Guo Shu-Li Han Li-Na

Linear Algebra and Its Applications in Programming

线性代数及其在规划中的应用

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Linear Algebra and Its Applications in Programming

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北京理工大学精品课程建设

线性代数及其在规划中的应用

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Introduction

In 1991, I first learned an undergraduate course on linear algebra. In 1995, I learned advanced linear algebra as a master degree course and in 1998, I learned linear algebra theory as a necessary Ph. D degree course. It was in 2009 that I first taught a graduate course in advanced linear algebra and its applications. Until 2013, hundreds of students already attended this course and benefited from it in their later control engineer careers. Over these years, I tried to choose different references and arranged a set of lecture notes for Engineering and Science students.

Prof. Han Li-Na first learned linear programming theory in 1998 and since then she focused on 3D cardiovascular system modelling and its medical analysis by using programming theory.

It is very necessary to edit a top-quality book about linear algebra and its applications in programming to meet our graduate students' needs now.

Here were some of my motives to write this book.

1) To have something as short and inexpensive as possible.

2) To organize the material in the most simple-minded, straightforward manner.

3) To order the material linearly.

4) To order many topics from fundamental linear algebra theory to matrix theory and its applications in linear and nonlinear programming. This is both a foundational course and a topic course.

5) To offer an alternative for control science majors to the linear and nonlinear programming courses.

This book may be divided into three parts.

Part I is a survey of abstract algebra with emphasis on linear algebra. Chapter 1 provides the background such that the basic concepts are products of sets, partial orderings, equivalence relations, functions, and the integers, especially that there are the properties of surjective, injective, bijective, and the notion of a solution of an equation. Chapter 2 is the most difficult part because groups are the central objects of algebra. In later chapters we will define rings and modules and see that they are special cases of groups. Also ring homomorphism and module homomorphism are special cases of group homomorphism. This chapter and the next two chapters are restricted to the most basic topics. The approach is to do quickly the fundamentals of groups, rings, and matrices, and to push forward to the chapter on linear algebra. In Chapter 3, rings are additive abelian groups with a second operation called multiplication. The connection between the two operations is provided by the distributive law. For example, ideals are also normal subgroups and ring homomorphisms are also group homomorphisms. In Chapter 4, many topics, such as invertible matrices, transpose, elementary matrices, systems of equations, and determinant, are all classical. Its highlights are these theorems, such that a square matrix is a unit in the matrix ring if its determinant is a unit in the ring, that similar matrices have the same determinant, trace, and characteristic polynomial, and that an endomorphism on a finitely generated vector space has a well-defined determinant, trace, and characteristic polynomial.

Part II presents the basic concepts of vector and tensor analysis, vector spaces, linear transformations, determinants and matrices, tensor algebra. Our intention is to present to Engineering and Science students a modern introduction to vectors and tensors and the systematic development of concepts. This part is intended as a text covering the central concepts of practical techniques on vector spaces, linear transformations, determinants and matrices, tensor algebra. It is designed for either undergraduates or graduates who have technical backgrounds in mathematics, engineering, or science. And this part should be useful to system analysts, operations researchers, numerical analysts, management scientists, and other practical specialists.

Part III provides several features as follows. Chapter 10 is devoted to a presentation of the theory and methods of polynomial-time algorithms for linear programming. These methods include, especially, interior point methods that have revolutionized linear programming. Chapter 11 includes an expanded treatment of necessary conditions, manifested by not only first- and second-order necessary conditions for optimality, but also by zeroth-order conditions that use no derivative information. This part continues to present the important descent methods for unconstrained problems, but there is new material on convergence analysis and on Newton's methods for both linear and nonlinear programming. Chapter 12 now includes the global theory of necessary conditions for constrained problems, expressed as zeroth-order conditions. Also interior point methods for general nonlinear programming are explicitly discussed within the sections on penalty and barrier methods. A significant addition to Chapter 12 is an expanded presentation of duality from both the global and local perspectives.

It is well known that mathematics theory and its applications are difficult and heavy subjects. Our styles are to make this book a little lighter. This book works best when viewed lightly and read as poems and songs, and I hope all readers enjoy it. Every effort has been extended to make every subject move rapidly and to make the sealing-in from one topic to the next as seamless as possible. The goal is to stay focused and go forward, because mathematics is learned in hindsight. We would have made the subjects short enough, every topic shorter, but I did not have any more time.

We wish to thank the many students who over the years have given us comments concerning this book and those who encouraged us to carry it out. It is difficult to do anything in life without help from friends, and many of my friends have contributed much to this text. My sincere gratitude goes especially to prof. Wu Qing-He, Prof. Wang Jun-Zheng and Prof. Zhang Bai-Hai.

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Part I

Chapter 1

Background and Fundamentals of Mathematics

The first chapter of this book shows the fundamental knowledge for both algebra and mathematics, which is very important for the future study. The basic concepts are set, relations, functions, integers and so on. We define some symbols firstly.

$\exists$ means "there exists."

$\exists\,!$ means "there exists a unique."

$\forall$ means "for each."

$\Rightarrow$ means "implies."

1.1 Basic Concepts

Any set called an index set is assumed to be non-void. Suppose T is an index set and for each $t \in T, A_t$ is a set.

$$\bigcup_{t \in T} A_t = \{x : \exists \ t \in T \text{ with } x \in A_t\}$$

$$\bigcap_{t \in T} A_t = \{x : \text{if } t \in T, x \in A_t\} = \{x : \forall t \in T, x \in A_t\}$$

Let $\varnothing$ be the null set. If $A \cap B = \varnothing$, then A and B are said to be disjoint.

Cartesian products: If X and Y are sets, $X \times Y = \{(x, y) : x \in X \text{ and } y \in Y\}$. In other words, the Cartesian product of X and Y is defined to be the set of all