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Interacting Systems far from Equilibrium

Quantum Kinetic Theory

KLAUS MORAWETZ



This book presents an up-to-date quantum kinetic theory based on the formalism of non-equilibrium Green's functions. It covers different applications ranging from solid state physics, plasma physics, cold atoms in optical lattices up to relativistic transport and heavy ion collisions. The basic sets of equations for these diverse systems are similar and approximations developed in one field can be adapted to another field. The central object is the selfenergy which includes all non-trivial aspects of the dynamics of a many-body system. The inductive part follows the concept of Ludwig Boltzmann to describe correlations by scattering of many particles from elementary principles up to refined approximations of many-body quantum systems. The deductive part provides a framework to develop and to apply the quantum many-body theory straight to versatile phenomena.

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INTERACTING SYSTEMS FAR FROM EQUILIBRIUM

Simplicity is the ultimate sophistication.
Leonardo da Vinci

Preface

With this book a current gap should be filled between standard textbooks on many-body theory and monographs on specific topics. The need for such a summarising overview arises first due to the current interest in describing nonequilibrium processes in different fields and secondly because of the inherent demand of theoretical physics to develop an easy and transparent language to condense many complicated developments into a straight and simple notation. The final aim is to provide a concise textbook which enables students to learn the method of nonequilibrium Green's functions.

Atoms in gases, electrons in crystals or nucleons in nuclear matter behave as independent particles in many aspects so that we can understand the properties of these systems from statistical considerations. We will follow the original question of Ludwig Boltzmann as to how one can describe the motion of many-body systems from individual collisions. In this respect the philosophy is to consider motion versus force and to distinguish between large- and small-angle collisions, where the first one leads to dissipation and collision delays and the second one to the mean field and more complex effects on the drift. In dense systems the motion of particles is not only affected by the surrounding medium but feeds back to the embedding medium which allows us to describe Bose-Einstein condensation and superconductivity on the same footing.

In the course of study there are two ways to become acquainted with a new subject. One consists of developing the theory from phenomenological findings as a bottom-up or inductive way towards the abstract formulation with the upmost simplicity in notation and compression. The second way is to learn the final theory directly and to deduce all different phenomenon as specific applications. Here both possibilities are provided, offering students the ability to learn the quantum kinetic theory in terms of Green's functions using their experience in different fields and at the same time experienced researchers offer a framework to develop and apply the theory straight to the phenomena. Reflecting the versatile applications of nonequilibrium Green's functions, examples are collected ranging from solid state physics (impurity scattering, systems with spin-orbit coupling and magnetic fields, diffraction on organic barriers, semiconductors, superconductivity, Bose-Einstein condensation, ultrafast phenomena, graphene), plasma physics (screening and ultrashort-time modes), quenches of cold atoms in optical lattices up to heavy ion collisions and relativistic transport.

Given the intricate matter and the versatile recent developments, many different views exist on how to present a theory consistently. As the old motto of the University of Rostock said: 'Doctrina multiplex veritas una'. Though preparing collectively to avoid misprints and errors, the alert reader is kindly thanked in advance for bringing any of them to my attention.

Klaus Morawetz

Dresden,
14 May 2017

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