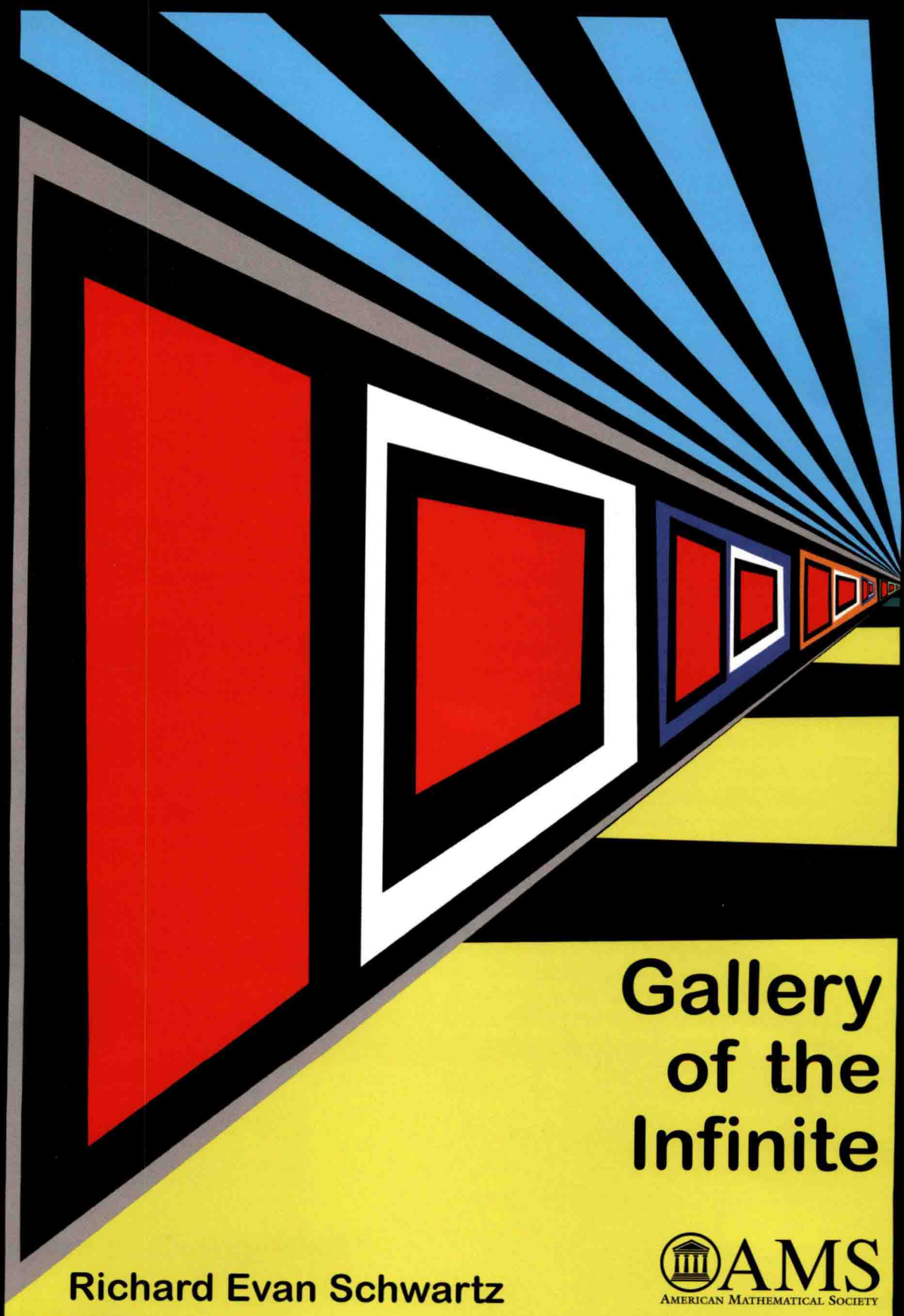




Gallery of the Infinite


Richard Evan Schwartz

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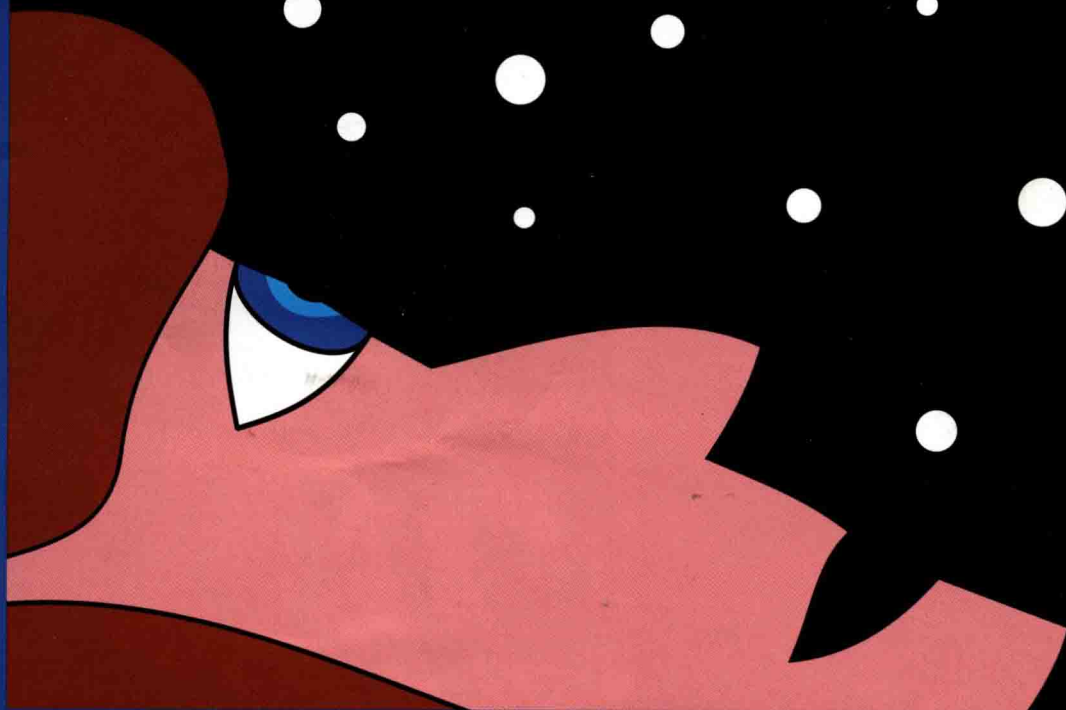
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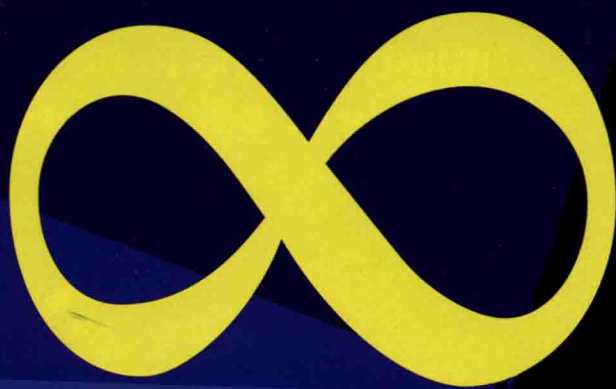
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10 9 8 7 6 5 4 3 2 1 21 20 19 18 17 16

Infinity seems to be a thing
outside of our universe ...



If you think about walking along a number line, with the numbers set out in front of you one after the other, then INFINITY...

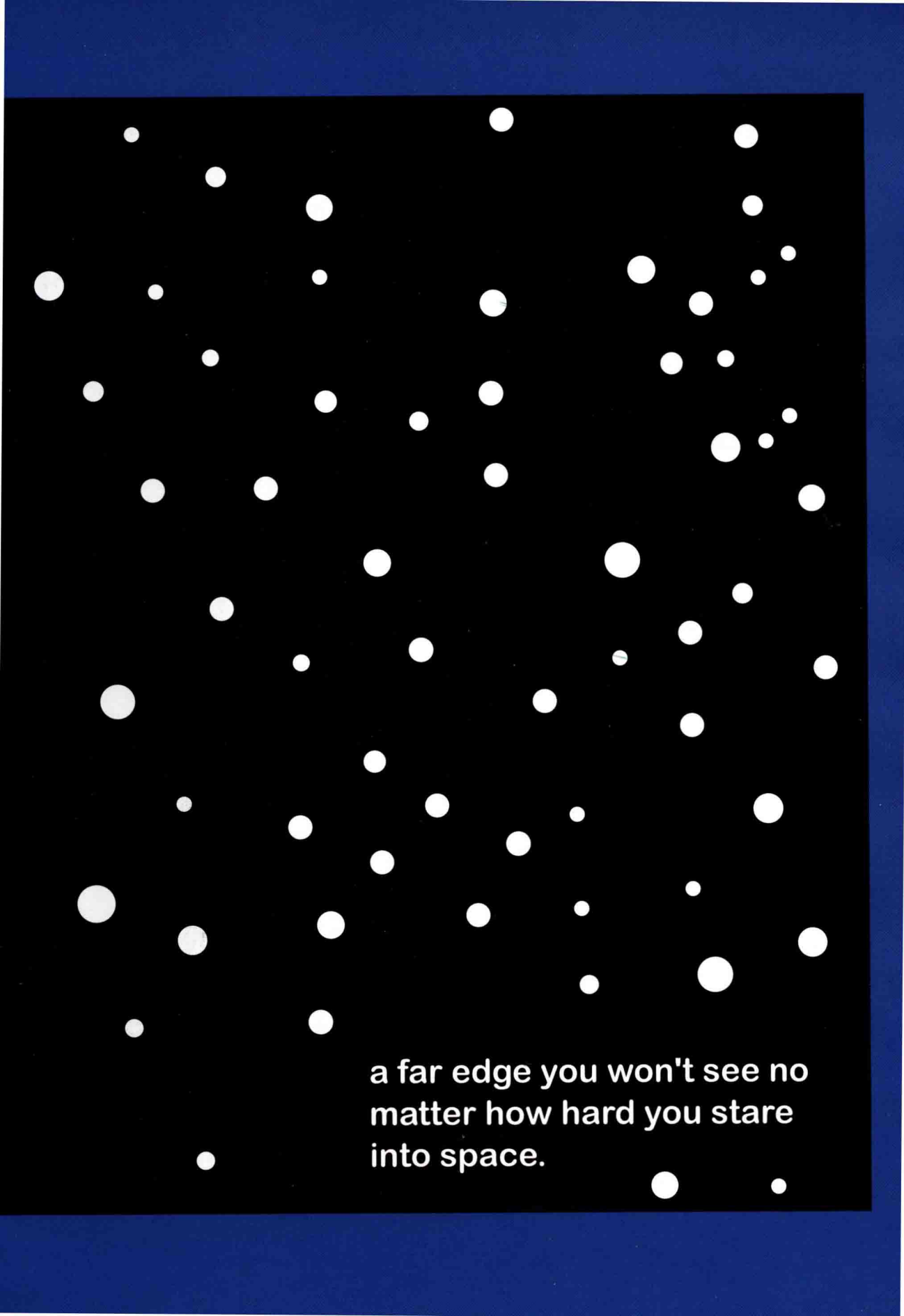


seems to be a long way off, a point on a horizon you will never reach, a height to which you can never climb.





I wrote this book
to explain how a
typical
mathematician
thinks about
infinity.



a far edge you won't see no
matter how hard you stare
into space.

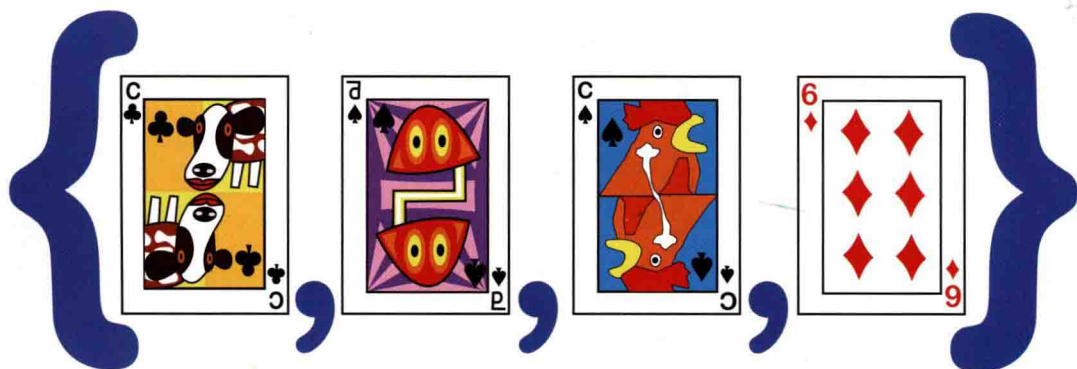
The approach takes some
getting used to, but
you'll see that a
mathematical view of
infinity leads to some
breathtaking surprises.



The first order of business is to talk about

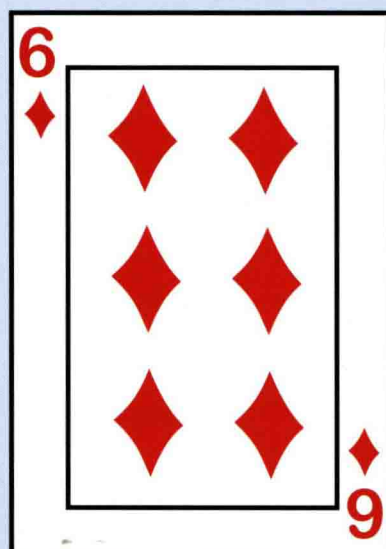
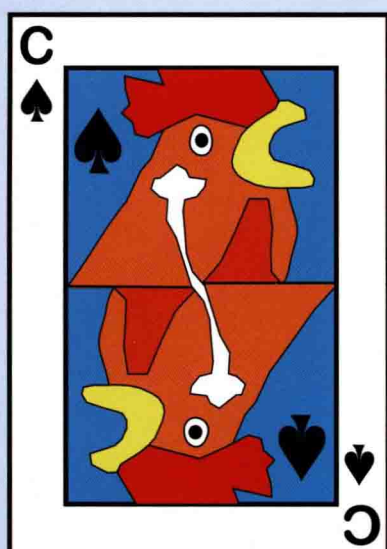
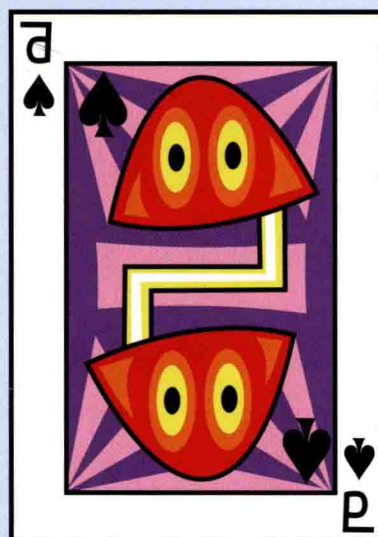
SETS.

A set is the name mathematicians have for collections of things. The things in the set are called the **MEMBERS** of the set.



Traditionally, mathematicians write the members of a set in symbols, in between two brackets and separated by commas. The brackets and commas are not part of the set. They are like a frame that goes around the outside of the picture.

I sometimes picture sets as things placed inside boxes, because then the box looks more clearly like a frame.



Informally, I like to picture the members of a set as all sorts of things, like playing cards...

or cats...



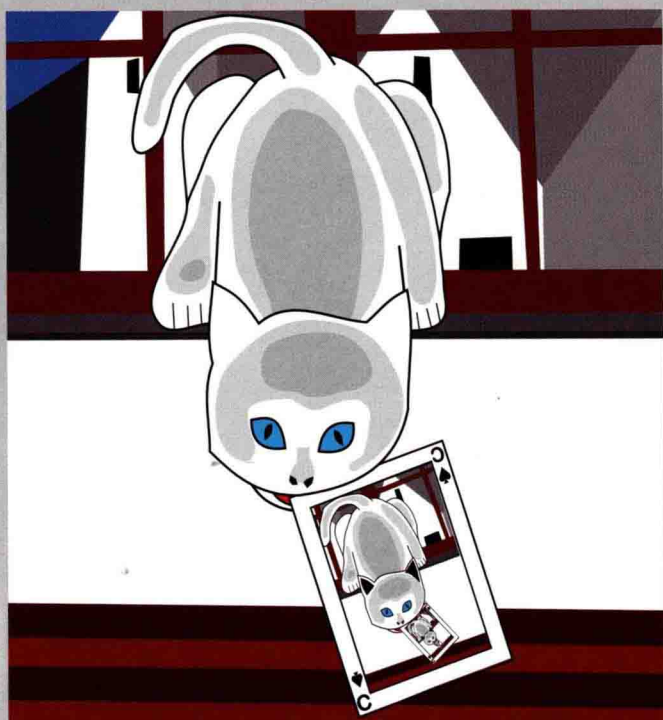


or aliens.



Formally, the
members of a
mathematical
set are not really
cards or cats
or aliens.

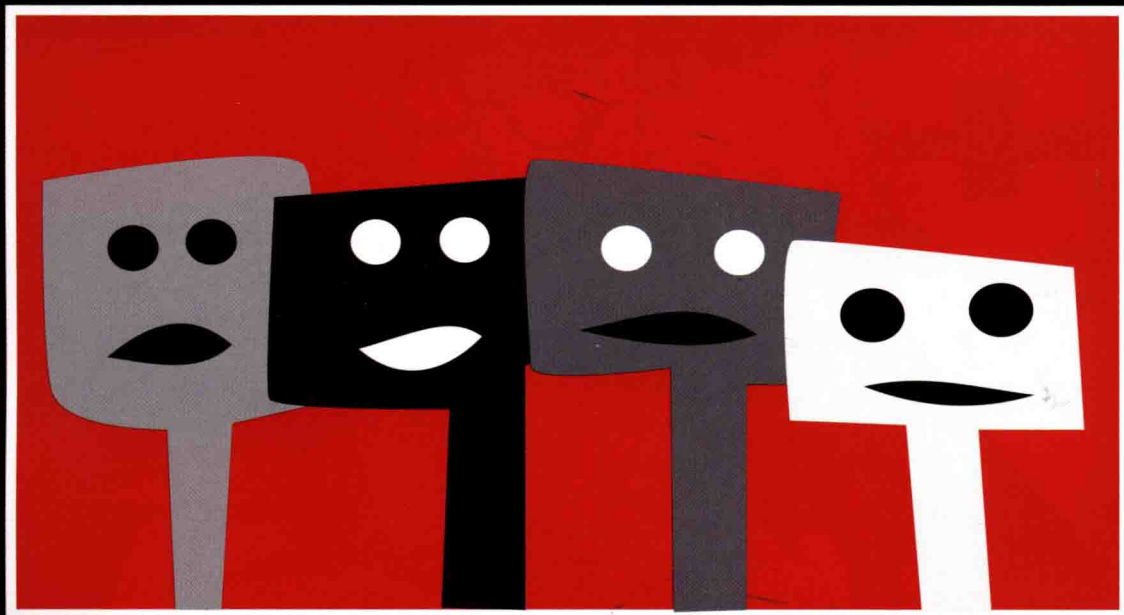
They are sets
themselves.



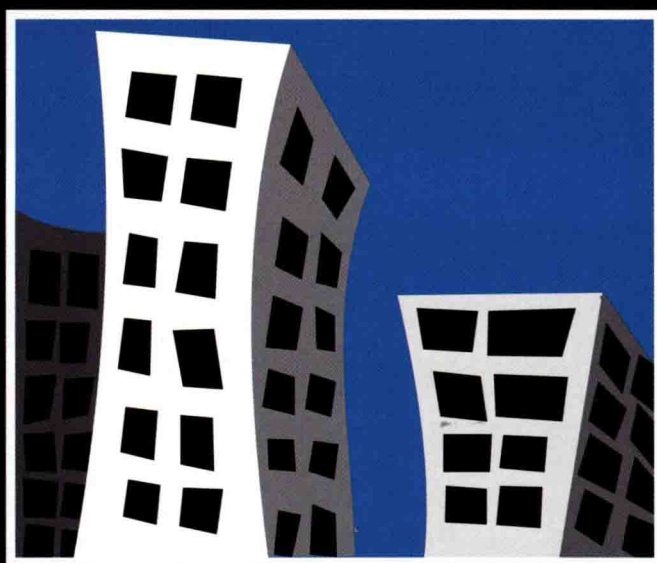


This gives mathematics a certain beauty and purity, but it does raise the question as to how the whole enterprise gets off the ground. Let's not get into these technical details just yet. For now, we'll think of sets as being all kinds of things.

Some sets are called FINITE.
Here are some examples.

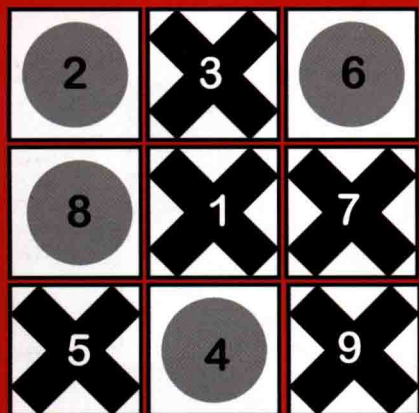
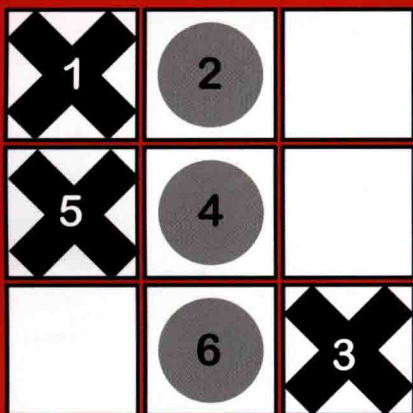
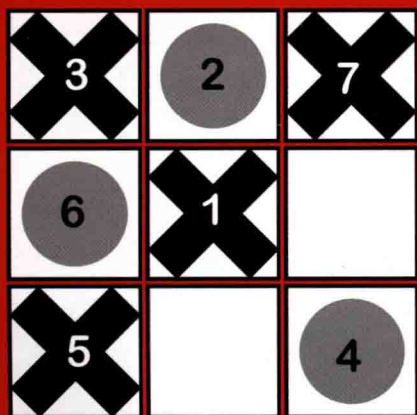


The set of pancake spatulas with faces drawn on them



The set of windows in Manhattan

The set of all
tic-tac-toe games



The set of seagulls
on the Rhode
Island coast

Of course,
I haven't
drawn all the
members of
these sets.

Intuitively, a set is finite if you can start counting its members and get to the end. But this isn't phrased quite right because sometimes you might not **ACTUALLY** be able to get all the way to the end. Consider the set of all chess games which last less than 200 moves ...

