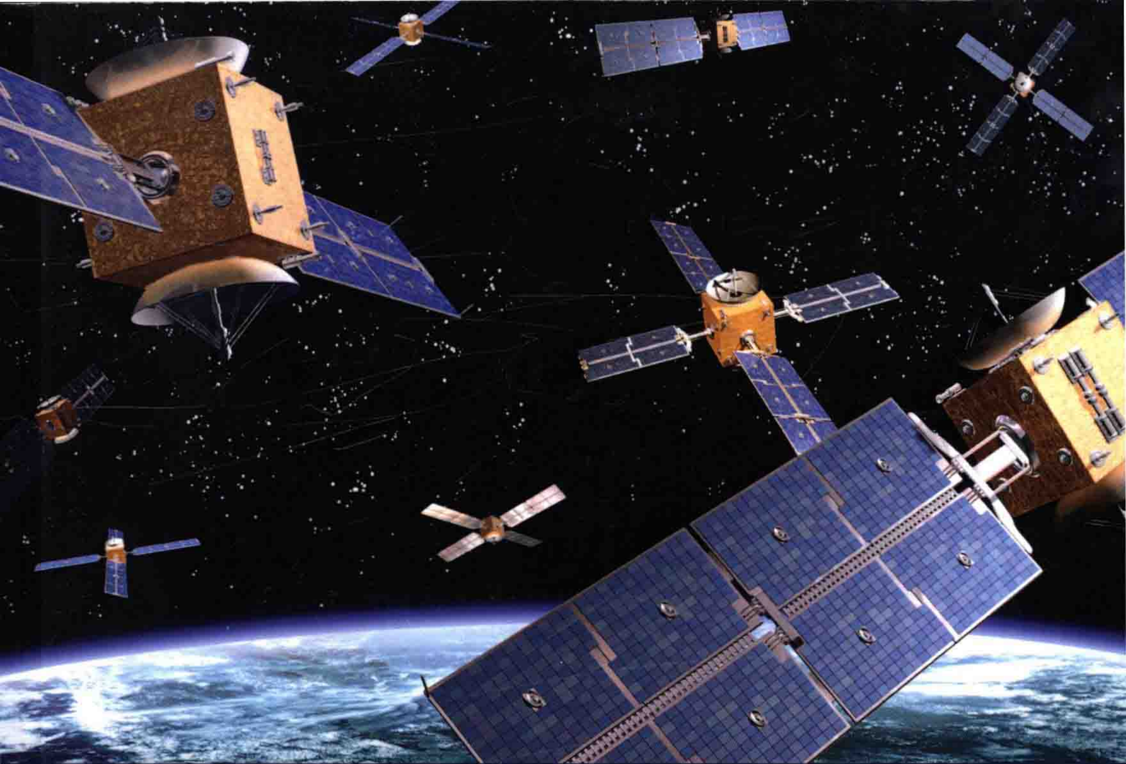
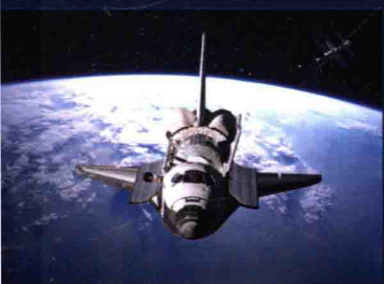
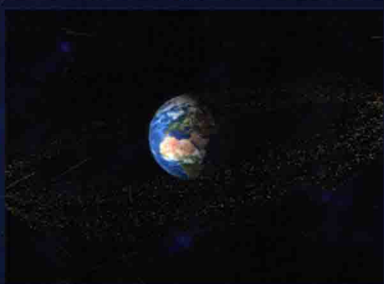
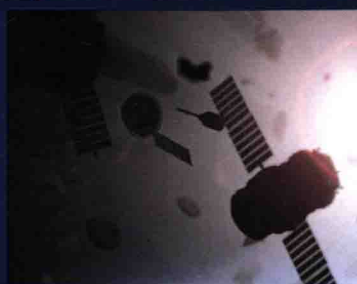

RIGID BODY DYNAMICS FOR SPACE APPLICATIONS

Vladimir S. Aslanov

RIGID BODY DYNAMICS FOR SPACE APPLICATIONS

Vladimir S. Aslanov

Head of Theoretical Mechanics Department, Samara National Research University, Russia

Most space systems can be considered as a system of rigid bodies, and in some cases, with additional elastic and viscoelastic elements and fuel residuals. *Rigid Body Dynamics for Space Applications* shows the nature of the phenomena and explains the behavior of space objects as a system of rigid bodies, based on the knowledge of classical mechanics, regular and chaotic dynamics.

This book covers existing problems of spaceflight mechanics, such as attitude dynamics of re-entry capsule in Earth's atmosphere, dynamics and control of coaxial satellite gyrostats, dynamics and control of a tether-assisted return mission, and removal of large space debris by a tether tow. Researchers working on spacecraft attitude dynamics or space debris removal as well as those in the fields of mechanics, aerospace engineering, and aerospace science will benefit from this book.

Key Features

- Provides a complete treatise of modeling attitude for a range of novel and modern attitude control problems of spaceflight mechanics
- Features chapters on the application of rigid body dynamics to atmospheric re-entries, on tethered assisted re-entry, and on tethered space debris removal
- Shows relatively simple ways of constructing mathematical models and analytical solutions describing the behavior of very complex material systems
- Uses modern methods of regular and chaotic dynamics to obtain results

About the author

Vladimir S. Aslanov graduated from the Kuibyshev Aviation Institute, Kuibyshev, USSR (KuAI) in 1972 with a specialty "Aircraft Engineer," and received his PhD in 1977. Since 1989, he has been the head of the Department of Theoretical Mechanics at Samara National Research University in Russia. His scientific interests are classical mechanics, nonlinear oscillations and chaotic dynamics, mechanics of space flight, dynamics of gyrostats, dynamics of tethered satellite systems, and spacecraft stability.

AEROSPACE
ENGINEERING

ISBN 978-0-12-811094-2

9 780128 110942

ELSEVIER

Butterworth-Heinemann
An imprint of Elsevier
elsevier.com/books-and-journals

RIGID BODY DYNAMICS FOR SPACE APPLICATIONS

Aslanov

RIGID BODY DYNAMICS FOR SPACE APPLICATIONS

VLADIMIR S. ASLANOV

ELSEVIER

Butterworth-Heinemann
An imprint of Elsevier

Butterworth-Heinemann is an imprint of Elsevier
The Boulevard, Langford Lane, Kidlington, Oxford OX5 1GB, United Kingdom
50 Hampshire Street, 5th Floor, Cambridge, MA 02139, United States

Notices
Knowledge and best practice in this field are constantly changing. As new research and experience broaden our understanding, changes in research methods, professional practices, or medical treatment may become necessary.

Practitioners and researchers must always rely on their own experience and knowledge in evaluating and using any information, methods, compounds, or experiments described herein. In using such information or methods they should be mindful of their own safety and the safety of others, including parties for whom they have a professional responsibility.

To the fullest extent of the law, neither the Publisher nor the authors, contributors, or editors, assume any liability for any injury and/or damage to persons or property as a matter of products liability, negligence or otherwise, or from any use or operation of any methods, products, instructions, or ideas contained in the material herein.

British Library Cataloguing-in-Publication Data
A catalogue record for this book is available from the British Library

Library of Congress Cataloging-in-Publication Data
A catalog record for this book is available from the Library of Congress

ISBN: 978-0-12-811094-2

Publisher: Joe Hayton
Acquisition Editor: Carrie Bolger
Editorial Project Manager: Carrie Bolger
Production Project Manager: Anusha Sambamoorthy
Cover Designer: Victoria Pearson

Typeset by SPi Global, India

RIGID BODY DYNAMICS FOR SPACE APPLICATIONS

DEDICATION

To my parents, and wife Lyudmila

–Vladimir S. Aslanov

PREFACE

Soviet sputnik ushered in the space era in Oct. 1957. This launch occurred almost 70 years ago and nonetheless interest in the creation and use of space technics continues unabated. New challenges formulate new ideas, which be implemented in the new space programs. Currently, global unexpected, including, and environmental concerns associated with big population of nonfunctional and abandoned satellites, spent upper stages and fragments. Each new space program is unique and requires new technologies and careful research based on mathematical modeling. Results of the mathematical simulation and the analytical solutions allow a better understanding of the phenomenon and processes of spacecraft functioning, and choose the conceptual design of future aerospace systems. Most of the space systems can be considered as a system of rigid bodies, and in some cases, with additional elastic and viscoelastic elements, and with fuel residuals.

The purpose of the book is to show the nature of the phenomena and to explain features of the behavior of space objects, as a system of rigid bodies, based on the knowledge of classical mechanics, regular and chaotic dynamics. The author tried to show relatively simple ways of constructing mathematical models and analytical solutions describing the behavior of very complex mechanical systems. The book contains many analytical and approximate analytical solutions that help to understand the nature of the studied phenomena. It is based on the recent papers of the author in international journals, which have been reviewed by leading scientists of the world, thus the results can be trusted. This book covers modern problems of spaceflight mechanics, such as attitude dynamics of reentry capsule in Earth's atmosphere, dynamics and control of coaxial satellite gyrostats, dynamics and control of a tether-assisted return mission, removal of large space debris by a tether tow.

The author hopes that this book will be helpful for a wide range of scientists, engineers, graduate students, university teachers, and students in the fields of mechanics, and aerospace science. Graduate students and researchers find in the book the new results of studies in a wide range of aerospace applications, and they can also use it as tool for obtaining new knowledge. Aerospace engineers can get engineering approaches to the development of new space systems. University teachers can use the text for preparation of new sections in the course of the mechanics of space flight and students will have updated courses of lectures.

The book consists of six chapters. It begins from the necessary fundamentals. Chapter 1 covers basic aspects of mathematics and mechanics, including elliptic functions, rigid body kinematics, Serret–Andoyer canonical variables, and Poincare and Melnikov's methods. Chapter 2 explores uncontrolled descent of the reentry capsule into an atmosphere by the averaging method and methods of chaotic dynamics. Chapter 3 deals with attitude motion of free dual-spin satellite gyrostats. Exact analytical solutions of the undisturbed motion are presented for all possible ratios of inertia moments of the gyrostats. Chapter 4 is devoted to a tether-assisted reentry capsule return mission. Chapter 5 considers a problem of removal of large space debris by a space transportation system, which is composed of a space tug connected by a tether with the space debris. Chapter 6 contains several separate issues of space flight mechanics, which are of great practical interest, but were not included in previous chapters: the problem of the gravitational stabilization of the satellite by a controlled motion of a point mass on board, dynamics of a space vehicle during retrorocket engine operating, and restoration of attitude motion of satellite using small numbers of telemetry measurements.

I would like to acknowledge brilliant Russian scientist in the field of Aeronautics and Astronautics Professor Vasiliy Yaroshevskiy for special attention and support in the beginning of my academic career, and my first research supervisor Professor Vitali Belokonov. I would like to thank all of my friends and colleagues who helped me make my researches, in particular Dr. Viktor Boyko, Professor Ivan Timbay, Dr. Anton Doroshin, Dr. Alexander Ledkov, and Dr. Vadim Yudintsev. Especially, I would like to express my appreciation to Dr. Alexander Ledkov and Dr. Vadim Yudintsev for their help in the work on this manuscript. I also thank Elsevier for their support and publication of this book, and Samara National Research University in the person of Rector Evgeniy Shakhmatov and President Viktor Soifer for productive environment and the opportunity to work with interesting people.

This book contains results of researches that has been supported in part by the Russian Science Foundation (project no. 16-19-10158, Chapter 5), the Russian Foundation for Basic Research (project no. 15-01-01456-A), and the Ministry Education and Science of Russia (Contract No. 9.540.2014/K).

Vladimir S. Aslanov

CONTENTS

CHAPTER 1

Mathematical Mechanical Preliminaries

1.1 MATHEMATICS

This section contains basic information on elliptic integrals and elliptical functions with examples of their use. Detailed information about elliptic integrals, elliptical functions, and their applications can be found in Refs. [1–6].

1.1.1 Elliptic Integrals

An elliptic integral is an integral that can be written in the form:

$$\int R\left(x, \sqrt{P(x)}\right) dx \tag{1.1}$$

where $R(x, y)$ is a rational function and $P(x)$ is a polynomial of the third or fourth degree in x.

Let us consider the following integral:

$$u = F(\varphi, k) = \int_0^{\varphi} \frac{dx}{\sqrt{1 - k^2 \sin^2 x}} \tag{1.2}$$

which is called incomplete integral of the first kind. The incomplete elliptic integral is a function of angle φ and the elliptical modulus $k (0 \leq k < 1)$.

When the amplitude $\varphi = \pi/2$, the incomplete integral of the first kind is said to be complete elliptic integral of the first kind and is denoted as $K(k)$:

$$K(k) = F(\pi/2, k) = \int_0^{\pi/2} \frac{dx}{\sqrt{1 - k^2 \sin^2 x}} \tag{1.3}$$

Definite integral

$$E(\varphi, k) = \int_0^{\varphi} \sqrt{1 - k^2 \sin^2 x}\, dx \tag{1.4}$$

Rigid Body Dynamics for Space Applications
http://dx.doi.org/10.1016/B978-0-12-811094-2.00001-7

is called incomplete integral of the second kind, and as the integral of the first kind, it also has complete form when $\varphi = \pi/2$

$$E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 x}\, dx \tag{1.5}$$

Incomplete elliptic integral of the third kind is

$$\Pi(\varphi, n, k) = \int_0^{\varphi} \frac{1}{1 - n\sin^2\theta} \frac{d\theta}{\sqrt{1 - k^2 \sin^2\theta}} \tag{1.6}$$

Complete elliptic integral of the third kind is

$$\Pi(n, k) = \int_0^{\pi/2} \frac{d\theta}{(1 - n\sin^2\theta)\sqrt{1 - k^2 \sin^2\theta}} \tag{1.7}$$

where n is called the characteristic.

The elliptic integrals satisfy the following relations:

$$F(-\varphi, k) = -F(\varphi, k) \tag{1.8}$$

$$F(n\pi \pm \varphi, k) = 2nK(k) \pm F(\varphi, k) \tag{1.9}$$

$$E(-\varphi, k) = -E(\varphi, k) \tag{1.10}$$

$$E(n\pi \pm \varphi, k) = 2nE(k) \pm E(\varphi, k) \tag{1.11}$$

For small $k^2 \ll 1$, complete elliptic integrals can be expanded into series as follows:

$$\frac{2}{\pi}K(\mathrm{k}) = 1 + \sum_{n=1}^{\infty} \left[\frac{(2n-1)!!}{2^n \cdot n!}\right]^2 k^{2n} \tag{1.12}$$

$$\frac{2}{\pi}E(\mathrm{k}) = 1 - \sum_{n=1}^{\infty} \left[\frac{(2n-1)!!}{2^n \cdot n!}\right]^2 \frac{k^{2n}}{2n-1} \tag{1.13}$$

where $(2n-1)!! = 1 \cdot 3 \cdot 5 \ldots (2n-1)$.

1.1.2 Elliptic Functions

The inverse functions of incomplete elliptic integral of the first kind form the elliptical functions. The amplitude function is defined as (see Eq. 1.2)

$$\varphi = \text{am}\,u \tag{1.14}$$

The elliptical sine function $\text{sn}(u, k)$ is given by

$$\text{sn}(u, k) = \sin\varphi = \sin(\text{am}\,u) \tag{1.15}$$

The elliptical cosine function $\text{cn}(u, k)$ is given by

$$\text{cn}(u, k) = \cos\varphi = \cos(\text{am}\,u) \tag{1.16}$$

Elliptic functions $\text{sn}(u, k)$ and $\text{cn}(u, k)$ have period $4K(k)$ for the argument u. Delta amplitude function $\text{dn}(u, k)$ is given by the expression:

$$\text{dn}(u, k) = \frac{d\varphi}{du} = \sqrt{1 - k^2 \sin^2\varphi} = \sqrt{1 - k^2 \text{sn}^2(u, k)} \tag{1.17}$$

This function has period $2K(k)$.

The elliptic functions satisfy the following relations:

$$\text{cn}^2 u + \text{sn}^2 u = 1 \tag{1.18}$$

$$\text{dn}^2 u + k^2 \text{sn}^2 u = 1 \tag{1.19}$$

The derivatives of the elliptic functions are given by the following expressions:

$$\frac{d\,\text{am}\,u}{du} = \text{dnu} \tag{1.20}$$

$$\frac{d}{du}\text{sn}\,u = \text{cn}\,u \cdot \text{dn}\,u \tag{1.21}$$

$$\frac{d}{du}\text{cn}\,u = -\text{sn}\,u \cdot \text{dn}\,u \tag{1.22}$$

$$\frac{d}{du}\text{dn}\,u = -k^2\,\text{sn}\,u \cdot \text{cn}\,u \tag{1.23}$$

Hyperbolic and trigonometric functions are the special cases of elliptical functions. For $k = 1$, Eq. (1.2) has the form:

$$u = \int_0^{\varphi} \frac{dx}{\sqrt{1 - \sin^2 x}} = \ln\left(\frac{1}{\cos\varphi} - \tan\varphi\right) = \ln\left(\frac{1 - \sin\varphi}{\cos\varphi}\right) \tag{1.24}$$

that means

$$e^u = \frac{1 - \sin\varphi}{\sqrt{1 - \sin^2\varphi}} \tag{1.25}$$

Solving Eq. (1.25) for $\sin\varphi$, we get

$$\mathrm{sn}\, u = \sin\varphi = \frac{e^u - 1}{e^u + 1} = \tanh u \tag{1.26}$$

Therefore, taking into account Eq. (1.18), we obtain

$$\mathrm{cn}\ u = \sqrt{1 - \mathrm{sn}^2 u} = \sqrt{1 - \tanh^2 u} = \frac{1}{\mathrm{ch}\, u} \tag{1.27}$$

Taking into account Eq. (1.19) for $k = 1$, we get

$$\mathrm{dn}\, u = \sqrt{1 - \mathrm{sn}^2 u} = \sqrt{1 - \tanh^2 u} = \frac{1}{\mathrm{ch}\, u} \tag{1.28}$$

For $k = 0$,

$$u = \int_0^{\varphi} \frac{d\theta}{\sqrt{1 - k^2 \sin^2\theta}} = \int_0^{\varphi} d\theta = \varphi \tag{1.29}$$

So we get

$$\mathrm{sn}\, u = \sin\varphi \tag{1.30}$$

and

$$\mathrm{cn}\, u = \cos u, \quad \mathrm{dn}\, u = 1$$

Thus, when $k = 0$, elliptical functions degenerate to trigonometric functions.

Elliptic integrals and elliptical functions are used in mechanics and engineering. For example, they used to describe nonlinear oscillations of mechanical systems. Let us consider the motion of a physical pendulum as an example of the application of elliptical functions [6].