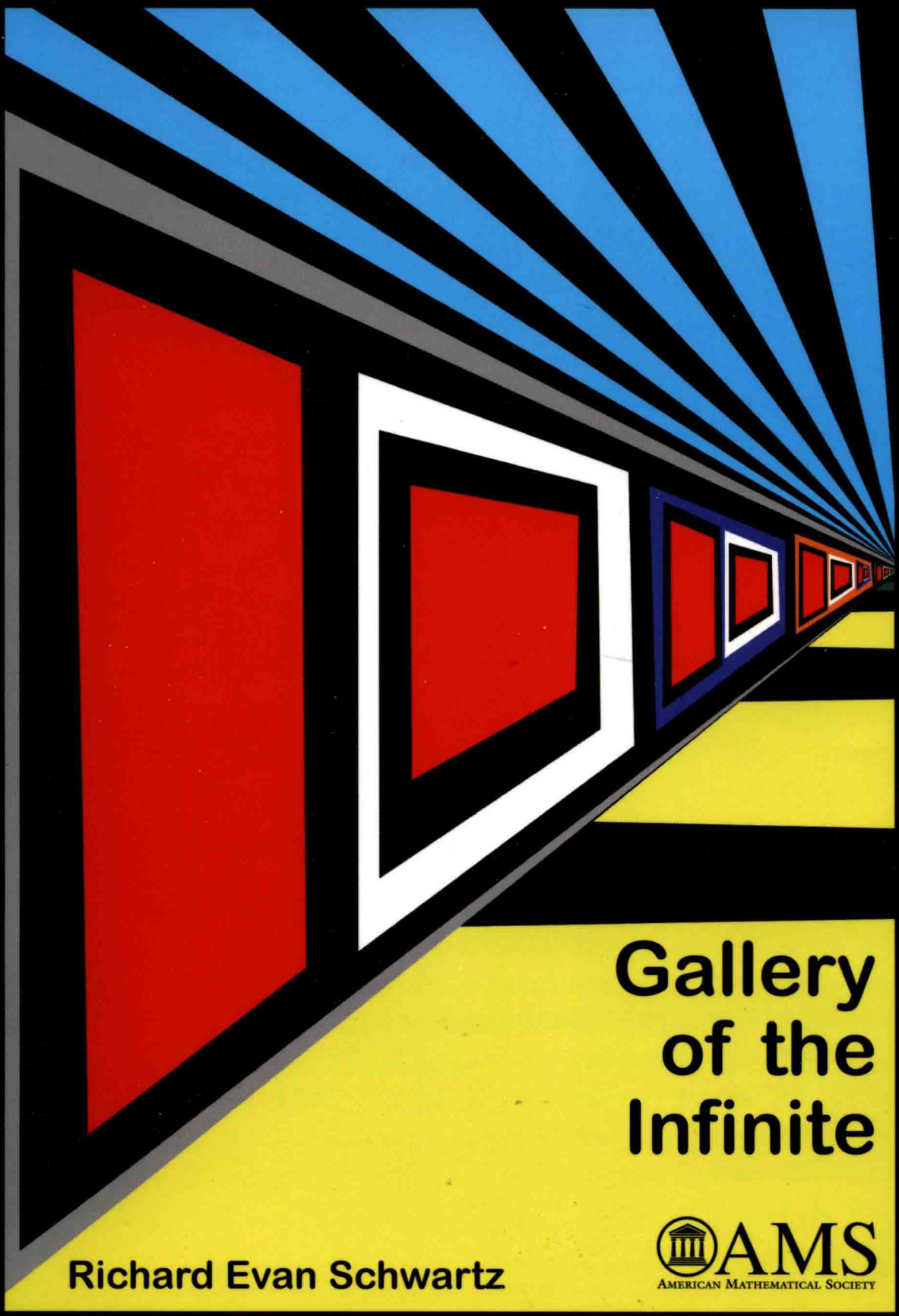
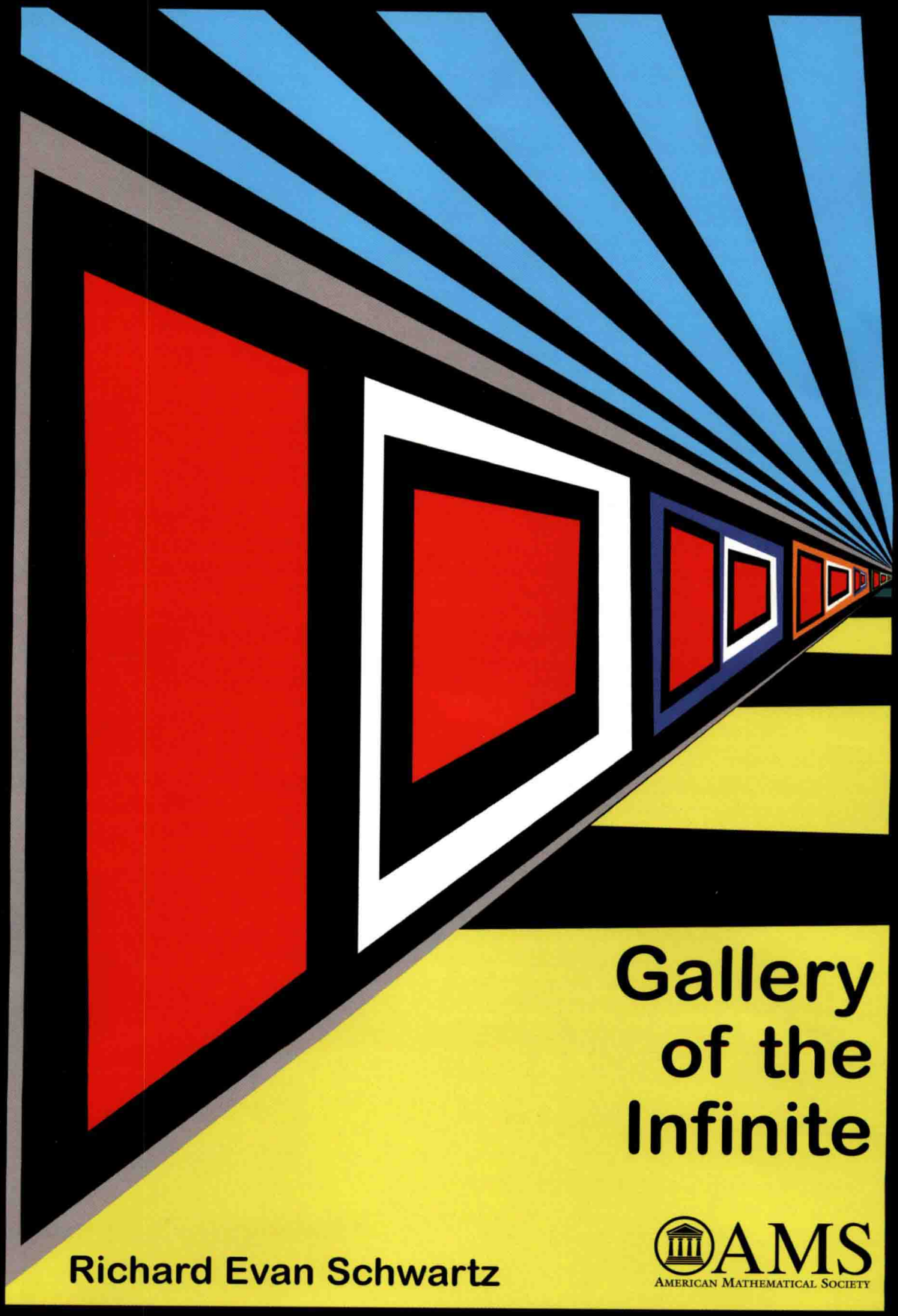


Gallery of the Infinite

Richard Evan Schwartz

AMS
American Mathematical Society



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For notes on the book, including answers to the questions, visit **www.ams.org/bookpages/mbk-97**

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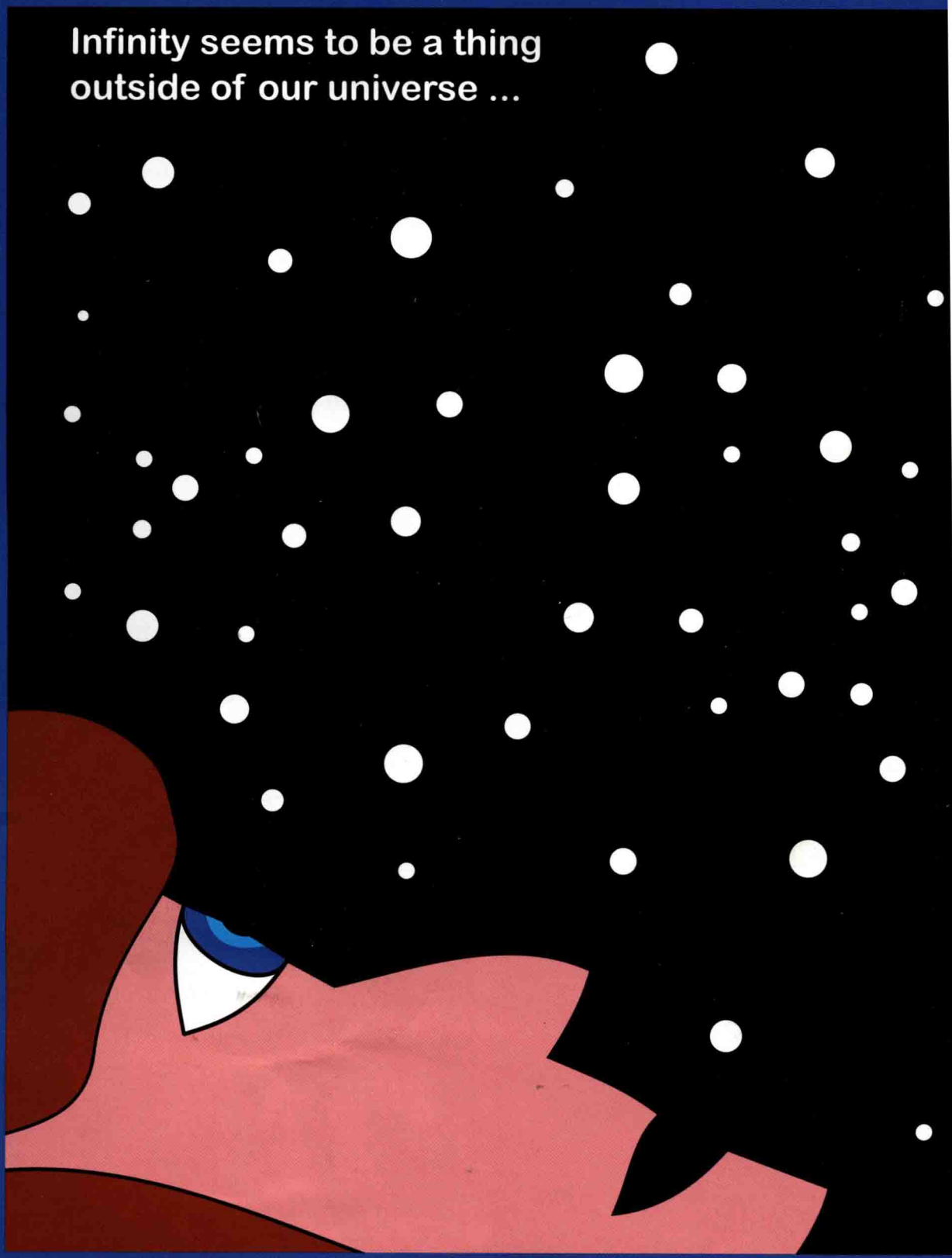
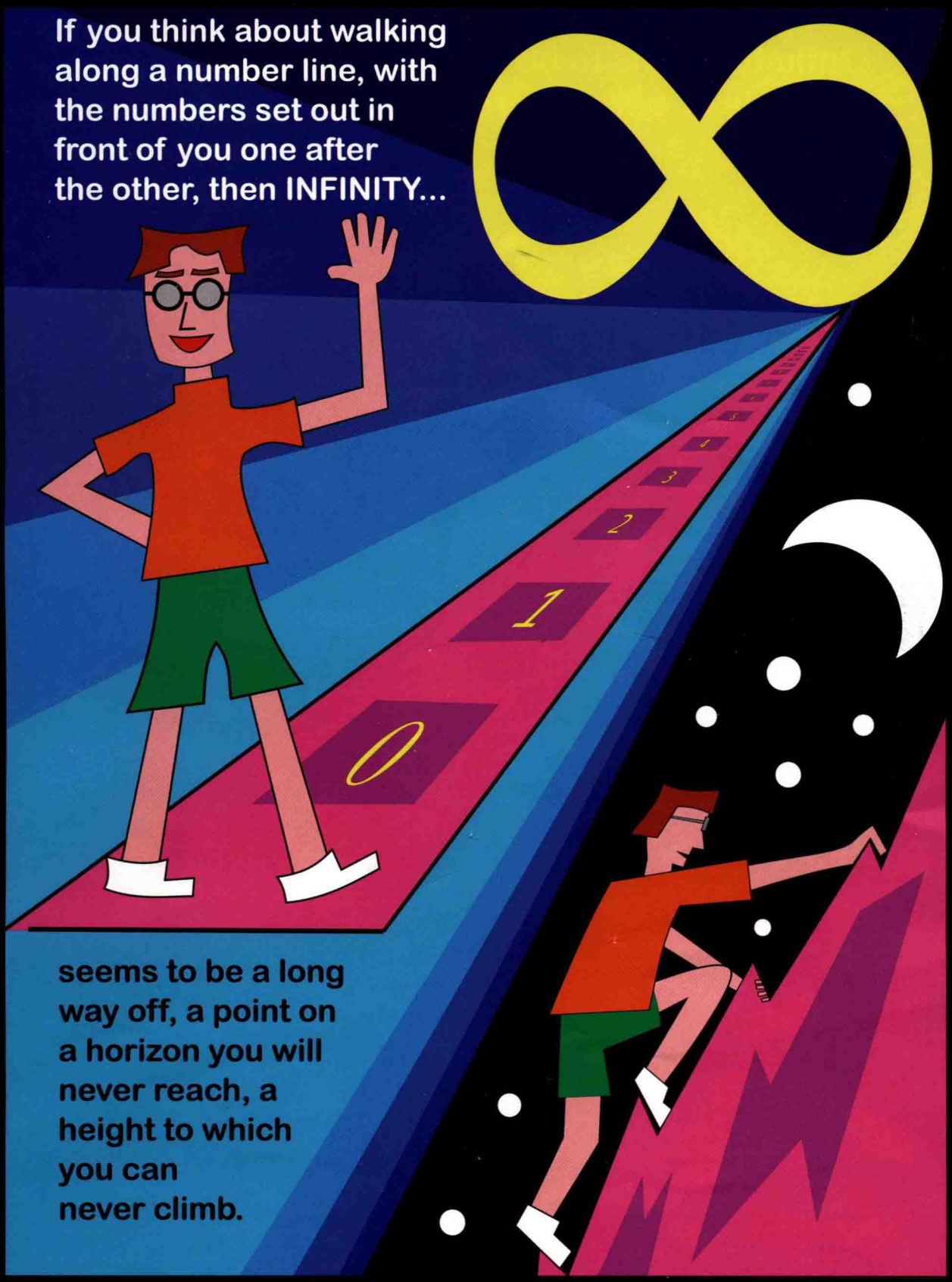
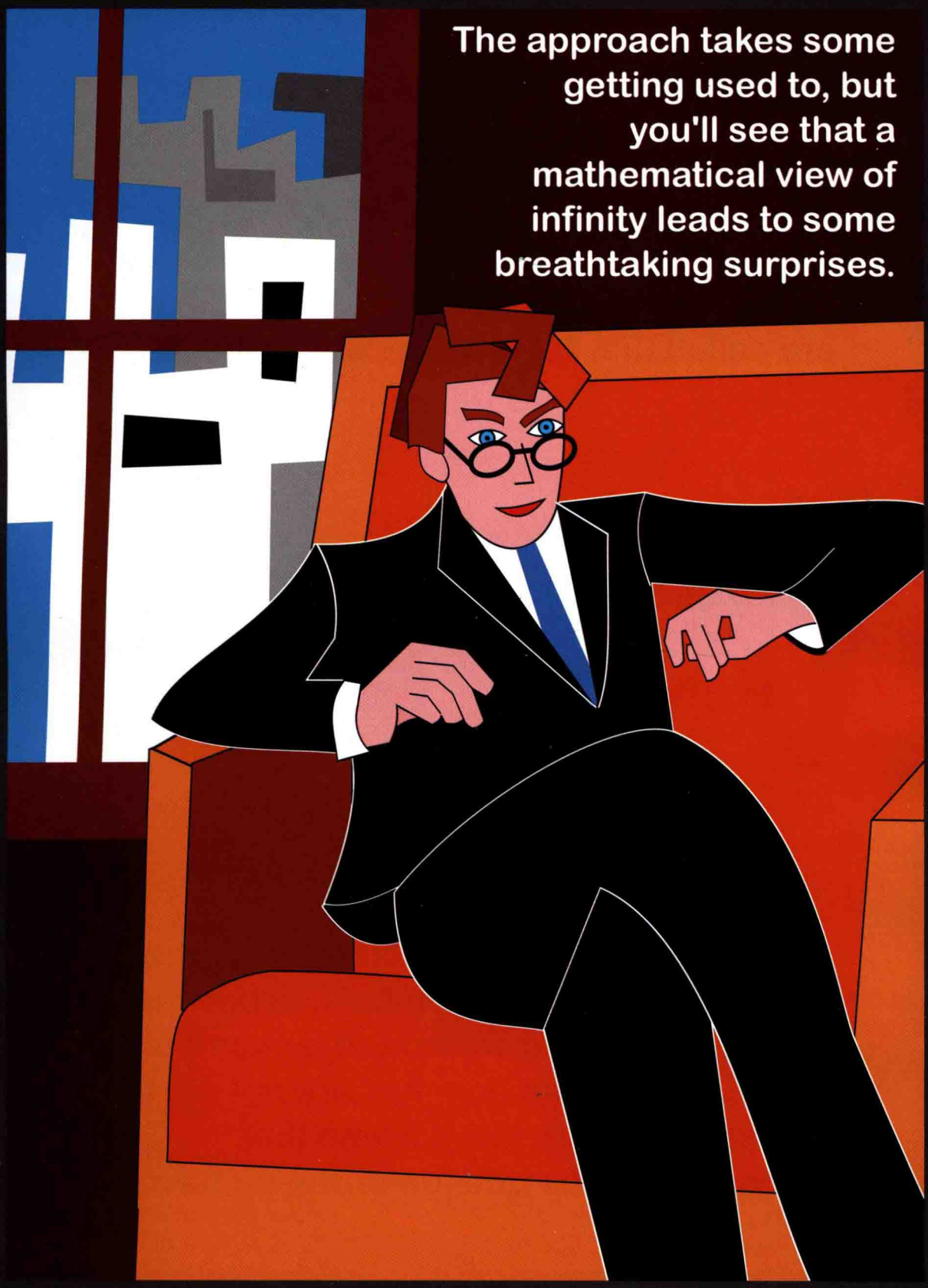

Infinity seems to be a thing
outside of our universe ...




I wrote this book
to explain how a
typical
mathematician
thinks about
infinity.

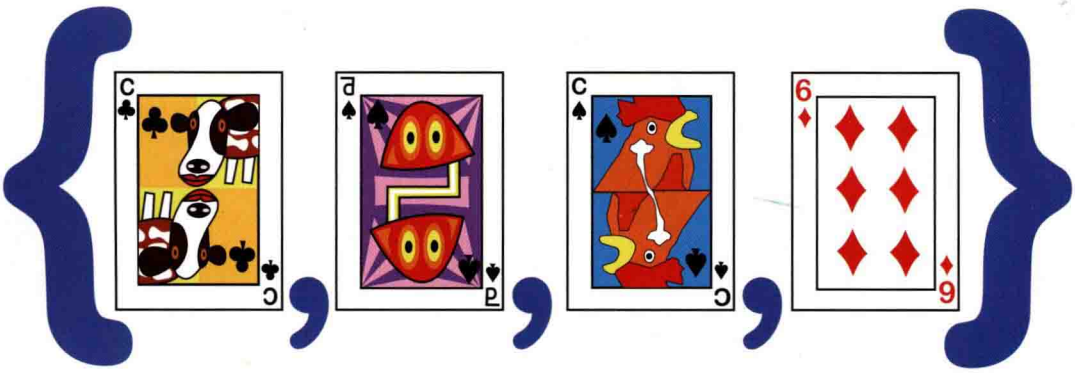

a far edge you won't see no
matter how hard you stare
into space.


The approach takes some getting used to, but you'll see that a mathematical view of infinity leads to some breathtaking surprises.

The first order of business is to talk about

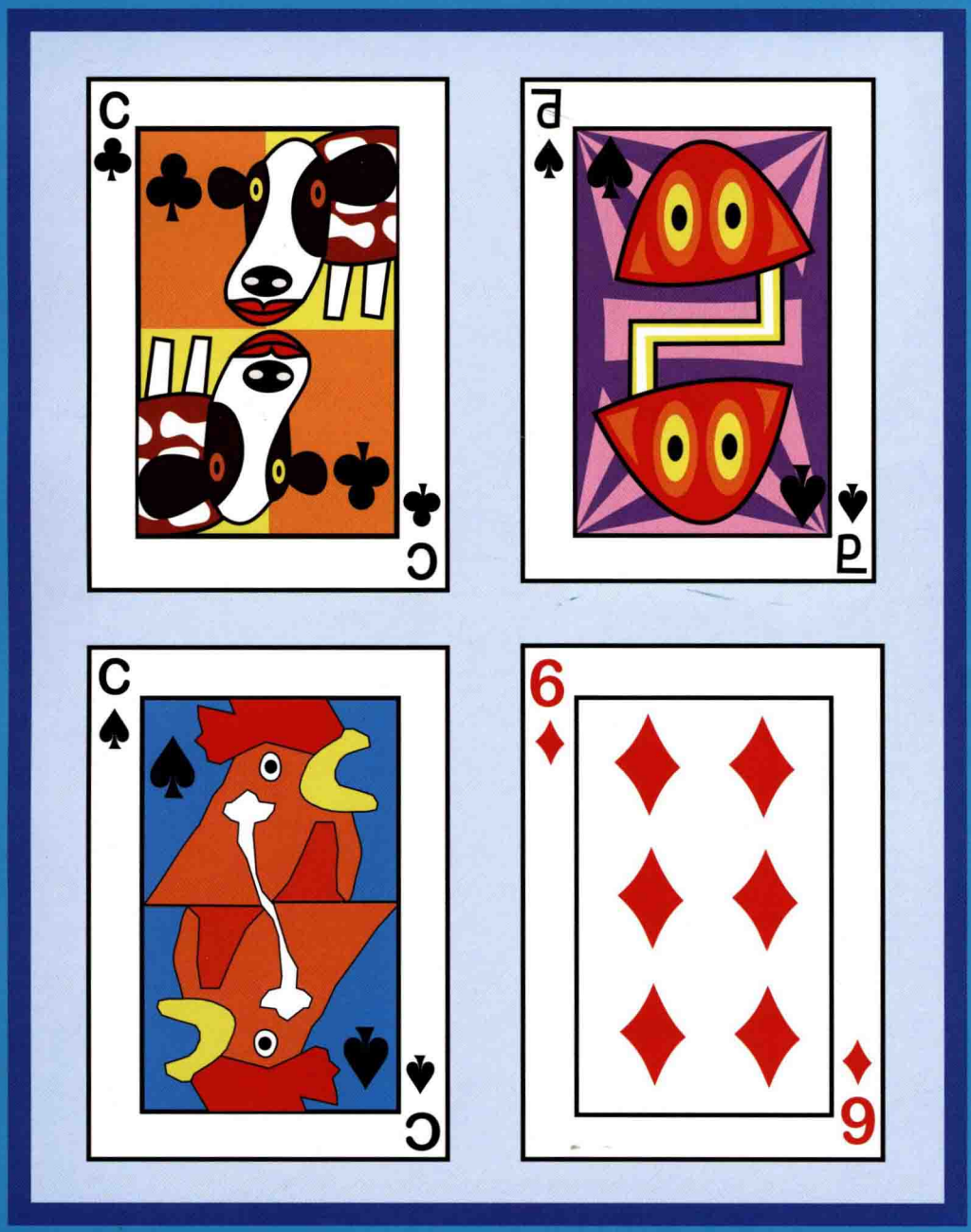
SETS.

A set is the name mathematicians have for collections of things. The things in the set are called the **MEMBERS** of the set.



Traditionally, mathematicians write the members of a set in symbols, in between two brackets and separated by commas. The brackets and commas are not part of the set. They are like a frame that goes around the outside of the picture.

I sometimes picture sets as things placed inside boxes, because then the box looks more clearly like a frame.



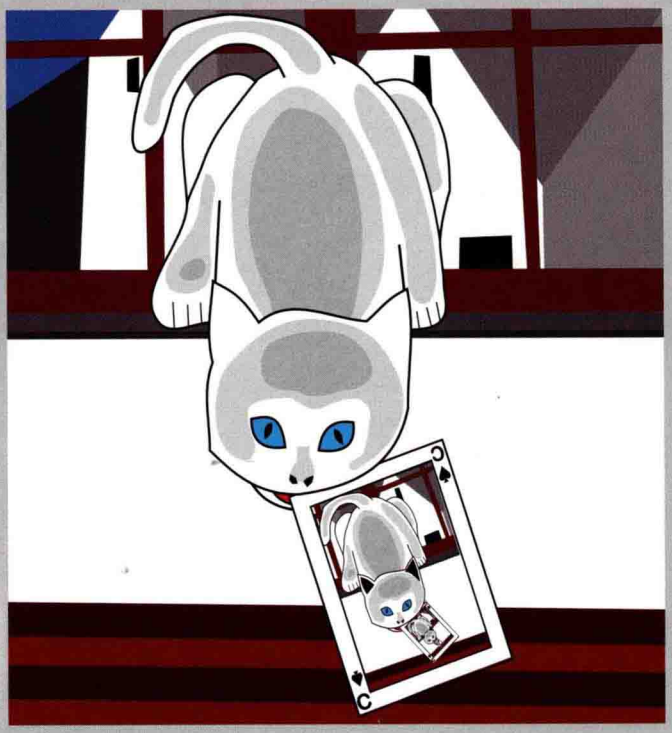
Informally, I like to picture the members of a set as all sorts of things, like playing cards...

or cats...


or aliens.

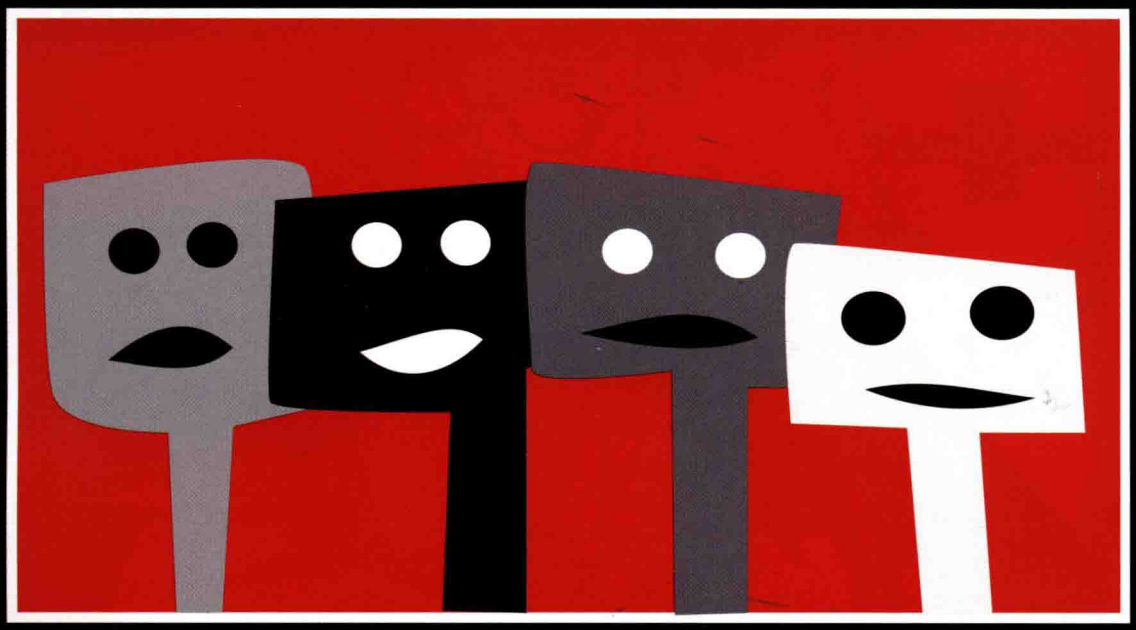
Formally, the members of a mathematical set are not really cards or cats or aliens.

They are sets themselves.



This gives mathematics a certain beauty and purity, but it does raise the question as to how the whole enterprise gets off the ground. Let's not get into these technical details just yet. For now, we'll think of sets as being all kinds of things.

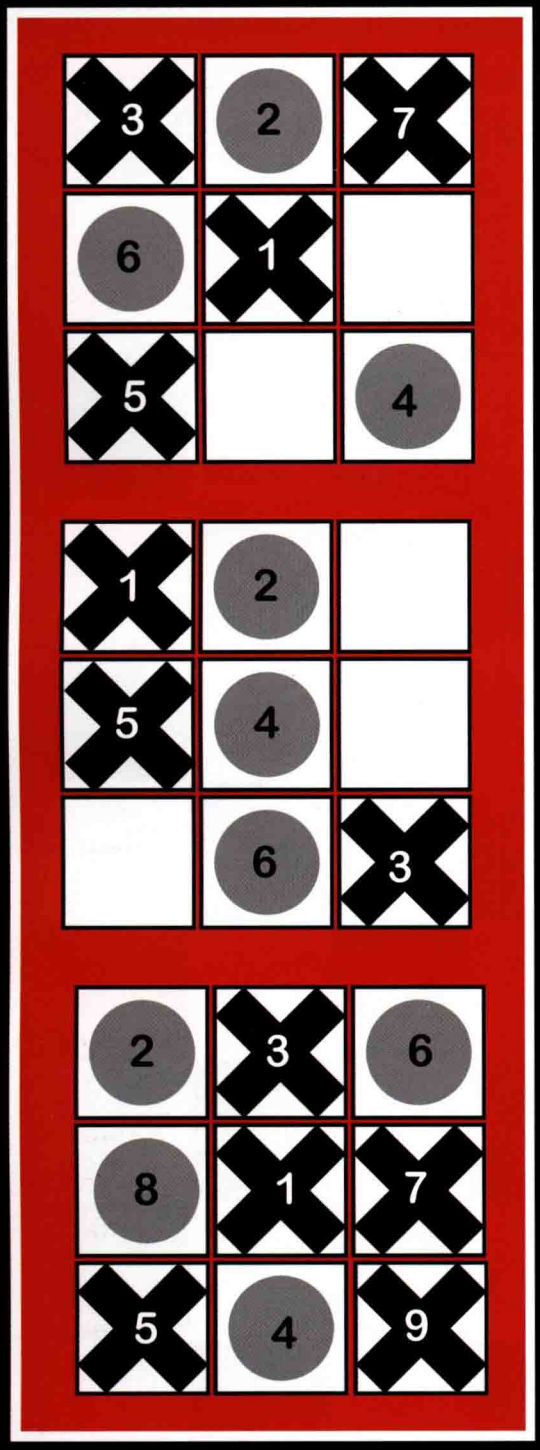
Some sets are called FINITE. Here are some examples.



The set of pancake spatulas with faces drawn on them

The set of windows in Manhattan

The set of all tic-tac-toe games



The set of seagulls on the Rhode Island coast

Of course, I haven't drawn all the members of these sets.

Intuitively, a set is finite if you can start counting its members and get to the end. But this isn't phrased quite right because sometimes you might not ACTUALLY be able to get all the way to the end. Consider the set of all chess games which last less than 200 moves ...