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Editors

# Masonry Structures: Between Mechanics and Architecture

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# Masonry Structures: Between Mechanics and Architecture

## Preface

We are pleased to present the book *Masonry Structures: Between Mechanics and Architecture*, sponsored by the Associazione Edoardo Benvenuto per la ricerca sulla Scienza e l'Arte del Costruire nel loro sviluppo storico in collaboration with the Dipartimento di Scienze per l'Architettura of the University of Genoa.

The idea of a book on masonry structures arises from the privileged context in which the Associazione Edoardo Benvenuto has carried out its activities in recent years. In fact the Associazione has been able to count on the participation of scholars of international prestige to its research and editorial initiatives, under the honorary presidency of Jacques Heyman. The book belongs to the series *Between Mechanics and Architecture*, born in 1995 from the collaboration of several internationally renowned scholars, including Edoardo Benvenuto. The first book in the series was *Entre Mécanique et Architecture/Between Mechanics and Architecture*, edited by Patricia Radelet-de Grave and Edoardo Benvenuto (Birkhäuser 1995).

As is well known, the topic of masonry structures is very complex and subject to multiple interpretations. In addition to historical studies, the mechanical behaviour of masonry arches and structures has been studied according to different lines of research (structural analysis, limit analysis, elastic analysis, plasticity, mathematical approaches, etc.), sometimes difficult to reconcile, sometimes intertwined with each other and complementary. Although we are aware that it is not possible to include in a single book the diversity of the studies on masonry structures, we have tried to represent the main approaches in order to make it easier for the reader to compare and evaluate their significance and interest.

In addition to selecting the papers published here, the editors have also played the role of reviewers of the manuscripts in conformance with the standards of peer review. In one case, in which one of the co-editors was also the co-author of a contribution, recourse was made to an external referee of international experience.

The introductory chapter, "Between Mechanics and Architecture: The Quest for the Rules of the Art" by Salvatore D'Agostino, addresses a fascinating topic: the quest for the "rules of the art", that is, the methods and procedures defined by complex experiences and verified by a practice which may be centuries old.

Continuing in the context of the search for such rules, in “Designing by “Expérience”: Lecreulx Model Tests for the Design of the Abutments of the Bridge of Fouchard”, Santiago Huerta investigates the role of experimentation to assess the stability of masonry arches. In the 1770s, the French engineer Jean-Rodolphe Perronet introduced a new type of masonry bridge, with very slender piers and extremely surbased segmental arches. Huerta examines the tests made by François Michel Lecreulx in 1774 during the construction of the bridge of Fouchard. The results demonstrated the enormous danger of a catastrophic failure by sliding. Huerta points out that Fouchard’s experiments must have been influential in the great increase of the size of buttresses from the original designs of the 1770s in all the bridges built (most of them completed after 1780). In the Appendix to his chapter, Huerta provides the transcription of the original Memoir by Lecreulx, never before published.

The complexity of the mechanics of masonry structures emerges clearly in the chapter by Mario Como, “Statics of Historic Masonry Constructions: An Essay”, author of *Statics of Historic Masonry Constructions* (Springer 2013). Como discusses the adopted hypotheses and the key passages of the main issues involved: the special features of the masonry behaviour, Heyman assumptions and their extension to the masonry continuum, the definition of the admissible equilibrium for the masonry solid by employing the principle of virtual work for masonry bodies.

From a historical point of view, the first approach to the study of mechanical behaviour is limit analysis, rooted in the contributions of Philippe de La Hire and Charles-Augustin de Coulomb. According to this line of reasoning, the masonry structures, in particular the arches, are conceived as a system of rigid blocks, focusing on the collapse mechanism and the determination of the ultimate load. In the twentieth century, this type of approach was taken up by various scholars from the point of view of the modern theory of plasticity. On the other hand, elastic analysis starts from the work of Claude-Louis Navier and from subsequent studies by Francesco Crotti, Carlo Alberto Castigliano, Ferdinand Gros de Perrodil and Antonio Signorini that little by little have contributed to define masonry structures as statically indeterminate elastic structures. This approach aims to describe the evolution of the stress and strain fields with increasing applied loads. If the solution of Castigliano is the outcome of nineteenth-century research on the statics of masonry vaults conceived as systems with linear elastic behaviour, in the twentieth century the issue about an adequate modelling of masonry material arises. This topic has led—even recently—to a renewed interest in the study of no-tension materials and in nonlinear elastic analysis of masonry arches.

The present volume contains some contributions focused on the mechanics of arches and masonry constructions, providing an overview of the recent state of the art on the matter.

In “Equilibrium Analysis”, Jacques Heyman underlines the fact that only rarely do deformations of a masonry structure need to be computed; deformations arise, almost without exception, from displacements imposed by movements of the environment (sinking of foundations, spread of abutments), and such deformations, notably cracking, do not depend on the elastic properties of the masonry.

Anna Sinopoli, in “A Semi-analytical Approach for Masonry Arch Dynamics”, proposes an analytical approach, firstly applied to the plane dynamics of a rectangular block simply supported on a moving base and then extended to the case of the arch, where each element is characterized at most by a double extended contact. This approach constitutes a first step for performing dynamic analysis through either an event-driven or a time-stepping numerical procedure.

The chapters that follow examine the mechanical behaviour of masonry structures found in historical buildings. In “On the Statics of the Dome of the Basilica of S. Maria Assunta in Carignano, Genoa”, Andrea Bacigalupo, Antonio Brencich and Luigi Gambarotta study in depth the sixteenth-century dome designed by Galeazzo Alessi, in which meridian cracking, rather common in masonry domes, requires the assessment of the dome’s safety. In order to set a general procedure for the assessment, limit analysis approaches are discussed and compared. On the basis of classic limit analysis, local (dome only) and global (dome-drum system) collapse mechanisms are examined considering the different behaviour of several structural elements (lantern, shells of the dome, drum, colonnade). Comparisons between the results obtained are carried out in order to discuss a general approach to the assessment of dome–drum systems based on both numerical tools and standard limit analyses approaches; they provide a first glance in the assessment of the dome of the Basilica.

In “The Panthéon’s Stability Already Questioned by Pierre Patte in 1770” Patricia Radelet-de Grave analyses some aspects of great historical interest related to the construction of the Panthéon in Paris. Conceived and initiated by Jacques Germain Soufflot, the construction of the Pantheon was continued after his death by Jean Baptiste Rondelet. This impressive structure was the object of various publications. As early as 1770, Pierre Patte pointed out stability problems in his *Mémoire*. Rondelet, a spokesman for Soufflot, does not answer to Patte, but writes a few notes on his copy of Patte’s *Mémoire*.

In the chapter that follows, “Transcription of Patte’s 1770 *Mémoire* on the Panthéon’s Stability Together with Rondelet’s *Marginalia*”, Radelet-de Grave provides her transcription of the historical text of 1770, along with the *Marginalia* written by Rondelet on Patte’s *Mémoire*.

Other authors of chapter in this volume use the approach of elastic analysis to study different types of masonry arches and structures.

In “Notes on Limit and Nonlinear Elastic Analyses of Masonry Arches”, Danila Aita, Riccardo Barsotti and Stefano Bennati suggest a parallel study of masonry arches via both non-linear elastic analysis, taking up the groundbreaking work of Signorini, and the so-called “method of stability areas”, originally proposed by Alfred Durand-Claye in 1867. Rather than offering two alternative paths, the two approaches may be considered complementary points of view on the same problem: the stability area method represents a particularly simple means for determining collapse load under conditions of limited material compressive strength, whereas the non-linear elastic analysis provides a helpful and, in some aspects, essential check of the former’s mechanical significance by following the evolution of the displacement field and the extension of the non-linear regions.

In his chapter “Some Aspects on the Statics of Masonry Arches”, Elio Sacco writes the equilibrium equations for the arch subjected to a distribution of point-wise forces acting on nodes lying on the line of thrust. He then determines the line of thrust for a prescribed arch geometry and loading distribution by solving a non-linear constrained minimization problem and formulates the problem of the elastic arch making use of the force method. Furthermore, the effects of the horizontal settlement of the impost of the arch are investigated.

Massimiliano Lucchesi, Miroslav Šilhavý and Nicola Zani, in “A Direct Approach to Membrane Reinforced Bodies”, deal with membrane reinforced bodies. The membrane is treated as a two-dimensional surface with concentrated material properties. Its response is linearized and depends linearly on the surface strain tensor. The response of the matrix is treated separately in three cases: as a non-linear material, as a linear material and finally as a no-tension material. An example presenting an admissible stress solution is given for a rectangular panel with membrane occupying the main diagonal plane.

The chapter by Piero Villaggio, “The Thrust of an Elastic Soil of Variable Density against a Rigid Wall”, is one of the last works written by the Professor, who passed away in January 2014, and it is a great honour for us to publish it. Villaggio examines the thrust of an elastic soil of variable density against a rigid wall, with reference to soil mechanics and complex variable method in elasticity. The theory of the equilibrium of a wall retaining earth masses was formulated by Coulomb in 1773. This topic is of great interest today, since Coulomb’s theory is still applied by engineers in order to design walls. However, while Coulomb assumes that the material is earthy, like sand or soft clay, in actual fact soil often behaves elastically, and thus the stress state inside the mass and the associated pressures on the retaining walls are different. Thus, the question arises of how to analyse the elastic stress state in a heavy medium in contact with a rigid plane, and how to determine the stress distribution at the interface. The chapter by Villaggio certainly provides an important perspective on this issue, which remains an open question to date.

We have chosen to conclude the book with a few pages written by Stefano Bennati to honour the memory of Piero Villaggio. Bennati, who worked with professor Villaggio for many years, offers us the opportunity to remember his selfless love for knowledge, his unconditional dedication to work and his rectitude and integrity. To Piero Villaggio, we are grateful for giving us a valuable paradigm of a scholar who is coherent, passionate and humble.

The present volume is intended to offer a useful tool and interesting insights for further research, since it contains important contributions to an overall picture of the state of the art on masonry structures. The reader is offered the possibility to compare different theoretical lines of inquiry (construction history, structural analysis, limit analysis, elastic analysis, plasticity, mathematical approaches, etc.) and is thus invited to go towards new horizons of research.



In closing, we wish to thank co-editor Kim Williams for her careful revision of all chapters following peer review.

Genova, Italy

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Orietta Pedemonte

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# Between Mechanics and Architecture: The Quest for the Rules of the Art

Salvatore D'Agostino

**Abstract** The ancient conception of construction, from the fourth millennium B.C. through the entire eighteenth century A.D., was based on the transmission of the 'rules of the art' of building. In the nineteenth century it was based on the development of mechanics applied to construction. It was revolutionised in the twentieth century by the creation of construction science and industrial material. Ancient architecture is now re-read in terms of mechanics, with the serious risk of betraying the ancient concepts. Instead, these should be examined with the aim of discerning the rules that governed the original construction.

**Keywords** Mechanics • Architecture • Masonry structures • Rules of the art

## 1 The Ancients' Concept of Construction

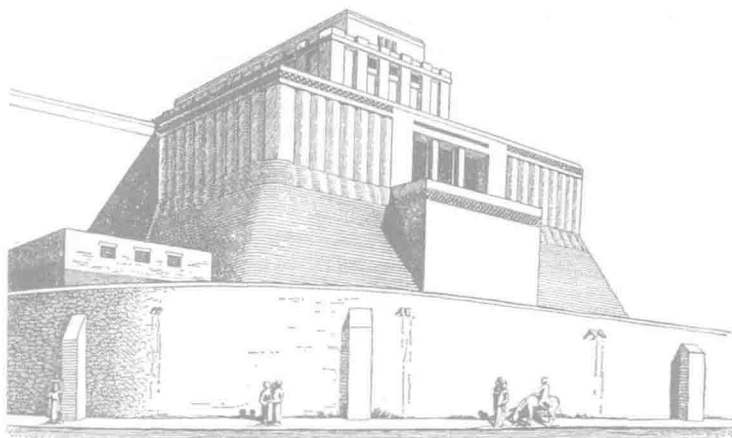
Living and building were the primordial requisites of *Homo sapiens* for a stable occupation of the territory and the construction of the earliest communities. He observed nature in its infinite configurations and continuous evolution, picking up ideas and hints about his own activities whether in hunting, agriculture, dwelling. The need to live together, grow produce and defend themselves prompted men to gather together in communities, which in turn tended to occupy the most strategic territories. In this long evolutionary process man drew on his powers of reasoning to conceive abstract forms suggested by natural shapes, and, in a lengthy rational process, man also drew on nature to tackle and solve his own needs.

In order to build, ancient man needed materials which he could only obtain from nature. Hence our use of those materials which, on account of their existence over millennia, we now define as traditional: earth, wood, the infinite variety of stone, followed by the first complex elaborations: mud and fired bricks, binding agents and metals. This is how the ancients' concept of construction evolved in its infinite formal varieties: volume conceived in space and defined by geometric forms which,

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**Fig. 1** Reconstruction of a temple at Eridu by Seton Lloyd, after a photograph by D. E. Woodall (Singer 1966)

through their dimensions, determined among other things by the quality of the materials, fulfil a range of functions in an indivisible unity, according to a spatial global conception in which load-bearing is just one of the functions that inform the project.

This conception spread with incredible tenacity from the fourth millennium B.C. through to the end of the eighteenth century (Fig. 1).

We can recall that this process also took the same course in civilizations whose practices developed in isolation, such as the pre-Columbian civilizations and those in the Far East. This lengthy process, which went hand in hand with man's historical development, could not have come about except through repeated experimentations and their constant rationalization: in this way the "rules of the art" developed in all sectors of man's activity, through failures, modifications, successes and evolutions, over the millennia (Cairolì Giuliani et al. 2007).

This process has left its mark on the evolution of human civilization; in particular, over five millennia it produced the built fabric and monumental constructions which form the material evidence of the evolution of the various civilizations. In fact, underlying the realization of both simple artefacts and of sophisticated monuments are the rules of the art (D'Agostino 2003).

## 2 The Rules of the Art

The rules of the art are methods and procedures defined by complex experiences and verified by practices which may be centuries old. They were formulated in response to material requirements and have informed everyday life since the dawn of time.

When they have specialised in the production of material products they have taken on a connotation of craftsmanship which gives rise to a professionalism that usually gained recognition as a specific trade.

A trade was acquired as standard practice by means of successive phases, and can aspire to ever higher levels of technical competence. When a craftsman both attains peaks of technical excellence and possesses a profound culture, he shows himself to be an artist capable of transmitting his own world view. This happened, and happens, in music, painting, sculpture, and so on. In architecture the process is the same, but in certain respects more complex, in that the realization of a work of architecture requires a range of processes which often take place over a lengthy period of time, involving a number of experts and kinds of expertise.

The rules of the art are still widespread and disseminated in today's world as "instructions for use". They take tangible form in a series of mechanical actions which cause a car, iPod or computer to function, without the user having to grasp the complex technical operations that enable him to exercise this control.

In the ancient world, on the contrary, the rules of the art developed through the slow, day by day acquisition of good practices passed down from one generation to the next, occasionally being improved by the genius of outstanding figures. Thus a trade was acquired not by means of an instructions manual, but through the everyday, laborious participation in the workshop or building site. In the artistic field this process actually survives in painting, sculpture and the so-called "minor arts", from ceramics to working with gold and silver, etc.

The slow acquisition of the rules of the art, together with outstanding personal abilities, created, as we have said, the "master craftsman" as well as, sometimes, the artist who realized an entire new work of art. In architecture the process was similar but not identical on account of the vast scale, complexity and often the lengthy time scale required for the completion of the work. In building the rules of the art sometimes manifested themselves in a simple, readily assimilable manner, and other times in a much more complex way, which may have involved strict secrecy. This gave rise, up until the mid-nineteenth century, to a widespread culture of building which enabled the peasant to make a house of his own, while complex, sophisticated rules, often revised in the course of operations by outstanding architects, informed the realization of large scale monumental complexes (Fig. 2).

This millennia-long process developed above all in the practice of construction, while with the advent of the Galilean revolution, both geometric forms and the resistance of the materials became objects of scientific interest, paving the way for the development of the disciplines of rational mechanics and building science (D'Agostino 2008).

**Fig. 2** Palazzo Ducale, Urbino. Photo courtesy of Gastone Segala, 2008



### 3 The Tradition of Manuals

In antiquity we know of no treatises that set out the rules of the art and construction methods, with the partial exception of Vitruvius, who makes passing reference to them. In the Renaissance there were great architects like Alberti and Palladio who, rather than systematic rules, bequeathed certain pieces of evidence and annotations on the art of building. Only Leonardo, typically, gave us some prodigious intuitions, such as the one concerning the behaviour of arches in which, with remarkable prescience, he demonstrated the fundamental presence of thrust.

In practice, for over five millennia (up to 1500 A.D.), through a constant succession of new construction methods and materials, the rules of the art of building remained quite deliberately confined to an oral and material tradition, which, as we have said, in the most significant cases were kept secret. It was only with the advent of the Galilean scientific method that, above all in the triangle formed by Italy, France and Britain, a scientific reflection began to develop focusing on the fundamental construction elements—columns, arches, vaults—while not as yet paying any attention to the way they were assembled into a built organism.

Galileo Galilei (1564–1641), who chose to enquire into the world of construction in his *Discourses* (1638), was responsible for the first reflections on the behaviour of columns and curved beams (Fig. 3). Thereafter, in the new scientific spirit of the age, numerous treatises were written which sought, on primarily geometric grounds, to define the static behaviour of the construction elements.

In the meantime, the development of architecture continued its prodigious course and, in view of the new cultural stimulus for a rational and systematic analysis of human activities, works began to circulate, alongside the treatises we have mentioned, which sought to describe the complex art of construction by pursuing knowledge, both experimental and rational, of building materials and construction elements. A first series of manuals, appearing from the mid-seventeenth to the



Fig. 3 Galileo's depiction of a beam (1638, p. 114)

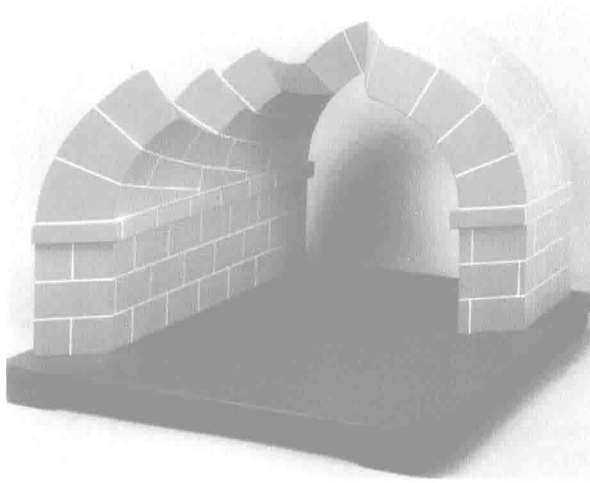
end of the eighteenth century, featured the study of construction elements through geometric constructions which in practice remained far removed from actual building.

The writings of Philippe de La Hire (1640–1718), in particular *Sur la construction des voûtes dans les édifices* of 1731, can be considered an important contribution to the rise of a theory embracing geometry and mechanics, but which has “no real practical counterpart and leads to various paradoxes” (Benvenuto 2006, p. 326, my trans.). In view of his geometric outlook La Hire can be considered the precursor of graphic statics, but over a century was to pass before this became, in the hands of engineers, a powerful method of calculation.

The manual by Bernard Forest de Bélidor (1693–1761), *Science des Ingénieurs dans la conduite des travaux de fortification et d'architecture civile*, published in Paris in 1719, proved to have more of an impact. In it, he developed de La Hire's theory of arches analytically, calculating the imposts and elaborating a first, incorrect, model of ground thrust. In addition he wrote about the construction of walls in fortifications, describing how the walls were erected.

The 1738 *Traité de la coupe des pierres* by J.B. de La Rue is full of interest for its analysis of the manufacture of the stone blocks that went into various construction elements, from vaulting to jack arches and flights of steps. The way in which stone was cut was highly important for the finished building but was even more crucial for





**Fig. 4** Model of a vault. Photo: Centro Interdipartimentale di Ingegneria per i Beni Culturali, Cabinet of Structural Models, reproduced by permission

its resistance, since a construction made from well hewn-stone could be considered practically as isodomic, with a minimum of joints—perhaps none at all—requiring fixing (Fig. 4).

Even though the manual by Vincenzo Lamberti (1740?–1790), *Statica degli edifici*, published in Naples in 1781, described itself as an eminently theoretical work, the author was aware of the mystification of the art of building and sought to make the mathematical principles and general formulae available to builders (Cirillo 2007; Lippiello 2008). Lamberti anticipated the methods of modern experimental science, carrying out trials with tufa, piperno, mortar and pozzolan. He was also probably the first author to deal with the origin of lesions and map the development of cracks (Fig. 5).

The weighty tome by Jules Dupuit (1804–1866), *Traité de l'équilibre des voûtes et de la construction des ponts en maçonnerie* dates from 1870. Published after its author's premature demise, the manual starts from the mechanical properties of masonry, showing the influence of form and height on stability. Describing the practical evolution of a vault, it sets out a theory on the way the stress curve varies and introduces, for the limit state, the concept of pivot point, which would in time lead to the plastic pivot, the key to limit state calculation. In addition, it elaborates the conditions of stability for a set of vaults, discussing the question of thickness in order to ensure stability. It analyses the problem of thermic variations and defines the thickness of the shoulders, providing formulae for the keystones of vaulting. Lastly it describes a series of major stone bridges, including the Pont de l'Alma, the Pont d'Austerlitz and the Pont Napoleone III over the Seine in Paris (Fig. 6).

These then are examples of the extensive production of manuals concerning the interpretation of the art of building based on geometrics and mechanics, from