Active Disturbance Rejection Control for Nonlinear Systems

An Introduction

Bao-Zhu Guo
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Preface

The modern control theory came to mathematics via N. Wiener's book *Cybernetics or the Science of Control and Communication in the Animal and the Machine*, published in 1948. In 1954, H.S. Tsien published the book *Engineering Cybernetics*, which brought the control theory to engineering.

Roughly speaking, the modern control theory has been developed through three stages. The first stage is the classical control or automatic principle of compensation developed from the 1940s to the 1960s. During this period the single-input and single-output linear time-invariant systems were studied by the frequency domain approach. The second stage was from the 1960s to the 1980s, during which time the multi-input and multi-output systems were studied by the state space or time domain approach. The state-space approach relies heavily upon the mathematical models of the systems. After the 1980s, many control theories were developed to cope with uncertainties in systems. Several powerful methods were developed, including the internal model principle for output regulation (which started in the 1970s), as well as adaptive control, roust control, high-gain feedback control, and sliding mode control (which started even earlier). In particular, the robust control theory was well-established by both the frequency domain approach and the time domain approach. A common feature for these methods was the worst case scenario regarding the disturbance. A different way of dealing with uncertainty may be found in adaptive control where the unknown parameters are estimated under the "exciting persistent condition" and in output regulation where a special class of external disturbance is estimated through the observer and internal model and is compensated for in the feedback-loop.

During the late 1980s and 1990s, Jingqing Han of the Chinese Academy of Sciences proposed a powerful unconventional control approach to deal with vast uncertainty in nonlinear systems. This new control technology was later called the active disturbance rejection control (ADRC). The uncertainties dealt with by the ADRC can be very complicated. They could include the coupling of the external disturbances, the system unmodeled dynamics, the zero dynamics with unknown model, and the superadded unknown part of control input. The key idea of the ADRC considers the "total disturbance" as a signal of time, which can be estimated by the output of the system. Basically, the ADRC consists of three main parts. The first part is the tracking differentiator (TD) that is relatively independent and is actually thoroughly discussed in the control theory. The aim of the TD is to extract the derivatives of the reference signal and is also considered as transient profile for output tracking. The second part of the ADRC is the extended state observer (ESO) which is a crucial part of the ADRC. In ESO, both the state and the "total disturbance" are estimated by the output of the system. This remarkable feature makes the ADRC a very different way of dealing with uncertainty. The ESO is the generalization of the traditional state observer where only the state of the system is estimated. The final part of the ADRC is the extended state observer-based feedback control. Since the uncertainty is estimated in the ESO and is compensated for in the feedback loop, the barriers between the time invariant and time varying, linear and nonlinear have been broken down by considering the time-varying part and the nonlinear part as uncertainty. At the same time, the control energy is significantly reduced. More importantly, in this way, the closed-loop systems look like linear time-invariant systems, for which a reliable result can be applied.

In the past two decades, the ADRC has been successfully applied to many engineering control problems such as hysteresis compensation, high pointing accuracy and rotation speed, noncircular machining, fault diagnosis, high-performance motion control, chemical processes, vibrational MEMS gyroscopes, tension and velocity regulations in Web processing lines and DC–DC power converters by many researchers in different contexts. In all applications of process control and motion control, compared with the huge amount of literature on control theory in dealing with the uncertainty such as system unmodeled dynamics, external disturbance rejection, and unknown parameters, the ADRC has shown its remarkable PID nature of an almost independent mathematical model, no matter the high accuracy control of micrometre grade or the integrated control on a very large scale.

On the other hand, although many successful engineering applications have been developed, the theoretical research on these applications lags behind. This book serves as an introduction to the ADRC from a theoretical perspective in a self-contained way. In Chapter 1, some basic background is introduced on nonlinear uncertain systems that can be dealt with by the ADRC. Chapter 2 presents convergence of the different types of tracking differentiators proposed by Han in his original papers. Chapter 3 is devoted to convergence of the extended state observer for various nonlinear systems. Chapters 2 and 3 can be considered as independent sections of the book. Chapter 4 looks at convergence of the closed-loop based on the TD and ESO. This can be considered as a separation principle of the ADRC for uncertain nonlinear systems. The numerical simulations are presented from here to where to illustrate the applicability of the ADRC. Finally, in Chapter 5, the ESO and stabilization for lower triangular systems are discussed.

Most of the material in this book is from the authors' published papers on this topic. However, the idea for the book comes from ideas published in Han's many original numerical experiments, engineering applications that appeared publicly in Chinese, and the numerous works done by the ADRC group, in particular the group led by Dr. Zhiqiang Gao at Cleveland State University in the United States.

The authors are deeply indebted to those who helped with the works presented in this book. These include Zhiqiang Gao, Yi Huang, and Wenchao Xue. This book is dedicated to Bao-Zhu Guo's memory of Professor Jingqing Han who passed away in 2008.

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Bao-Zhu Guo and Zhi-Liang Zhao
October, 2015

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1

Introduction

In this chapter, we introduce some necessary background about the active disturbance rejection control (ADRC). Some notation and preliminary results are also presented.

1.1 Problem Statement

In most control industries, it is hard to establish accurate mathematical models to describe the systems precisely. In addition, there are some terms that are not explicitly known in mathematical equations and, on the other hand, some unknown external disturbances exist around the system environment. The uncertainty, which includes internal uncertainty and external disturbance, is ubiquitous in practical control systems. This is perhaps the main reason why the proportional–integral–derivative (PID) control approach has dominated the control industry for almost a century because PID control does not utilize any mathematical model for system control. The birth and large-scale deployment of the PID control technology can be traced back to the period of the 1920s–1940s in response to the demands of industrial automation before World War II. Its dominance is evident even today across various sectors of the entire industry. It has been reported that 98% of the control loops in the pulp and paper industries are controlled by single-input single-output PI controllers [18]. In process control applications, more than 95% of the controllers are of the PID type [9].

Let us look at the structure of PID control first. For a control system, let the control input be $u(t)$ and let the output be $y(t)$. The control objective is to make the output $y(t)$ track a reference signal $v(t)$. Let $e(t) = y(t) - v(t)$ be the tracking error. Then PID control law is represented as follows:

$$u(t) = k_0 e(t) + k_1 \int_0^t e(\tau)d\tau + k_2 \dot{e}(t), \tag{1.1.1}$$

where k_0, k_1, and k_2 are tuning parameters. The PID control is a typical error-based control method, rather than a model-based method, which is seen from Figure 1.1.1 for its advantage

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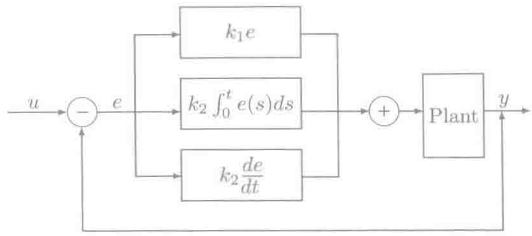

Figure 1.1.1 PID control topology.

of easy design. The nature of independent mathematical model and easy design perhaps have explained the partiality of control engineers to PID.

However, it is undeniable that PID is increasingly overwhelmed by the new demands in this era of modern industries where an unending efficiency is pursued for systems working in more complicated environments. In these circumstances, a new control technology named active disturbance rejection control (ADRC) was proposed by Jingqing Han in the 1980s and 1990s to deal with the control systems with vast uncertainty [58, 59, 60, 62, 63]. As indicated in Han's seminal work [58], the initial motivation for the ADRC is to improve the control capability and performance limited by PID control in two ways. One is by changing the linear PID (1.1.1) to nonlinear PID and the other is to make use of "derivative" in PID more efficiently because it is commonly recognized that, in PID, the "D" part can significantly improve the capability and transient performance of the control systems. However, the derivative of error is not easily measured and the classical differentiation most often magnifies the noise, which makes the PID control actually PI control in applications, that is, in (1.1.1), $k_2 = 0$.

In automatic principle of compensation, the differential signal for a given reference signal $v(t)$ is approximated by $y(t)$ in the following process:

$$\hat{y}(s) = \frac{s}{Ts+1}\hat{v}(s) = \frac{1}{T}\left(\hat{v}(s) - \frac{1}{Ts+1}\hat{v}(s)\right), \tag{1.1.2}$$

where $\hat{L}(s)$ represents the Laplace transform of $L(t)$, T is a constant, and $\frac{1}{Ts+1}\hat{v}(s)$ represents the inertial element with respect to T (see Figure 1.1.2).

The time domain realization of (1.1.2) is

$$y(t) = \frac{1}{T}(v(t) - v(t-T)). \tag{1.1.3}$$

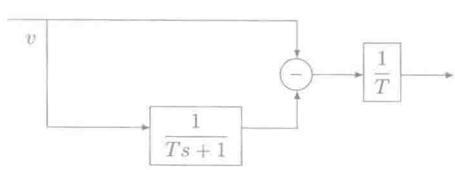

Figure 1.1.2 Classical differentiation topology.

If $v(t)$ is contaminated by a high-frequency noise $n(t)$ with zero expectation, the inertial element can filter the noise ([62], pp. 50–51):

$$y(t) = \frac{1}{T}(v(t) + n(t) - v(t-T)) \approx \dot{v}(t) + \frac{1}{T}n(t). \tag{1.1.4}$$

That is, the output signal contains the magnified noise $\frac{1}{T}n(t)$. If T is small, the differential signal may be overwhelmed by the magnified noise.

To overcome this difficulty, Han proposed a noise tolerant tracking differentiator:

$$\hat{y}(s) = \frac{1}{T_2 - T_1}\left(\frac{1}{T_1 s + 1} - \frac{1}{T_2 s + 1}\right)\hat{v}(s), \tag{1.1.5}$$

whose state-space realization is

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = -\dfrac{1}{T_1 T_2}(x_1(t) - v(t)) - \dfrac{T_2 - T_1}{T_1 T_2}x_2(t), \\ y(t) = x_2(t). \end{cases} \tag{1.1.6}$$

The smaller T_1/T_2 is, the quicker $x_1(t)$ tracks $v(t)$. The abstract form of (1.1.6) is formulated by Han as follows:

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = r^2 f\left(x_1(t) - v(t), \dfrac{x_2(t)}{r}\right), \end{cases} \tag{1.1.7}$$

where r is the tuning parameter and $f(\cdot)$ is an appropriate nonlinear function. Although a convergence of (1.1.7) is first reported in [59], it is lately shown to be true only for the constant signal $v(t)$. Nevertheless, the effectiveness of a tracking differentiator (1.1.7) has been witnessed by many numerical experiments and control practices [64, 147, 152, 153]. The convergence proof for (1.1.7) is finally established in [55 and 52]. In Chapter 2, we analyze this differentiator, and some illustrative numerical simulations and applications are also presented.

The second key part of the ADRC is the extended state observer (ESO). The ESO is an extension of the state observer in control theory. In control theory, a state observer is a system that provides an estimate of the internal state of a given real system from its input and output. For the linear system of the following:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t), \end{cases} \tag{1.1.8}$$

where $x(t) \in \mathbb{R}^n (n \geq 1)$ is the state, $u(t) \in \mathbb{R}^m$ is the control (input), and $y(t) \in \mathbb{R}^l$ is the output (measurement). When $n = 1$, the whole state is measured and the state observer is unwanted. If $n > 1$, the Luenberger observer can be designed in the following way to recover the whole state by input and output:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - C\hat{x}(t)), \tag{1.1.9}$$

where the matrix L is chosen so that $A - LC$ is Hurwitz. It is readily shown that the observer error $x(t) - \hat{x}(t) \to 0$ as $t \to \infty$. The existence of the gain matrix L is guaranteed by the detectability of system (1.1.8). If it is further assumed that system (1.1.8) is stabilizable, then there exists a matrix K such that the closed-loop system under the state feedback $u(t) = Kx(t)$ is asymptotically stable: $x(t) \to 0$ as $t \to \infty$. In other words, $A + BK$ is Hurwitz. When the observer (1.1.9) exists, then under the observer-based feedback control $u(t) = K\hat{x}(t)$, the closed-loop system becomes

$$\begin{cases} \dot{x}(t) = (A + BK)x(t), \\ \dot{\hat{x}}(t) = (A - LC + BK)\hat{x}(t) + LCx(t). \end{cases} \tag{1.1.10}$$

It can be shown that $(x(t), \hat{x}(t)) \to 0$ as $t \to \infty$ and, moreover, the eigenvalues of (1.1.10) are composed of $\sigma(A + BK) \cup \sigma(A - LC)$, which is called the separation principle for the linear system (1.1.8). In other words, the matrices K and L can be chosen separately.

The observer design is a relatively independent topic in control theory. There are huge works attributed to observer design for nonlinear systems; see, for instance, the nonlinear observer with linearizable error dynamics in [87 and 88], the high-gain observer in [84], the sliding mode observer in [24, 26, and 130], the state observer for a system with uncertainty [22], and the high-gain finite-time observer in [103, 109, and 116]. For more details of the state observer we refer to recent monograph [14].

A breakthrough in observer design is the extended state observer, which was proposed by Han in the 1990s to be used not only to estimate the state but also the "total disturbance" that comes from unmodeled system dynamics, unknown coefficient of control and external disturbance. Actually, uncertainty is ubiquitous in a control system itself and the external environment, such as unmodeled system dynamics, external disturbance, and inaccuracy in control coefficient. The ubiquitous uncertainty in systems explains why the PID control technology is so popular in industry control because PID control is based mainly on the output error not on the systems' mathematical models. Since the ESO, the "total disturbance" and the state of the system are estimated simultaneously, we can design an output feedback control that is not critically reliant on the mathematical models. Let us start from an nth order SISO nonlinear control systems given by

$$\begin{cases} x^{(n)}(t) = f(t, x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)) + w(t) + u(t), \\ y(t) = x(t), \end{cases}$$

which can be rewritten as

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = x_3(t), \\ \quad \vdots \\ \dot{x}_n(t) = f(t, x_1(t), x_2(t), \ldots, x_n(t)) + w(t) + u(t), \\ y(t) = x_1(t), \end{cases} \tag{1.1.11}$$

where $u(t) \in C(\mathbb{R}, \mathbb{R})$ is the control (input), $y(t)$ is the output (measurement), $f \in C(\mathbb{R}^n, \mathbb{R})$ is the system function, which is possibly unknown, and $w \in C(\mathbb{R}, \mathbb{R})$ is unknown external disturbance; $f(\cdot, t) + w(t)$ is called the "total disturbance" or "extended state" and

$\alpha_i \in \mathbb{R}, i = 1, 2, \ldots, n+1$ are the tuning parameters. The ESO designed in [60] is as follows:

$$\begin{cases} \dot{\hat{x}}_1(t) = \hat{x}_2(t) - \alpha_1 g_1(\hat{x}_1(t) - y(t)), \\ \dot{\hat{x}}_2(t) = \hat{x}_3(t) - \alpha_2 g_2(\hat{x}_1(t) - y(t)), \\ \quad \vdots \\ \dot{\hat{x}}_n(t) = \hat{x}_{n+1}(t) - \alpha_n g_n(\hat{x}_1(t) - y(t)) + u(t), \\ \dot{\hat{x}}_{n+1}(t) = -\alpha_{n+1} g_{n+1}(\hat{x}_1(t) - y(t)). \end{cases} \tag{1.1.12}$$

By appropriately choosing the nonlinear functions $g_i \in C(\mathbb{R}, \mathbb{R})$ and tuning the parameters α_i, we expect that the states $\hat{x}_i(t), i = 1, 2, \ldots, n+1$ of the ESO (1.1.12) can approximately recover the states $x_i(t), i = 1, 2, \ldots, n$ and the extended state $f(\cdot, t) + w(t)$, that is,

$$\hat{x}_i(t) \approx x_i(t), i = 1, 2, \ldots, n, \hat{x}_{n+1}(t) \approx f(\cdot, t) + w(t).$$

In Chapter 3, we have a principle of choosing the nonlinear functions $g_i(\cdot)$ and tuning the gain parameters α_i. The convergence of the ESO is established. We also present some numerical results to show visually the estimations of state and extended state. In particular, if the functions $g_i(\cdot)$ in (1.1.12) are linear, the ESO is referred to as the linear extended state observer (LESO). The LESO is also called the extended high-gain observer in [35].

The final key part of the ADRC is the TD and the ESO-based feedback control. In the feedback loop, a key component is to compensate (cancel) the "total disturbance" by making use of its estimate obtained from the ESO. The topology of the active disturbance rejection control is blocked in Figure 1.1.3.

Now we can describe the whole picture of the ADRC for a control system with vast uncertainty that includes the external disturbance and unmodeled dynamics. The control purpose is to design an output feedback control law that drives the output of the system to track a given reference signal $v(t)$. Generally speaking, the derivatives of the reference $v(t)$ cannot be measured accurately due to noise. The first step of the ADRC is to design a tracking differentiator

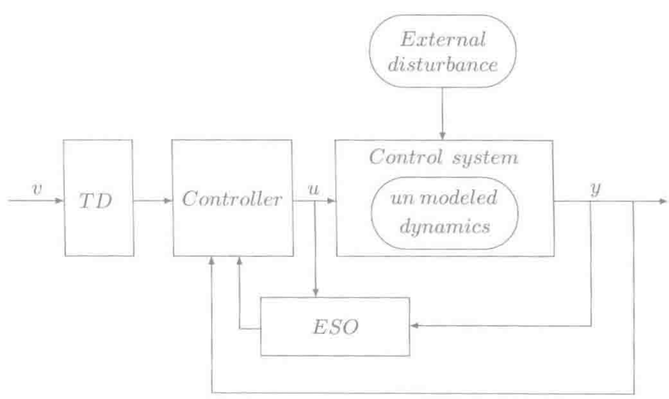

Figure 1.1.3 Topology of active disturbance rejection control.

(TD) to recover the derivatives of $v(t)$ without magnifying measured noise. Tracking differentiator also serves as transient profile for output tracking. The second step is to estimate, through the ESO, the system state and the "total disturbance" in real time by the input and output of the original system. The last step is to design an ESO-based feedback control that is used to compensate the "total disturbance" and track the estimated derivatives of $v(t)$. The whole ADRC design process and convergence are analyzed in Chapter 4.

The distinctive feature of the ADRC lies in its estimation/cancelation nature. In control theory, most approaches like high-gain control (HGC) and sliding mode control (SMC) are based on the worst case scenario, but there are some approaches that use the same idea of the ADRC to deal with the uncertainty. One popular approach is the internal model principle (IMP) [33, 34, 77, 99] and a less popular approach is the external model principle (EMP) [66, 104, 129, 149]. In the internal model principle and external model principle, the dynamic of the system is exactly known and the "external disturbance" is considered as a signal generated by the exogenous system, which follows exactly known dynamics. The unknown parts are initial states. However, in some complicated environments, it is very difficult to obtain the exact mathematical model of the exogenous system, which generates "external disturbance". In the ADRC configuration, we do not need a mathematical model of external disturbance and even most parts of the mathematical model of the control system itself can be unknown. This is discussed in Section 4.5.

The systems dealt with by the ADRC can also be coped with by high-gain control [128] and sometimes by sliding mode control [94, 130, 131]. However, control law by these approaches is designed in the worst case of uncertainty, which may cause unnecessary energy waste and may even be unrealizable in many engineering practices. In Section 4.6, three control methods are compared numerically by a simple example.

1.2 Overview of Engineering Applications

Nowadays, the ADRC is widely used in many engineering practices. It is reported in [166] that the ADRC control has been tested in the Parker Hannifin Parflex hose extrusion plant and across multiple production lines for over eight months. The product performance capability index (Cpk) is improved by 30% and the energy consumption is reduced by over 50%.

The Cleveland state university in the USA established a center for advanced control technologies (CACT) for further investigation of the ADRC technology. Under the cooperation of CACT and an American risk investment, the industrial giant Texas Instruments (TI) has adopted this method. In April 2013, TI issued its new motor control chips based on the ADRC. The control chips can be used in almost every motor such as washing machines, medical devices, electric cars and so on.

There is a lot of literature on the application of the ADRC. In what follows, we briefly overview some typical examples. In the flight and integrated control fields, an ESO and non-smooth feedback law is employed to achieve high performance of flight control [72]. In [126], the ADRC is adopted to tackle some problems encountered in pitch and roll altitude control. The ADRC is used for integrated flight-propulsion control in [135], and the coupling effects between altitude and velocity and attenuates measurement noise are eliminated by this method. In [169], the ADRC is applied to altitude control of a spacecraft model that is nonlinear in dynamics with inertia uncertainty and external disturbance. The ESO is applied to estimate the disturbance and the sliding mode control is designed based on the ESO to achieve the control purpose. The safe landing of unmanned aerial vehicles (UAVs) under various wind conditions has been a challenging task for decades. In [143], by using the ADRC method, an auto-landing control system consisting of a throttle control subsystem and an altitude control subsystem has been designed. It is indicated that this method can estimate directly in real time the UAV's internal and external disturbances and then compensate in the feedback. The simulation results show that this auto-landing control system can land the UAV safely under wide range wind disturbances (e.g., wind turbulence, wind shear). The application of the ADRC on this aspect can be found in monograph [139].

In the energy conversion and power plant control fields, [28] presents a controller for maximum wind energy capture of a wind power system by employing the ADRC method. The uncertainties in the torque of turbine and friction are both considered as an unknown disturbance to the system. The ESO is used to estimate the unknown disturbance. The maximum energy capture is achieved through the design of a tracking-differentiator. It is pointed out that this method has the merits of feasibility, adaptability, and robustness compared to the other methods. The paper [102] summarizes some methods for capturing the largest wind energy. It is indicated that the ADRC method captures the largest wind energy. The ADRC is used for a thermal power plant, which is characterized by nonlinearity, changing parameters, unknown disturbances, large time-delays, large inertia, and highly coupled dynamics among various control loops in [167]. In [121], the ADRC method is developed to cope with the highly nonlinear dynamics of the converter and the disturbances. The ADRC method is used for a thermal power generation unit in [69]. It is reported that the real-time dynamic linearization is implemented by disturbance estimation via the ESO and disturbance compensation via the control law, instead of differential geometry-based feedback linearization and direct feedback linearization theory, which need an accurate mathematical model of the plant. The decoupling for an MIMO coordinated system of boiler–turbine unit is also easily implemented by employing the ADRC. The simulation results on STAR-90 show that the ADRC coordinated control scheme can effectively solve problems of strong nonlinearity, uncertainty, coupling, and large time delays. It can also significantly improve the control performance of a coordinated control system. To eliminate the total disturbance effect on the active power filter (APF) performance, the ADRC is adopted in [95]. It is reported that the ADRC control has the merits of strong robustness, stability, and adaptability in dealing with the internal perturbation and external disturbance. In [151], the ADRC is used to regulate the frequency error for a three-area interconnected power system. As the interconnected power system transmits the power from one area to another, the system frequency will inevitably deviate from a scheduled frequency, resulting in a frequency error. A control system is essential to correct the deviation in the presence of external disturbances and structural uncertainties to ensure the safe and smooth operation of the power system. It is reported in [151] that the ADRC can extract the information of the disturbance from input and output data of the system and actively compensate for the disturbance in real time. Considering the difficulty of developing an accurate mathematical model for active power filters (APF), [168] uses the ADRC to parallel APF systems. It is reported that the analog signal detected in the ADRC controller is less than other control strategies. In [27], the ADRC is applied to an electrical power-assist steering system (EPAS) in automobiles to reduce the steering torque exerted by a driver so as to achieve good steering feel in the presence of external disturbances and system uncertainties. With the proposed ADRC, the driver can turn the steering wheel with the desired steering torque, which is independent of load torques, and tends to vary, depending on driving conditions.

As to motor and vehicle control, in [127], the ADRC is used to ensure high dynamic performance of a magnet synchronous motor (PMSM) servo system. It is concluded that the proposed topology produces better dynamic performance, such as smaller overshoot and faster transient time, than the conventional PID controller in its overall operating conditions. A matrix converter (MC) is superior to a drive induction motor since it has more attractive advantages than a conventional pulse width modulation (PWM) inverter such as the absence of a large dc-link capacitor, unity input power factor, and bidirectional power flow. However, due to the direct conversion characteristic of an MC, the drive performance of an induction motor is easily influenced by input voltage disturbances of the MC, and the stability of an induction motor drive system fed by an MC would be affected by a sudden change of load as well. In [105], the ADRC is applied to the MC fed induction motor drive system to solve the problems successfully. In [31], the ADRC is developed to ensure high dynamic performance of induction motors. In [123], the ADRC is developed to implement high-precision motion control of permanent-magnet synchronous motors. Simulations and experimental results show that the ADRC achieves a better position response and is robust to parameter variation and load disturbance. Furthermore, the ADRC is designed directly in discrete time with a simple structure and fast computation, which makes it widely applicable to all other types of drives. In [96], an ESO-based controller is designed for the permanent-magnet synchronous motor speed-regulator, where the ESO is employed to estimate both the states and the disturbances simultaneously, so that the composite speed controller can have a corresponding part to compensate the disturbances. Lateral locomotion control is a key technology for intelligent vehicles and is significant to vehicle safety itself. In [115], the ADRC is used for the lateral locomotion control. Simulation results show that, within the large velocity scale, the ADRC controllers can assist the intelligent vehicle to accomplish smooth and high precision on lateral locomotion, as well as remaining robust to system parameter perturbations and disturbances. In [146], the ADRC is applied to the anti-lock braking system (ABS) with regenerative braking of electric vehicles. Simulation results indicate that this method can regulate the slip rate at expired value in all conditions and, at the same time, it can restore the kinetic energy of a vehicle to an electrical source. In [142], the ADRC is applied to the regenerative retarding of a vehicle equipped with a new energy recovery retarder. Considering the railway restriction and comfort requirement, the ADRC is applied to the operation curve tracking of the maglev train in [100].

There is also a lot of literature on the ADRC's application in ship control. In [113], the ADRC is applied to the ship tracking control by considering the strong nonlinearity, uncertainty, and typical underactuated properties, as well as the restraints of the rudder. The simulation results show that the designed controller can achieve high precision on ship tracking control and has strong robustness to ship parameter perturbations and environment disturbances. In [108], the ADRC is used on the ship's main engine for optimal control under unmatched uncertainty. The simulation results show that the controller has strong robustness to parameter perturbations of the ship and environmental disturbances.

In robot control [73], the ESO is used to estimate and compensate the nonlinear dynamics of the manipulator and the external disturbances for a complex robot systems motion control. [120] applies the ADRC to the lateral control of tracked robots on stairs. The simulation results show that this algorithm can keep the robot smooth and precise in lateral control and effectively overcome the disturbance. In [114], the ADRC is applied to the rock drill robot joint hydraulic drive system. The simulation results show that the ADRC controller has ideal robustness to the system parameters' disturbances and the large load disturbance and a rapid and smooth control process and high steady precise performances can be implemented.

As to gyroscopes, [162] applies the ADRC to control two vibrating axes (or modes) of vibrational MEMS gyroscopes in the presence of the mismatch of natural frequencies between two axes, mechanical-thermal noises, quadrature errors, and parameter variations. The simulation results on a Z-axis MEMS gyroscope show that the controller is very effective by driving the output of the drive axis to a desired trajectory, forcing the vibration of the sense axis to zero for a force-to-rebalance operation, and precisely estimating the rotation rate. In [29], the ADRC is used for both vibrating axes (drive and sense) of vibrational gyroscopes, in both simulation and hardware tests on a vibrational piezoelectric beam gyroscope. The proposed controller proves to be robust against structural uncertainties and it also facilitates accurate sensing of time-varying rotation rates. [154] uses the ADRC and fuzzy control method for stabilizing circuits in platform inertial navigation systems (INS) based on fiber optic gyroscopes (FOGs).

1.3 Preliminaries

In this section, we first present a canonical form of active disturbance rejection control (ADRC). To make the book self-contained, we also present some notation and results about Lyapunov stability, asymptotical stability, finite-time stability, and weighted homogeneity.

1.3.1 Canonical Form of ADRC

As pointed out in the previous section, the ADRC can deal with nonlinear systems with vast uncertainty. However, for the sake of clarity, we first limit ourselves to a class of nonlinear systems that are canonical forms of the ADRC. Let us start with some engineering control systems.

Firstly, we consider micro-electro-mechanical systems (MEMS). The mechanical structure of the MEMS gyroscope can be understood as a proof mass attached to a rigid frame by springs and dampers, as shown in Figure 1.3.1. As the mass is driven to resonance along the drive (*X*) axis and the rigid frame is rotating along the rotation axis, a Coriolis acceleration will be produced along the sense (*Y*) axis, which is perpendicular to both drive and rotation axes. The Coriolis acceleration is proportional to the amplitude of the output of the drive axis and the unknown rotation rate. Therefore, we can estimate the rotation rate through measuring the vibration of the sense axis. To measure accurately the rotation rate, the vibration magnitude of the drive axis has to be regulated to a fixed level. Therefore, the controller of the drive axis is mainly used to drive the drive axis to resonance and to regulate the output amplitude.

The vibrational MEMS gyroscope can be modeled as follows:

$$\begin{cases} \ddot{x}(t) + 2\zeta\omega_n^2 x(t) + \omega_{xy} y(t) - 2\Omega\dot{y}(t) = \dfrac{k}{m} u(t) + N_x(t), \\ \ddot{y}(t) + 2\zeta\omega_n \dot{y}(t) + \omega_{xy} x(t) + 2\Omega\dot{x}(t) = N_y(t), \end{cases} \tag{1.3.1}$$

where $x(t)$ and $y(t)$ are the outputs of the drive and sense axes, $2\Omega\dot{x}(t)$ and $2\Omega\dot{y}(t)$ are the Coriolis accelerations, Ω is the rotation rate, ω_n is the natural frequency of the drive and sense axes, $\omega_{xy} y(t)$ and $\omega_{xy} x(t)$ are quadrature errors caused by spring couplings between two axes,