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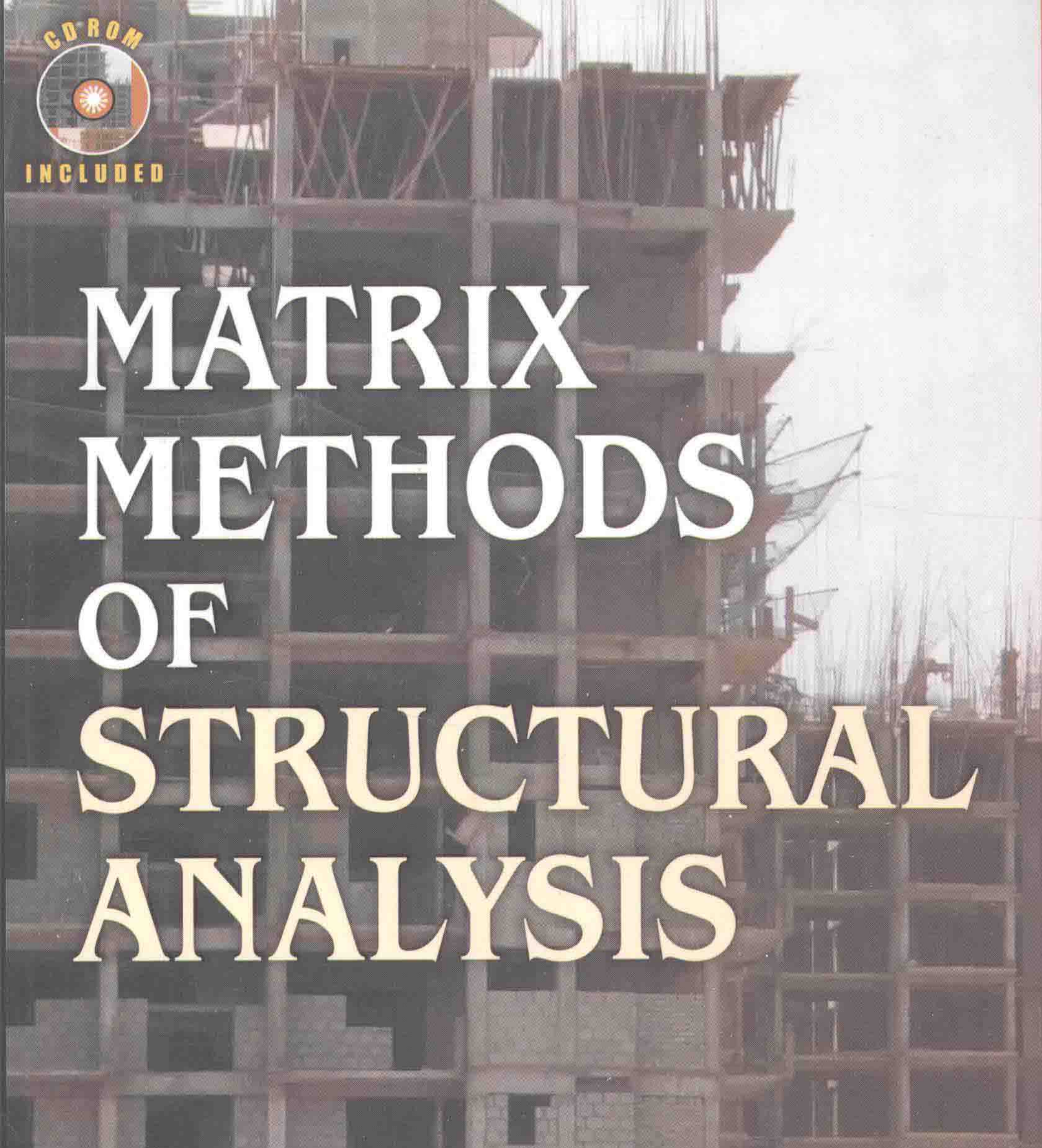

MATRIX METHODS OF STRUCTURAL ANALYSIS

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PHI

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PREFACE

The book describes in great detail the Matrix Methods of Structural Analysis used extensively for the analysis of skeletal or framed structures. The concepts of flexibility method and stiffness method are given, but the emphasis is placed on stiffness (displacement) method which is a systematic procedure for analysis, is suitable for computer programming and is available in present-day software packages. For practicing engineers, research scientists, academicians and students it is necessary to have basic understanding of the subject. Keeping this view in mind, this subject has been included in the Civil Engineering Curriculum as a core subject in M.Tech. programs and in undergraduate programs as a part or full credit course.

The main objective has been to present the Matrix Methods of Structural Analysis in simple and systematic way so that the book can serve as a basic learning tool for undergraduate and postgraduate students of civil engineering. However, the concepts have been presented in quite general form so that the book serves as a learning aid for students with other backgrounds as well as practicing engineers.

Chapter 1 of the book presents general introduction to the subject of Matrix Methods of Structural Analysis followed by Chapters 2–14 which are grouped in four parts. The parts can be read together or selectively depending upon the requirement of the reader.

Part 1 Basics

It includes three chapters. Chapters 2, 3 and 4 discuss the matrix algebra, solution of equations, stiffness and flexibility respectively, the knowledge of which is prerequisite for the better understanding of the subject. In Chapter 4 flexibility and stiffness methods are explained by taking simple examples of a spring and a bar.

Part 2 Structures Approach

This part contains two chapters in which the analysis of various structures by flexibility and stiffness matrix method using structure or system approach are discussed. Chapter 5 discusses the flexibility matrix (force) method in which it is necessary to obtain the flexibility coefficients to formulate the compatibility equations for the complete structure. Chapter 6 discusses the stiffness matrix (displacement) method which requires the determination of stiffness coefficients to formulate the equilibrium equations for the structure. As complete structure is involved, for large and complex structures the formulation does not remain simple nor is amenable to computer programming.

Part 3 Stiffness Matrix (Displacement) Method

This part contains seven chapters and discusses the analysis of various skeletal structures by the stiffness matrix (displacement) method using the member approach. Chapter 7 discusses the basic step of the method considering an elastic spring and its assemblage as a structure. It is shown that final set of equilibrium equations can be assembled considering behaviour of individual spring at a time in an assemblage leading to member approach. This makes the method simple, systematic and suitable for computer programming. Also it is shown that there are set of steps which need to be followed for the analysis of a structure. The application of stiffness matrix method with member approach is explained in great detail to various types of framed structures, e.g. beams, plane trusses, plane frames, grids and space structures in Chapters 8 to 12.

Additional topics on use of symmetry and anti-symmetry, inclined (oblique) supports, beams with shear deformation, member end releases in beams and frames, temperature changes and prestrains are covered in Chapter 13.

Part 4 Educational Program

This part contains an educational program for the analysis of framed structure by stiffness matrix method written in FORTRAN/C. The general layout of the program, description of various segments, guidelines for preparation of Input data and the Output details are discussed. Illustrative examples on beam, plane and space truss, plane frames and grid structures are given. A CD appended with the book contains source code, explanation of INPUT/OUTPUT and test examples.

The experience gained by the authors in teaching this subject for large number of years at respective institutions has contributed a lot in bringing this book to the final shape. The authors would like to express their appreciation to their colleagues and students for useful comments and suggestions. Authors appreciate the meticulous efforts of the editorial team of PHI Learning in the processing of script for this volume.

The encouragement from the families of the author in bringing this book to the final shape is warmly acknowledged.

Suggestions/corrections are most welcome for improving the content of the book and may be sent to the authors.

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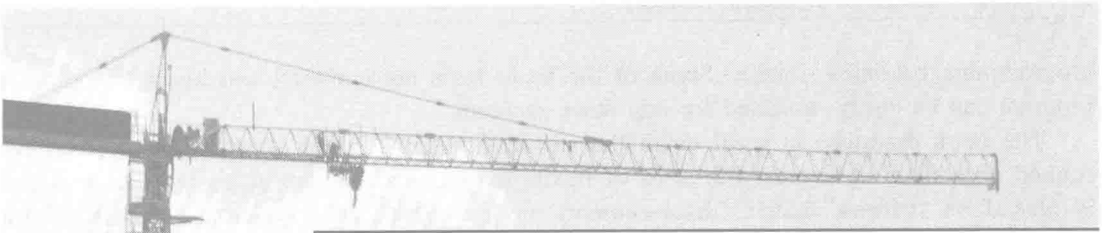
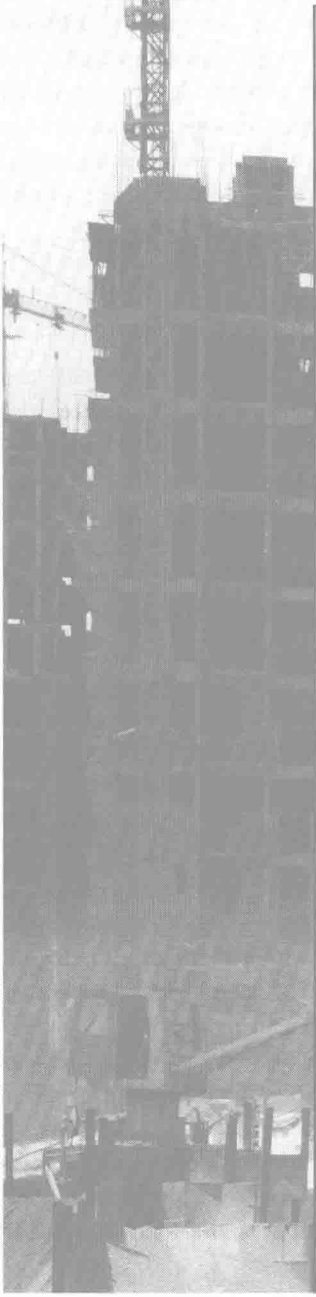

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INTRODUCTION

1.1 WHY MATRIX METHODS

The invention of computers in the late 1940s and subsequent development has revolutionised the technological world. The digital computers have huge memory, fantastic speed and absolute reliability for performing the computations. However, for computer implementation it is necessary that the problem is expressed in a systematic and precise manner so that it can be programmed easily. The methods using matrix algebra, which express the problem in concise form, are ideally suited for computer implementation.

Matrix method is being extensively used in the analysis of framed or skeletal structures which are an assemblage of long straight members such as bar or a beam. A large number of software packages are available which are routinely used in design offices for the analysis of commonly used structures such as beams, plane and space trusses and rigid frames. Matrix methods with digital computer become a very powerful tool for the structural engineer for the analysis of skeletal structures. Matrix methods are systematic and general and can analyse a large size problem, with high degree of indeterminacy. The solution procedure requires a set of steps which once understood for simple problem can be easily extended to complex problem or any other structure as the same steps are repeated. As the procedure is systematic, computer programming becomes simpler. Some of the basic steps are common and hence a computer program can be easily modified for any other problem.

The book describes in great detail the stiffness matrix method of structural analysis for framed or skeletal structures. Concepts of flexibility and stiffness are given but the emphasis is placed on stiffness matrix (displacement) method which is a systematic procedure for analysis and is suitable for computer programming and is available in present-day software packages. The present-day computers with high speed and storages with most advanced input and output facilities have been a tremendous asset to the structural engineers. It has stimulated important theoretical advances; it has permitted the use of greatly refined structural models in stress analysis; it has freed the engineers from long hours of hand calculations; it has enabled broad spectrum of input conditions (loads, thermal conditions, boundary conditions, etc.) to be investigated; and finally it has reduced errors in the analysis of complex structures.

1.2 TYPES OF FRAMED STRUCTURES

Framed or skeletal structures are an assemblage of members (usually straight) which are long in comparison to their cross sectional dimensions. The joints of framed structures are point of intersection of members, as well as points of support and free end of members. Various types of framed structures are shown in Figure 1.1. The framed structures can be divided in six categories: beams, plane truss, space truss, plane frames, grid and space frames. These categories are selected because each represents a class having special characteristics. Furthermore, while the basic principles of the flexibility or stiffness methods are same for all types of structures, the analyses for these six categories are sufficiently different in details, which require separate discussion for each of them.

Beams are long slender members subjected to transverse loading and most common structural element used in buildings, bridges and many other structures. Most commonly beam is idealised as a one-dimensional member [Figure 1.1(a)] having one or more points of support, such as points *A*, *B* and *C*. The beam is subjected usually to transverse loads and with the assumption of simple bending theory, the beam deflects and develops bending moment and shear force at a cross section.

A ***Truss*** is defined as a two/three-dimensional framework of long straight prismatic members connected at their ends by frictionless hinged joints and subjected to loads and reactions that act only at the joints. A plane truss [Figure 1.1(b)] is a structure in which members, loads and reactions lie in one plane (usually $X - Y$ plane). Under the action of loads the members of plane truss develop axial forces which may be tensile or compressive in nature.

A ***Space Truss*** [Figure 1.1(d)] is similar to plane truss, except that the structure lies in space. The members, loads and reactions may have arbitrary orientations in space. The member of space truss also develops axial force—tensile or compressive, under the action of loads.

A ***Frame*** is a structure consisting of long, straight, prismatic members connected at their ends by rigid joints. A plane frame [Figure 1.1(c)] is a structure in which the members, loads and reactions lie in one plane (usually $X - Y$ plane). Under the action of the load a member of plane frame develops bending moment, shear force and axial force.

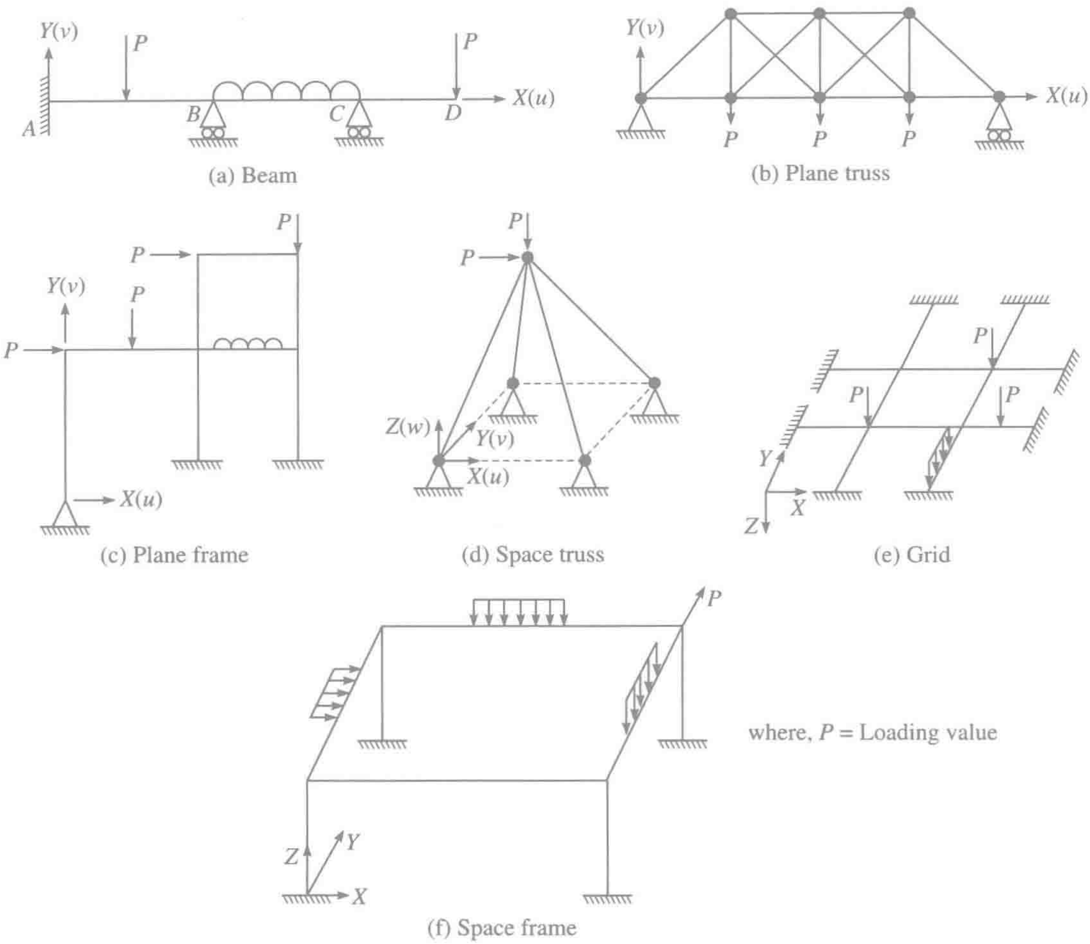


Figure 1.1 Types of framed (skeletal) structures.

A ***Grid*** [Figure 1.1(e)] is a structure similar to plane frame but with a difference. In case of grid the members connected at rigid joints lie in a plane ($X - Y$ plane) and the applied forces are normal to the structure (Z direction). The grid members are thus subjected to bending and twisting moment and shear force at a section.

The ***Space Frame*** [Figure 1.1(f)] is the most generalised case of framed structures. In the space frame, the joints, the members and loads all lie in space and can have any orientations. The structure is referred to $X - Y - Z$ axis, and members of space frame develops axial force, torsional moment, bending moment and shear force in both principle direction of the cross section.

1.3 FORCES AND DISPLACEMENTS

The types of framed structures described in Section 1.2 are elastic in nature, and develop force and displacement under the action of external loads. The term force and displacement is used in the text in a generalised way. The term force includes forces and moments; similarly the term displacement includes the translations and rotations at a point in a structure. In Table 1.1 the forces and displacements in members and joints of a framed structure are given.

Table 1.1 Forces and displacements in members and joints of a framed (skeletal) structure

Sr. No.	Structure	Member (local axis)		Joint (global axis)	
		Forces	Displacements	Forces	Displacements
1	Beam	Bending moment (BM) and shear force (SF)	Normal displacement and rotation	Moment and force	Displacement and rotation
2	Plane truss	Axial force	Axial displacement	Force in X and Y directions	Displacement in X and Y directions
3	Space truss	Axial force	Axial displacement	Force in X, Y and Z directions	Displacement in X, Y and Z directions
4	Plane frame	Axial force, SF and BM	Axial and transverse displacement and rotation	Force in X and Y directions and moment	Displacement in X and Y directions and rotation
5	Grid	SF, BM and twisting moment	Normal displacement and rotation about $\bar{X}$ and $\bar{Y}$ axes	Force in Z direction, moment about X and Y axes.	Normal displacement and Rotation about X and Y axes
6	Space frame	Forces along $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ directions; Bending moment about $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ axes	Displacement in $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ axes; rotation about $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ axes	Force in X, Y and Z directions moment about X, Y and Z axes.	Displacement in X, Y and Z directions; rotation about X, Y and Z axes

Note:

1. Forces and displacements in members are usually with respect to local axis—$\bar{X}$, $\bar{Y}$ and $\bar{Z}$ with $\bar{X}$ axis along the longitudinal direction.
2. Forces and displacements at joints are usually with respect to global axis—X, Y and Z.
3. The forces and displacements of local and global axis are related by transformation matrix.

1.4 BASIC STRUCTURAL PRINCIPLES

The simplest structural problems are statically determinate and may be solved by applying the equations of static equilibrium. For example, a statically determinate pin connected truss may be analysed completely for the member forces and reactions by simply writing the total set of applicable equilibrium equations. The essential point is that the internal member forces can be determined without considering structural deformation. For the designer this means that cross sectional areas and modulus of elasticity of members need not be specified prior to calculating member forces.

For redundant truss the equilibrium equations are not sufficient for the calculation of member forces. Additional equations must be used, and these are available from consideration of geometry of structural deformation. Continuity or the compatibility of deformations leads to required additional equations. Furthermore, introducing compatibility in terms of consistent deformation of the structure requires that a force-displacement relationship be specified. For conventional elastic framed structures this is well-known Hooke's law.

Therefore, the analysis of indeterminate structures, which are most commonly encountered, the analysis is involved as compared to determinate structures. The analysis includes the consideration of geometry of deformation, which means the cross sectional area and material modulii must be specified beforehand.

Analysis of framed structure must satisfy the following conditions:

1.4.1 Condition of Equilibrium

One of the objectives of any structural analysis is to determine the forces in the structure, such as reactions at the supports and internal stress resultants (bending moment, shear force, axial forces, etc.). A correct solution for any of these quantities must satisfy all the condition of static equilibrium, not only for the entire structure, but also for any part of the structure taken as free body.

In a free body subjected to set of forces the resultant will be a force, couple or both. If the free body is in static equilibrium, the resultant vanishes, that is, the resultant force vector and the resultant moment vector are both zero. If the structure is referred to set of X, Y and Z axes then for resultant force vector to be zero, the components along axes must be zero, gives rise to condition of static equilibrium as

$$\sum F_X = 0 \quad \sum F_Y = 0 \quad \sum F_Z = 0 \tag{1.1a}$$

In these equation $\sum F_X = 0$, $\sum F_Y = 0$ and $\sum F_Z = 0$ are algebraic sums of X, Y and Z components, respectively, of all the force vectors acting on the free body. Similarly, if the resultant moment vector equals zero, the moment equation of static equilibrium are

$$\sum M_X = 0 \quad \sum M_Y = 0 \quad \sum M_Z = 0 \tag{1.1b}$$

In which $\sum M_X = 0$, $\sum M_Y = 0$ and $\sum M_Z = 0$ are the algebraic sums of the moments about X, Y and Z axes, respectively, of all the forces and couples acting on the free body. The six relations of Eq. (1.1) are conditions of static equilibrium for a body in space. They may be applied to any free body such as entire structure, a part of structure, a single member or a joint in structure.

When a structure is planar, i.e., the structure and loads lie in one plane as is the case with beams, plane trusses and plane frames, only three of the six equilibrium equations will be useful. Assuming the structure and the loads are lying in $X - Y$ plane then the conditions of static equilibrium become

$$\sum F_X = 0 \quad \sum F_Y = 0 \quad \sum M_Z = 0 \tag{1.2}$$

In the stiffness method of analysis, the basic equations to be solved are those which express the equilibrium conditions at the joints of the structure.

1.4.2 Compatibility of Deformations

In any structural analysis, in addition to the condition of equilibrium, it is necessary to satisfy the conditions of compatibility of deformations. These conditions refer to continuity of displacement throughout the structure and sometimes referred to as condition of geometry. Compatibility condition must be satisfied at all points throughout the interior of structure, in particular, the joints of the structure and the points of support. For instance, at a fixed support there can be no translation or rotation. So also at a joint where two members are rigidly connected, the displacement (translation and rotation) for both members must be same.

In flexibility method of analysis the basic equations to be solved are equations that express the compatibility of displacements.

1.5 STATIC AND KINEMATIC INDETERMINACY

There are two types of indeterminacy that must be considered in structural analysis, depending upon whether forces or displacements are of interest. When forces are the basic unknowns in the analysis, as in the flexibility (force) method then static indeterminacy must be considered. However, when displacements are considered as basic unknowns in the analysis as in the stiffness (displacement) method, kinematic indeterminacy must be considered.

1.5.1 Static Indeterminacy

Framed structures are either statically determinate or statically indeterminate. In statically determinate structures, it is possible to determine all external reaction components and also internal resisting forces in members by equation of static equilibrium [Eqs. (1.1a), (1.1b), and (1.2)]. Analysis of such structures is simple and straightforward.

Statically indeterminate structures are those for which equations of static equilibrium are not sufficient to calculate external reactions and the internal forces in the members. In these structures the support and/or members are more than the minimum required to keep it statically determinate. A structure can be statically indeterminate externally, internally or both.

The unknown forces in excess of those that can be found out by equations of static equilibrium are known as static redundant, and the number of such redundant represents the degree of static indeterminacy of the structure. The available conditions of equilibrium for the entire structure are three for beams, plane truss, plane frame, grids and six for space truss and space frame. Thus, the degree of static indeterminacy can be obtained by summing, the number of external reaction components to be removed, and, the number of internal forces required to be released, to make the structure statically determinate. Thus, the static indeterminacy (I_s) can be expressed as

$$I_s = I_{se} + I_{si} \tag{1.3}$$

where I_{se} is the external indeterminacy (number of excess reaction component) and I_{si} is the internal indeterminacy (number of internal forces need to be released).

Figure 1.2 shows some framed structures and their static and kinematic indeterminacy.

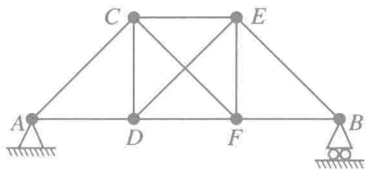
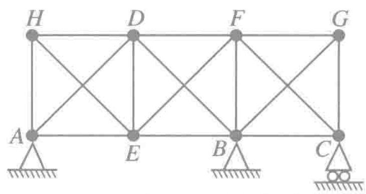
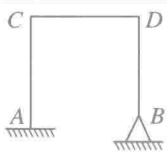
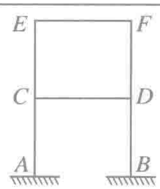
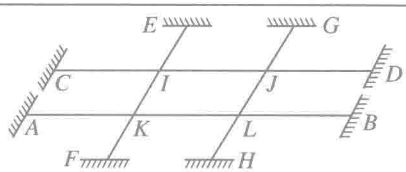
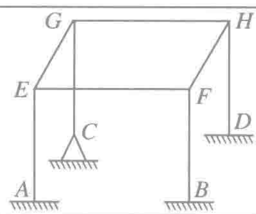
Type		Framed structure	$I_{se} + I_{si} = I_s$			I_k
			I_{se}	I_{si}	I_s	
Beam	(a)		1	0	1	1 (neglect axial deformation)
	(b)		2	0	2	4 (neglect axial deformation)
Plane truss	(c)		0	1	1	9
	(d)		2	3	5	11
Plane frame	(e)		2	0	2	7
	(f)		3	3	6	12
Grid	(g)		21	3	24	12
Space frame	(h)		15	6	21	27

Figure 1.2 Static and kinematic indeterminacy in framed structures.